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# FINITE ELEMENT SIMULATION OF INCOMPRESSIBLE AIRFLOW THROUGH HUMAN VOCAL FOLDS

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**Abstract:** *This study investigates incompressible airflow through the human vocal folds. The fluid motion is formulated within the arbitrary Lagrangian–Eulerian (ALE) framework to account for the moving computational domain. The governing equations are discretized using the finite element method (FEM) with Taylor–Hood elements. To ensure numerical stability in convection-dominated regimes, streamline upwind Petrov–Galerkin (SUPG) and grad–div stabilization techniques are employed. The proposed numerical framework is applied to simulations with prescribed vocal fold motion based on a kinematic model. Numerical results are validated against available experimental data for the static M5 vocal folds geometry, showing good agreement. In addition, a simulation of the soft voice regime is presented.*

**Keywords: Finite element method, Taylor-Hood element, Vocal folds, Soft voice**

## 1. Introduction

Fluid–structure interaction (FSI) plays a key role in many scientific and engineering applications, including biomechanics, where it is essential to understand airflow–tissue interaction during human phonation. In this process, incompressible airflow from the lungs induces vibrations of the vocal folds, and accurate numerical modeling is crucial to studying voice production mechanisms and supporting experimental and clinical research, see Kumar and Svec (2019).

This paper focuses on the numerical simulation of incompressible airflow interacting with the vocal folds in the M5 geometry, see Scherer et al. (2001). The flow is governed by the incompressible Navier–Stokes equations, while the motion of the vocal folds is prescribed using a kinematic mucosal wave model, see Kumar and Svec (2019). The interaction with the moving boundaries is treated within the arbitrary Lagrangian–Eulerian (ALE) framework, allowing a consistent description of the time-dependent computational domain, see Vacek and Sváček (2025) and John (2016). Challenges such as convection-dominated flow and strong outflow effects are addressed.

The governing equations are discretized using the finite element method with Taylor–Hood elements. To ensure the numerical stability under phonation conditions, the streamline upwind Petrov–Galerkin (SUPG) and the grad–div stabilization are employed, see e.g. John (2016). The proposed framework is applied to soft-voice phonation simulations and validated against experimental pressure measurements for the M5 geometry, demonstrating its suitability for analyzing airflow-induced loading in vocal folds dynamics.

## 2. Governing equations

This section presents the mathematical formulation of the fluid-structure interaction problem considered in this study.

Let $\Omega_t \subset \mathbb{R}^2$ denote a bounded, time-dependent computational 2D domain with boundary $\partial\Omega_t = \Gamma_D \cup \Gamma_{D,\text{wall}} \cup \Gamma_O \cup \Gamma_{W_t}$, where $\Gamma_D$, $\Gamma_{D,\text{wall}}$, $\Gamma_O$, and $\Gamma_{W_t}$ represent the inlet, walls, outlet, and movement of the vocal fold, respectively, see Fig. 1.

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The fluid motion is described within the arbitrary Lagrangian-Eulerian (ALE) framework, see Vacek and Sváček (2025). The incompressible Navier-Stokes equations in ALE form read: find the velocity $\boldsymbol{u} : \Omega_t \to \mathbb{R}^2$ and pressure $p : \Omega_t \to \mathbb{R}$ such that

$$\frac{D^A}{Dt}\boldsymbol{u} + ((\boldsymbol{u}-\boldsymbol{w})\cdot\nabla)\boldsymbol{u} - \nu\Delta\boldsymbol{u} + \nabla p = 0, \qquad \nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\nu$ is the kinematic viscosity, $\frac{D^A}{Dt}$ denotes the ALE time derivative, and $\boldsymbol{w}$ is the domain velocity.

