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ENTROPY GENERATION ANALYSIS FOR PRANDTL POWER LAW VELOCITY PROFILES IN CIRCULAR PIPES

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Abstract: *The purpose of this study is to verify whether calculating total entropy generation can serve as a method for evaluating flow quality when the velocity profile is determined using theoretical formulas, such as Prandtl's power law. Current methods for assessing flow straighteners rely mainly on qualitative analysis of the velocity profile shape, which impedes objective comparisons. The investigation involved calculations for a pipe with a diameter of 100 mm at a Reynolds number of 10,000, analysing power-law exponents n ranging from 5 to 12 as well as laminar flow. The results indicate that specific generated entropy increases with the exponent n, implying that a more turbulent profile generates higher entropy. Although the total entropy for n=5 was found to be lower than for laminar flow, literature suggests such low exponents are practically impossible in real-world scenarios. This confirms that the assumption of using generated entropy to evaluate flow conditioning is valid for practical applications.*

Keywords: Velocity profile, Inner flow, Channels, Flow conditioning

1. Introduction

Stream straighteners are used for flow conditioning in many applications, such as wind tunnels, biomedical engineering, and flow rate measurements. While the use of these devices is widespread, there are still no good methods for comparing their performance.

Jurga et al. (2024) investigated the impact of flow straighteners on the flow behind 90° pipe bend. Evaluation of the impact of the straightener was based mainly on the velocity profile analysis. However, the swirl intensity and nondimensional pressure coefficient were also taken into account. Smyk et al. (2025) analyzed velocity and turbulence intensity profile, kinetic energy correction factor, and pressure drop on the straightener. The investigation was conducted for a straight duct with a fan as a disturbance generator. Analysis of other articles shows that researchers often rely on analysis of the shape of the speed profile (Spearman et al., 1996, Kühnen et al., 2018, 2019), which does not allow for clear comparisons of the performance of straighteners. This data is often more qualitative than quantitative and relates to a specific case.

It seems that a good way to evaluate the flow objectively and therefore the operation of the straightener is to measure the generated entropy (related to the dissipation of energy through viscous friction), which can be defined by the formula (Kock and Herwig, 2004):

$$\begin{aligned}\dot{s}_{gen} = \frac{\Phi}{T} = \frac{\mu}{T}\Bigg[& 2\left\{\left(\frac{\partial\bar{u}}{\partial x}\right)^2 + \left(\frac{\partial\bar{v}}{\partial y}\right)^2 + \left(\frac{\partial\bar{w}}{\partial z}\right)^2\right\} \\ & + \left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{v}}{\partial x}\right)^2 + \left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{w}}{\partial x}\right)^2 + \left(\frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial x}\right)^2\Bigg] \\ & + \frac{\mu}{T}\Bigg[2\left\{\overline{\left(\frac{\partial u`}{\partial x}\right)^2} + \overline{\left(\frac{\partial v`}{\partial y}\right)^2} + \overline{\left(\frac{\partial w`}{\partial z}\right)^2}\right\}\end{aligned} \tag{1}$$

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$$+\overline{\left(\frac{\partial u`}{\partial y}+\frac{\partial v`}{\partial x}\right)^2}+\overline{\left(\frac{\partial u`}{\partial z}+\frac{\partial w`}{\partial x}\right)^2}+\overline{\left(\frac{\partial v`}{\partial z}+\frac{\partial w`}{\partial y}\right)^2}\Bigg]$$

where Φ is the viscous dissipation function dependent on the strain tensor [Pa/s], T is the absolute temperature of the flowing fluid [K], μ is the dynamic viscosity [Pa·s], u, v and w are the velocity components in x, y, z directions [m/s], respectively. The symbol ‾ denotes the time-averaged value, while ` represents the fluctuating component.

