32nd International Conference
ENGINEERING MECHANICS 2026
Medlov, Czech Republic, May 11 – 13, 2026

WAVE-BASED CONTROL FOR MECHANICAL SYSTEMS WITH FORCE FEEDBACK

Neusser Z.[1]

Abstract: *This paper presents wave-based control with force feedback and its incorporation into wave model and subsequently into the wave-based control. The force feedback allows the suppression of the influence of external forces acting on the system, which is a problem in wave-based control with position feedback only.*

Keywords: Wave-Based Control, Force Feedback, Vibration Suppression

1. Introduction

The concept of wave-based control (WBC) was presented in O'Connor and Lang (1998). The idea is based on longitudinal waves propagating in a compliant structure. In a flexible system, due to an imposed force (actuator, disturbance), the effect of a so-called launched wave is induced, which is at the same time accompanied by a returned (reflected) wave in the system, schematically shown in Figure 1a.

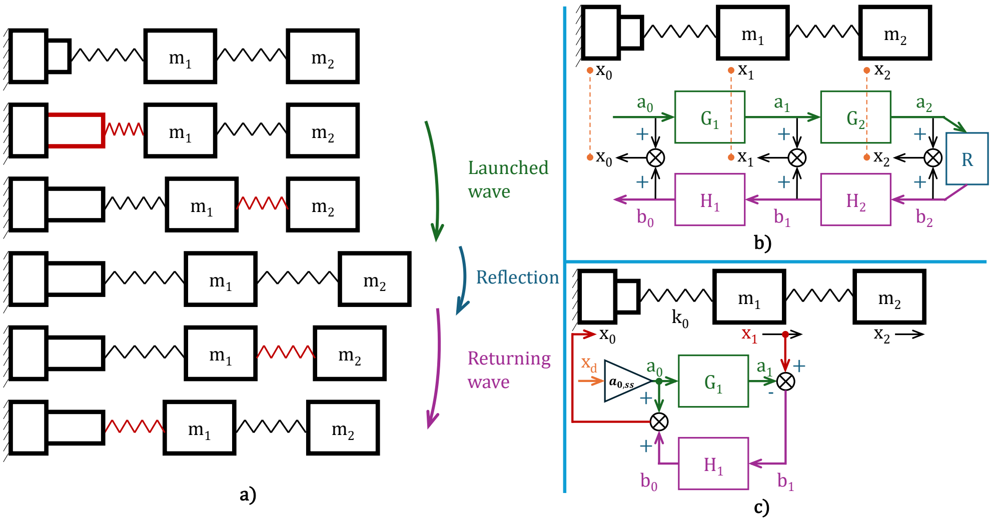

Fig. 1: The idea of a launched and returned wave in a mechanical system.

Based on this inspiration, the so-called wave model is developed, which describes the above wave behavior using transfer functions.

Wave-based control is typically used for flexible, multibody systems. The WBC is used for one-dimensional position control of mechanical systems O'Connor (2004); O'Connor (2006); O'Connor (2007b) as well as for angular position control (O'Connor, 2005; Marek and Valasek, 2009). Flexible systems are treated by WBC in O'Connor et al. (2009); Beneš et al. (2018). Modal properties are investigated in O'Connor and McKeown (2008); O'Connor (2011). There are some other applications suitable for WBC or control inspired by wave-based technique. Control of synchronous machine Valášek et al. (2016), mass-rope system O'Connor (2002); O'Connor (2004), adaptive cruise control for vehicles Martinec et al. (2014); Valasek et al. (2016) are some examples of WBC usage.

[1] Ing. Zdeněk Neusser, Ph.D.: Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4; 166 00, Prague; CZ, zdenek.neusser@fs.cvut..cz

ISSN: 1805-8256, doi: 10.21495/em2026-89

2. Wave model and wave-based control

An alternative model to the mechanical model is developed; Figure 1b shows schematically the wave and mechanical models. The wave model is composed of transfer functions describing the launched (G_i) and reflected wave (H_i). The mechanical model is connected to the wave model through the coordinates of the individual bodies, as shown in Figure 1b. The interconnection is written in the equation (1).

$$x_i = a_i + b_i \tag{1}$$

Wave-based control is based on feedback, in this case measuring the position of the first mass. We cut out everything behind the measured mass from the wave model, see Figure 1c. The reflected wave can be determined based on the knowledge of the reflected wave and the aforementioned feedback. The control law can be constructed based on the first mass measurement and wave model.