The system is supplemented with the initial condition $\boldsymbol{u}(x,0) = \boldsymbol{u}_0$ in $\Omega_0$ and boundary conditions $\boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma_D, \qquad \boldsymbol{u} = \boldsymbol{w} \quad \text{on } \Gamma_{W_t}$, together with a directional "do-nothing" condition at the outlet, $-(p - p_{\text{ref}})\boldsymbol{n} + \nu\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{n}} - \frac{1}{2}(\boldsymbol{u}\cdot\boldsymbol{n})_-\boldsymbol{u} = 0 \quad \text{on } \Gamma_O$, which stabilizes backflow while preserving physically consistent outlet behavior, see John (2016).

The motion of the vocal fold is prescribed using a kinematic mucosal wave model based on the M5 geometry, see Scherer et al. (2001). The model describes phase-delayed oscillatory motion of surface points to represent upward propagation of the mucosal wave and is suitable for soft voice conditions, see Kumar and Svec (2019). The vertical wave velocity is given by $c = \frac{360 f_0 T}{\phi}$, where $f_0$ is the fundamental frequency, $T$ the vertical thickness of the vocal fold, and $\phi$ the phase delay.

The trajectory of a surface point $P_i = (x_i, y_i)$ is prescribed as

$$x_i(t) = x_i^0 + A_i \sin\left(2\pi f_0\left(t - \frac{d}{c}i\right)\right), \qquad y_i(t) = y_i^0 + aA_i \cos\left(2\pi f_0\left(t - \frac{d}{c}i\right)\right), \tag{2}$$

where $(x_i^0, y_i^0)$ are the initial coordinates, $A_i$ the local amplitude, $a$ the amplitude ratio and $d$ the spacing between the surface points, for more details see Kumar and Svec (2019).

## 3. Discretization method

Problem (1) is semi-discretized in time using a constant step $\Delta t$. At times $t_n = n\Delta t$, the velocity, pressure and domain velocity are approximated by $\boldsymbol{u}^n(x) \approx \boldsymbol{u}(x,t_n)$, $p^n(x) \approx p(x,t_n)$, and $\boldsymbol{w}^{n+1}(x) \approx \boldsymbol{w}(x,t_{n+1})$. Using the BDF2 scheme, the ALE formulation yields

$$\frac{3\boldsymbol{u}^{n+1} - 4\tilde{\boldsymbol{u}}^n + \tilde{\boldsymbol{u}}^{n-1}}{2\Delta t} + ((\boldsymbol{u}^{n+1} - \boldsymbol{w}^{n+1})\cdot\nabla)\boldsymbol{u}^{n+1} - \nu\Delta\boldsymbol{u}^{n+1} + \nabla p^{n+1} = 0,$$
$$\nabla\cdot\boldsymbol{u}^{n+1} = 0, \tag{3}$$

where $\tilde{\boldsymbol{u}}^i = \boldsymbol{u}^i \circ A_{t_i} \circ A_{t_{n+1}}^{-1}$ denotes the ALE mapping, see Vacek and Sváček (2025).

The weak formulation is considered at $t_{n+1}$ with $\boldsymbol{u} = \boldsymbol{u}^{n+1}$, $\boldsymbol{w} = \boldsymbol{w}^{n+1}$, $p = p^{n+1}$, and $\Omega = \Omega_{t_{n+1}}$. The test spaces are

$$\boldsymbol{\mathcal{V}} = \{\boldsymbol{\varphi} \in \boldsymbol{H}^1(\Omega) : \boldsymbol{\varphi} = 0 \text{ on } \Gamma_D \cup \Gamma_W\}, \qquad \mathcal{Q} = L^2(\Omega).$$

Defining the bilinear and linear forms

$$\begin{aligned} a(U^*, U, V) &= \tfrac{3}{2\Delta t}(\boldsymbol{u},\boldsymbol{v})_\Omega + \nu(\nabla\boldsymbol{u},\nabla\boldsymbol{v})_\Omega + c(\boldsymbol{u}^* - \boldsymbol{w}, \boldsymbol{u}, \boldsymbol{v}) \\ &\quad - (p, \nabla\cdot\boldsymbol{v})_\Omega - (\nabla\cdot\boldsymbol{u}, q)_\Omega, \end{aligned} \tag{4}$$

$$F(V) = \tfrac{1}{2\Delta t}(4\tilde{\boldsymbol{u}}^n - \tilde{\boldsymbol{u}}^{n-1}, \boldsymbol{v})_\Omega, \tag{5}$$

the weak problem reads: find $U = (\boldsymbol{u}, p) \in \boldsymbol{\mathcal{V}} \times \mathcal{Q}$ such that $a(U,U,V) = F(V) \quad \forall V \in \boldsymbol{\mathcal{V}} \times \mathcal{Q}$. The nonlinear convective term is treated by Oseen linearization, see John (2016).