If we assume that the flow is axisymmetric and the velocity depends only on the radius, then Eq. (1) can be significantly simplified and written as:

$$\dot{s}_{gen}=\frac{\Phi}{T}=\frac{\mu}{T}\left[\left(\frac{\partial\bar{u}}{\partial r}\right)^2+\overline{\left(\frac{\partial u`}{\partial r}\right)^2}\right] \qquad (2)$$

where r is a radial coordinate [m]. If we omit the part concerning flow fluctuations, Eq. (2) takes the form:

$$\dot{s}_{gen}=\frac{\mu}{T}\left[\left(\frac{\partial\bar{u}}{\partial r}\right)^2\right] \qquad (3)$$

The Eq. (3) is the entropy distribution in the pipe, which, although important, is difficult to interpret case by case. In the case of stream analysis, it seems better to measure the total entropy, which can be understood as the amount of entropy generated per meter of length. This value can be expressed by the formula:

$$\dot{s}_{total,gen}=2\pi\int_0^R \dot{s}_{gen}r\,dr \qquad (4)$$

Where R is the radius of pipe [m].

The purpose of this study is to verify whether Eq. (4) can be used to evaluate flow if the velocity profile is calculated using theoretical formulas (Prandtl's power law). Since the calculations will be theoretical, entropy generated by fluctuations will be ignored. The calculations will be performed for an assumed flow rate and various velocity profiles, which reflect the state that occurs after flow conditioning or relaminarization.

2. Methods

The velocity profile in a round pipe can be calculated with the Prandtl power law for laminar flow or with a modified Prandtl law for turbulent flow (Salama, 2021):

$$\frac{\bar{u}}{u_{max}}=1-\left(\frac{r}{R}\right)^2 \qquad (5)$$

$$\frac{\bar{u}}{u_{max}}=\left(1-\left(\frac{r}{R}\right)^2\right)^{\frac{1}{n}} \qquad (6)$$

where u_{max} is the maximal velocity [m/s], and n is an exponent that depends on the Reynolds number.

The calculations were performed for the pipe with a diameter 100 mm (with step 1 mm), and for Reynolds number 10 000. Air flow for a temperature of 21°C (294.15 K), atmospheric pressure of 1025hPa, and air humidity of 37 %. For such conditions, the air and flow parameters are: density $\rho = 1.21$ kg/m^3, dynamic viscosity $\mu = 1.818 \cdot 10^{-5}$ Pa·s, mean velocity $\bar{u} = 1.5$ m/s, and volume flow rate $Q = 42.5$ m^3/h.

Salama (2021) indicates that the n coefficient is most often equal to 7, while experimental data presented by Smyk et al. (2025) show that it can be much larger. Therefore, it was decided to analyze flows for n coefficients from 5 to 12 with a step size of 1.

3. Results and discussion

To better illustrate the relationship between the velocity profile exponent n and entropy generation, the analytical formulas can be explicitly derived. By substituting Eq. (6) into Eq. (3) and introducing the dimensionless radial coordinate $\rho = r/R$, the specific generated entropy is proportional to:

$$\dot{s}_{gen}(\rho,n)\sim\frac{4\rho^2}{n^2}(1-\rho^2)^{2\frac{1-n}{n}} \qquad (7)$$

Consequently, integrating this local generation over the pipe cross-section (Eq. 4) yields an analytical expression for the total entropy generation:

$$\dot{s}_{total,gen} \sim \frac{2\pi}{n(n-2)} \left[(1-\rho^2)^{\frac{2}{n}-1} \cdot (n(\rho^2-1) - 2\rho^2) \right]_0^1 \tag{8}$$

This analytical formulation reveals the continuous nature of the dependence on n. However, it should be noted that for exponents $n > 2$, the gradient of the pure Prandtl power-law profile approaches infinity at the wall ($\rho \rightarrow 1$), which mathematically leads to a divergence of the total entropy integral. For this reason, in the practical engineering approach adopted in this study, the total generated entropy was evaluated using numerical integration with a discrete step size of 1 mm (as mentioned in methods section). This allows for a robust, objective comparison of the bulk flow properties without the unbounded wall singularity artificially dominating the results.