$$x_0 = a_0 + b_0 \tag{2}$$

where

$$a_0 = a_{0,ss} x_d \tag{3}$$

$$a_1 = G_1 a_0 \qquad b_0 = H_1 b_1 \tag{4}$$

$$b_1 = x_1 - a_1 \tag{5}$$

with k_0 representing the spring between actuator x_0 and first mass x_1 and m_1 is first mass weight. The launched wave a_0 in equation (3) is composed of the desired position of the first mass x_d and the coefficient $a_{0,ss}$, whose value is derived in Marek (2013); Valasek et al. (2019). The launched wave, reflected wave and the relationships between their quantities are in (4); these are the relationships of the wave model. The b_1 magnitude in (5), used for determination of b_0, is obtained from the launched wave a_1 and from the measured value of the first mass position x_1. After substituting and adjusting, the control law is obtained

$$x_0 = H_1 x_1 + (1 - H_1 G_1)\, a_{0,ss} x_z \tag{6}$$

$$G_1 = \frac{\omega^2}{s^2 + \omega s + \omega^2} \qquad H_1 = G_1 \qquad \omega = \sqrt{\frac{k_0}{m_1}}$$

where $H_1 x_1$ represents output feedback and $(1 - H_1 G_1)\, a_{0,ss} x_z$ forward control with input x_z as the desired position of the system. Such control is called absolute wave-based control (WBC-A). The transfer functions G_1 and H_1 in (6) are determined as second order transfer functions according to O'Connor (2007a).

3. Wave-based control with force feedback

Force and position feedback control, first used in the article O'Connor and Fumagalli (2009), is placed in the context of the philosophy of the wave approach using force interaction, nested in the wave model. The feedback in the control can also be implemented by force measurement (Fig. 2), which complements the position measurement.

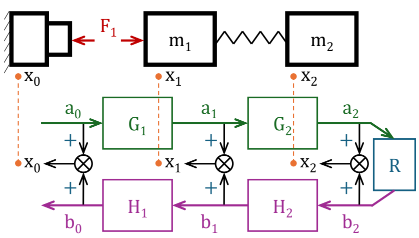

Fig. 2: The wave model and the force between bodies.

From the expression of the relative force acting between the bodies of the mass chain in Figure 2, the relation for the reflected wave (quantity b_0), can be obtained. The force acting on mass 1 is determined as:

$$F_1 = k_0 (x_0 - x_1) \tag{7}$$

By introducing substitutions according to (1) we obtain the force expressed in the coordinates of the wave model:

$$F_1 = k_0 (a_0 + b_0 - a_1 - b_1) \tag{8}$$

From the equations of the launched wave (4), a_0 is eliminated, then, thanks to the x_1 position measurement (5), the equation is converted to the coordinates of the reflected wave. Using the equations of the reflected part of the wave model (4), the expressions are converted to a function of the variable b_0.

$$\begin{aligned} F_1 &= k_0 \left(\left(\frac{1}{G_1} - 1 \right) (x_1 - b_1) + b_0 - b_1 \right) \\ F_1 &= k_0 \left(\frac{1 - G_1}{G_1} x_1 + \frac{G_1 H_1 - 1}{G_1 H_1} b_0 \right) \end{aligned} \tag{9}$$

Express the reflected wave on the actuator (b_0):

$$b_0 = H_1 \left(\frac{G_1}{(G_1 H_1 - 1) k_0} F_1 - \frac{1 - G_1}{G_1 H_1 - 1} x_1 \right) \tag{10}$$

The control law of WBC with position and force feedback is following:

$$x_0 = a_{0,ss} x_d + H_1 \left(\frac{G_1}{(G_1 H_1 - 1) k_0} F_1 - \frac{1 - G_1}{G_1 H_1 - 1} x_1 \right) \tag{11}$$

with the measured force F_1 and the position x_1 on the mass m_1. This control is called absolute, force wave-based control (F-WBC-A).

4. Simulation experiment

The system of three masses with springs and dampers is subjected to a step demand on position, force pulses on the individual masses (in ascending order) and a constant force on the first mass.

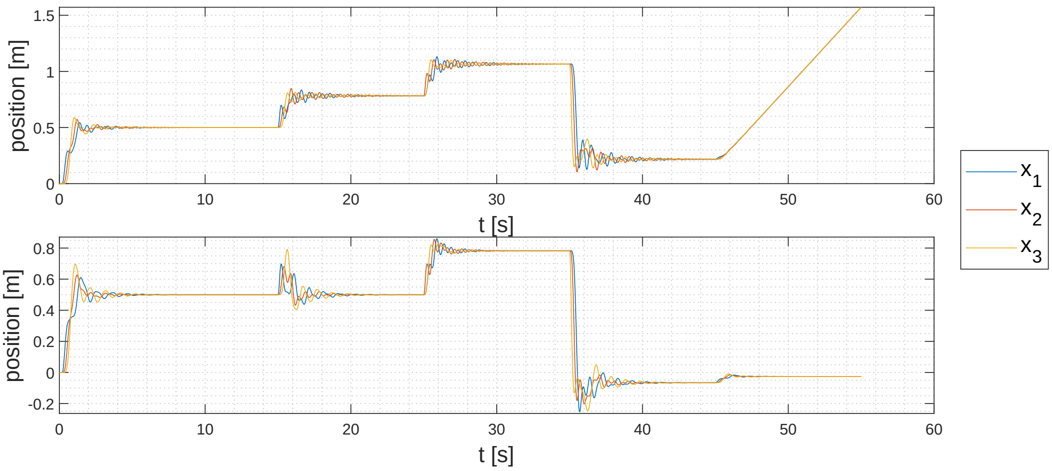

Fig. 3: Three mass system position behavior, WBC-A on the top, F-WBC-A (with force feedback) on the bottom.