Let $\tau_h$ be an admissible triangulation of $\Omega$. The spatial discretization employs the Taylor–Hood element with quadratic ($P_2$) velocity and linear ($P_1$) pressure approximations: $\boldsymbol{\mathcal{V}}_h \subset \boldsymbol{\mathcal{V}}$, $\mathcal{Q}_h \subset \mathcal{Q}$. This element pair satisfies the inf-sup condition, though incompressibility is enforced only in a discrete sense, see John (2016) (this issue is treated using the well-known grad–div stabilization). The discrete problem reads: find $U_h \in \boldsymbol{\mathcal{V}}_h \times \mathcal{Q}_h$ such that

$$a(U_h^*, U_h, V_h) = F(V_h), \quad \forall V_h \in \boldsymbol{\mathcal{V}}_h \times \mathcal{Q}_h. \tag{6}$$

For flows dominated by convection, SUPG stabilization is applied to the convective term, see John (2016).

## 4. Numerical results

This section presents numerical results obtained with the proposed framework. The approach is first validated for airflow at ambient temperature through a static vocal fold configuration by comparing the pressure distribution along the fold surface with experimental data from Scherer et al. (2001) (the symmetric case of M5 geometry with divergent angle 10°). The method is then applied to a soft-voice phonation regime to investigate unsteady flow features. The computational domain and boundary conditions are shown in

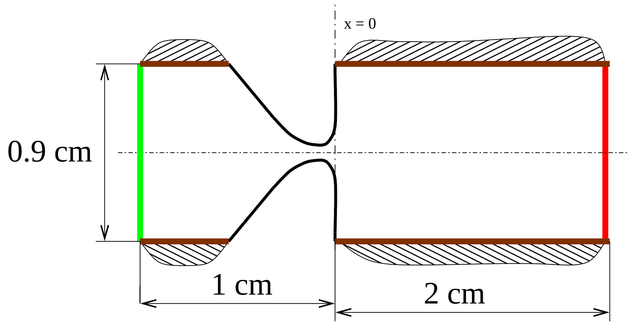



*Fig. 1: Computational domain and boundary conditions. Green indicates the inlet $\Gamma_D$ with prescribed velocity, brown the rigid walls $\Gamma_{D,\mathrm{wall}}$, black the moving vocal folds $\Gamma_{W_t}$, and red the outlet $\Gamma_O$ with a directional do-nothing condition.*

Fig. 1, where the channel depth is 1.1 cm. The discretization consists of 40 572 elements and 20 653 vertices, resulting in 184 407 degrees of freedom. Boundary layer refinement is applied near the vocal fold surfaces. The M5 geometry represents an idealized convergent–divergent glottal channel capturing the essential features of human phonation.

For the static configuration, Fig. 2 shows the pressure distribution along the vocal-fold surface for inlet-to-outlet pressure drops of 306 Pa, 510 Pa, and 1020 Pa ($x = 0$ is at the glottal exit). The numerical results reproduce the characteristic pressure drop in the glottal constriction and show good qualitative agreement with the experimental measurements.