Figure 1a presents a maximum velocity as a function of the exponent *n*. The exponent *n* equal to 1 means the laminar flow. The calculations were based on theoretical profiles calculated from Eqs. (5) and (6), and the maximum velocity value for the laminar profile was equal to the two average velocities (in accordance with theory, see Salama 2021). The maximum velocity for the remaining *n* exponent values was found through optimization to achieve the assumed flow rate for a given profile. The higher the *n* exponent value, the smaller the maximum velocity. In the turbulent flow range, this relationship is linear ($R^2 = 0.938$). However, it should be noted that the dependence presented in Fig. 1a is a continuous function in the entire interval and therefore the approximation function is only informative.

Figure 1b shows the specific generated entropy as a function of the exponent n. As expected, the generated entropy increases with increasing exponent n. This means that the more turbulent the velocity profile, the greater the generated entropy. It should be noted that since the calculations were based on theoretical data, the calculations were performed based on Eq. (4), and therefore, the energy of velocity fluctuations was not taken into account. The generated entropy is positively corelated with the *n* exponent and the dependence between this two value can be approximated by a quadratic function with a very good approximation ($R^2 = 0.99$). It can therefore be expected that the entropy values generated for $n < 5$ also slope along the line shown in Fig.1b. However, such a low exponent value as $n = 5$ is rarely encountered in practical engineering applications. A literature review did not reveal any studies reporting an exponent as low as 5. For instance, Smyk et al. (2025) reported a minimum value of $n = 5.28$, corresponding to $\dot{s}_{total,gen} = 1.01 \cdot 10^{-5}$ J/(K·m·s) for analyzed theoretical model.

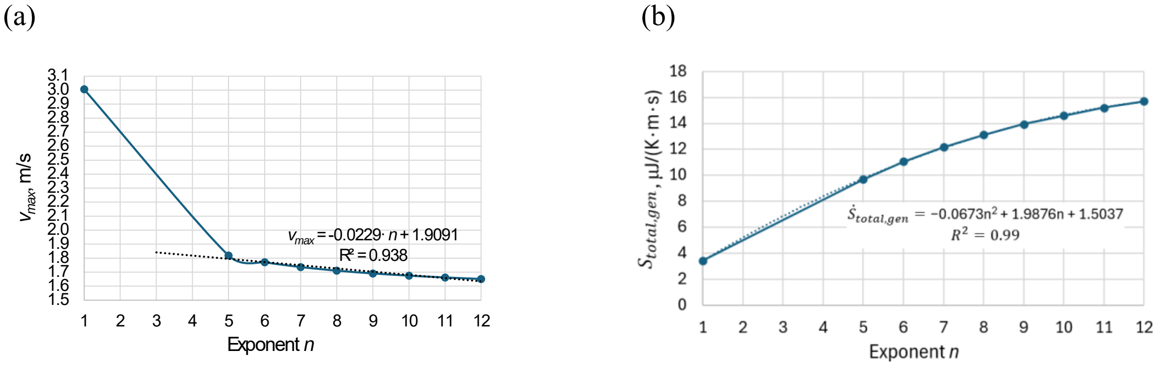

Fig. 1: The maximum velocity (a) and the specific generated entropy (b) in function of exponent n, where n=1 means the laminar flow.

Figure 2b presents the distribution of generated entropy as a function of distance from the axis. To unify the scales, the ratio of generated entropy to the maximum value of generated entropy is presented as the y-axis. It should be noted that the distribution of entropy generated in the duct cross-section is logarithmic for laminar flow, reflecting the logarithmic nature of the velocity profile. In the case of turbulent flow, entropy is generated primarily at the wall. It should be noted that a 1 mm step was used during the calculations, which corresponds to $r/R = 0.02$ for the assumed parameters. Note that increasing the exponent *n* does not cause a noticeable shape change in the generated entropy distribution. Even with a significant fine step size near the wall, the largest jump in entropy generation is observed at the wall itself. This is related to the shape of the velocity profile (see Smyk et al. 2025, Salama 2021). The velocity profiles are illustrated in Figure 2b.