Figure 3 shows the response of the system with wave-based control (WBC-A on top) and wave-based control with force feedback (F-WBC-A on bottom).

5. Conclusions

The extension of wave-based control with force feedback is presented in this paper. In contrast to existing work, the implementation of force in control is derived. A simulation comparison of WBC-A and F-WBC-A (with force feedback) is performed. Both approaches suppress vibrations, and force feedback also eliminates the influence of external forces acting on the body where the force is measured. Further research will focus on removing the influence of forces on all bodies.

Acknowledgments

This work was possible thanks to the grant released within the Framework Programme for Research and Innovation (2021–2027), grant n∘ 101159722, with project name "Twinning for Excellence in Morphing and Aeromechatronic Wing Control: A Leap Towards Eco-Smart Aviation" acronym (MechaTwing). I would like to thank professor Michael Valášek for his guidance and valuable advice in wave-based control research.

References

Beneš, P., Neusser, Z., Sika, Z., Valášek, M., and Zavrel, J. (2018) Wave-based control of a planar mechanical structure by piezoelectric actuators. In *Proceedings of ISMA 2018 - International Conference on Noise and Vibration Engineering and USD 2018 - International Conference on Uncertainty in Structural Dynamics*. p. 303 – 307.

Marek, O. (2013) *Řízení polohy poddajných struktur vlnovou metodou*. dizertační práce, České Vysoké Učení Technické v Praze, Praha, Česká republika.

Marek, O. and Valasek, M. (2009) Wave-based control of motion of flexible bodies. In *Multibody Dynamics 2009*. p. 1 – 6.

Martinec, D., Herman, I., Hurák, Z., and Šebek, M. (2014) Refinement of a bidirectional platooning controller by wave absorption at the leader. In *2014 European Control Conference (ECC)*. pp. 2845–2850.

O'Connor, W. (2002) Gantry crane control: a novel solution explored and extended. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*. 1, pp. 250–255.

O'Connor, W. (2004) Wave-echo position control of flexible systems: towards an explanation and theory. In *Proceedings of the 2004 American Control Conference*. 5, pp. 4837–4842.

O'Connor, W. (2005) Excellent control of flexible systems via control of the actuator-system interface. In *Proceedings of the 44th IEEE Conference on Decision and Control*. pp. 6181–6186.

O'Connor, W. J. (2007a) Control of flexible mechanical systems: wave-based techniques. In *2007 American Control Conference*. pp. 4192–4202.

O'Connor, W. J. (2007b) Wave-based analysis and control of lump-modeled flexible robots. *IEEE Transactions on Robotics*, 23, 2, pp. 342–352.

O'Connor, W. J., Ramos de la Flor, F., McKeown, D. J., and Feliu, V. (2009) Wave-based control of non-linear flexible mechanical systems. *Nonlinear Dynamics*, 57, 1-2, pp. 113–123.

O'Connor, W. and Lang, D. (1998) Position control of flexible robot arms using mechanical waves. *Journal of Dynamic Systems, Measurement, and Control*, 120, 3, pp. 334–339.

O'Connor, W. J. (2004) A gantry crane problem solved. *Journal of Dynamic Systems, Measurement, and Control*, 125, 4, pp. 569–576.

O'Connor, W. J. (2006) Wave-echo control of lumped flexible systems. *Journal of Sound and Vibration*, 298, 4, pp. 1001–1018.

O'Connor, W. J. (2011) Wave-like modelling of cascaded, lumped, flexible systems with an arbitrarily moving boundary. *Journal of Sound and Vibration*, 330, 13, pp. 3070–3083.

O'Connor, W. J. and Fumagalli, A. (2009) Refined wave-based control applied to nonlinear, bending, and slewing flexible systems. *Journal of Applied Mechanics*, 76, 4, pp. 041005.

O'Connor, W. J. and McKeown, D. J. (2008) A new approach to modal analysis of uniform chain systems. *Journal of Sound and Vibration*, 311, 3, pp. 623–632.

Valasek, M., Marek, O., Olgac, N., and Neusser, Z. (2019) Rigorous treatment of wave-based control concept, structured procedures and critical observations. *IET Control Theory & Applications*, 13, pp. 2620–2629.

Valasek, M., Neusser, Z., and Gordon, T. (2016) Wave-based control for intelligent longitudinal traffic column. In *Dynamics of Vehicles on Roads and Tracks*. pp. 181–188.

Valášek, M., Neusser, Z., Neuman, P., and Nečas, M. (2016) Wave-based modeling and control of interconnected synchronous machines-application on mechanical model. In *IFAC Workshop on Control of Transmission and Distribution Smart Grids CTDSG 2016*. 49, pp. 352–357.