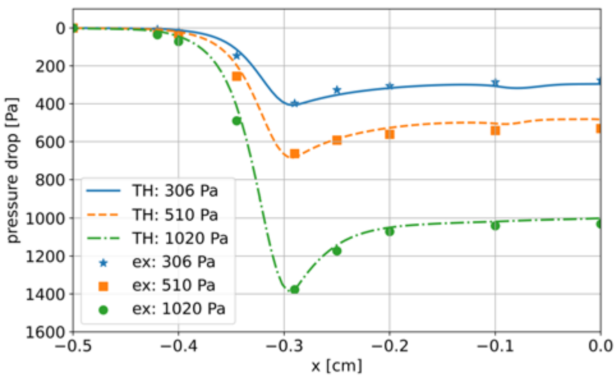
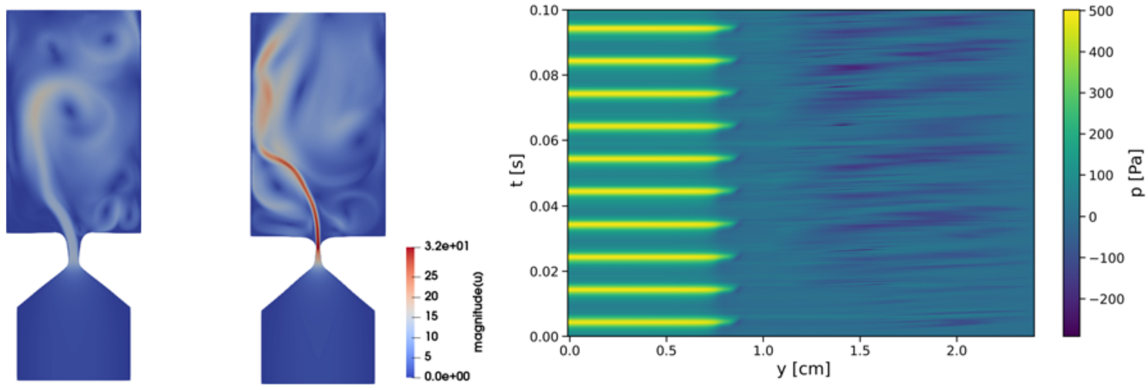



*Fig. 2: Local pressure drop along the vocal fold surface in the static M5 geometry, with $\Delta p = 0$ at the inlet. Taylor–Hood (TH) simulations are shown as lines, and experimental data as symbols, for inlet pressures of 306 Pa, 510 Pa, and 1020 Pa.*

The validated model is subsequently applied to a soft-voice phonation case with prescribed vocal fold motion based on a kinematic model, see Kumar and Svec (2019). The prescribed inlet velocity leads to the 0.1 l/s flow rate. Fig. 3 shows snapshots of the velocity magnitude in a closed channel and a fully open configuration. In addition, there is the pressure along the centerline over time. The channel is displayed in a vertical orientation to facilitate physical interpretation; gravity effects are neglected and therefore the orientation does not influence the flow dynamics. A high-speed glottal jet downstream of the folds becomes unstable in the supraglottal region, generating large-scale vortical structures. The occurrence of backflow motivates the use of a directional "do-nothing" outlet boundary condition, which improves numerical stability while preserving physically consistent flow behavior.




*(a)* *(b)* *(c)*

*Fig. 3: Velocity magnitude $|\mathbf{u}|$ at (a) $t = 0.210$ and (b) $t = 0.21135$, and the pressure along the centerline as a function of time (c).*

## 5. Conclusions

A numerical framework for simulating incompressible airflow in human phonation has been presented, focusing on flow through the M5 vocal fold geometry under soft-voice conditions with prescribed vocal fold motion, see Kumar and Svec (2019). The fluid problem is formulated within the arbitrary Lagrangian–Eulerian (ALE) framework and discretized using the finite element method with Taylor–Hood elements, complemented by streamline upwind Petrov–Galerkin (SUPG) and grad–div stabilization to ensure numerical robustness John (2016); Vacek and Sváček (2025).

The approach was validated on a static M5 configuration, where the computed pressure distribution along the vocal fold surface shows good agreement with available experimental measurements Scherer et al. (2001). It was then applied to simulations with prescribed vocal fold motion based on a kinematic mucosal wave model Kumar and Svec (2019). The numerical results capture the formation of a high-speed glottal jet and the development of large-scale vortical structures in the supraglottal region, together with intermittent reverse flow near the outlet.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