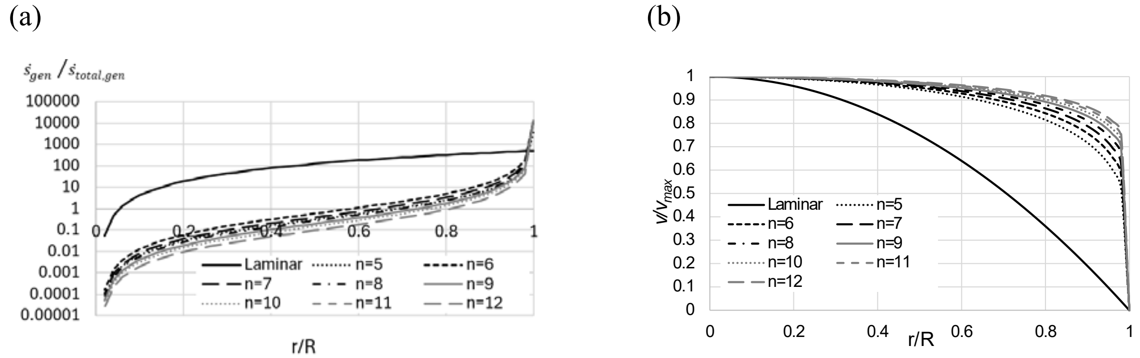

Fig. 2: The generated entropy profile (a) and the velocity profile calculated from Eq. (5) for laminar flow and from Eq. (6) for turbulent flow(b).

As can be seen, for turbulent profiles, the greatest velocity increase occurs between the wall and the first measurement point. Therefore, this will also correspond to the highest value of entropy generated, see Eq. (3).

4. Conclusions

The paper analysed the utility of measuring total entropy generation to evaluate flow after conditioning, based on theoretical velocity profiles. A linear relationship was demonstrated between the maximum velocity and the exponent n within the turbulent flow range. A key finding is that generated entropy rises as the exponent n increases, with the most significant generation occurring at the pipe wall due to the shape of the velocity profile. It was determined that the theoretical drop in entropy below the laminar value for $n = 5$ is an anomaly not encountered in practice, where the lowest observed exponents are higher. It is also crucial to emphasize that the presented analysis focused solely on the entropy generated by the mean velocity gradients (viscous dissipation), omitting the turbulent dissipation component caused by velocity fluctuations. Therefore, the calculated values should be interpreted as a lower bound estimate of the total entropy generation. In real flows, the contribution of turbulent fluctuations would further increase the total entropy production, thereby reinforcing the observed trend that profiles with higher Reynolds numbers (and higher exponents) result in greater energy losses. Ultimately, the study confirms that the method based on integrating entropy generation over the pipe radius allows for a correct and quantitative assessment of flow characteristics.

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References

Jurga, A., Janocha, M., Ong, M., Yin, G. (2024) Numerical Investigations of Turbulent Flow Through a 90-Degree Pipe Bend and Honeycomb Straightener. *Journal of Fluids Engineering*, 146, 021307.

Kock, F.; Herwig, H. (2004) Local Entropy Production in Turbulent Shear Flows: A High-Reynolds Number Model with Wall Functions. *International Journal of Heat and Mass Transfer*, 47, 2205–2215.

Kühnen, J.; Scarselli, D.; Schaner, M.; Hof, B. (2018) Relaminarization by Steady Modification of the Streamwise Velocity Profile in a Pipe. *Flow, Turbulence and Combustion*, 100, 919–943.

Kühnen, J.; Scarselli, D.; Hof, B. (2019) Relaminarization of Pipe Flow by Means of 3D-Printed Shaped Honeycombs. *Journal of Fluids Engineering*, 141, 111105.

Salama, A. (2021) Velocity Profile Representation for Fully Developed Turbulent Flows in Pipes : A Modified Power Law. *Fluids*, 6, 369

Smyk, E., Stopel, M., Rachwalski, A. (2025) Impact of honeycomb straightener parameters on operation in a straight duct. *Journal of Theoretical and Applied Mechanics*, 63(4), 781-794.

Spearman, E.P.; Sattary, J.A.; Reader-Harris, M.J. (1996) Comparison of Velocity and Turbulence Profiles Downstream of Perforated Plate Flow Conditioners. *Flow Measurement and Instrumentation*, 7, 181–199