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A NEW SIMPLE BELT FRICTION MODEL

Frydrýšek K.[1], Hrabovský L.[2], Čepica D.[3]

Abstract: *Article focuses on the belt friction which obeys Coulomb's Law. It presents a new model, which is compared to the traditional simple Euler-Eytelwein (Capstan) model. Derivations are based on the differential equilibrium of forces in a cylindrical coordinate system on an elementary segment of a rope. Compared to the Euler-Eytelwein model, our new model additionally respects the weight of the rope, the radius of the cylindrical surface, the rope's geometry, and the rope's position relative to gravitational force in space. Experiment using a pulley with a semicircular groove wrapped with a loaded rope and the measuring of the forces in the rope was used to evaluate it. Our new model is slightly more complicated and precise than the Euler-Eytelwein model. Finite Element Method is applied too.*

Keywords: Belt friction, Analytical solutions, Numerical solutions, Experiments

1. Introduction

Belt friction is a classic engineering concept that explains the resistance to motion between a flexible belt (or rope) and a cylindrical surface of radius R /m/. When a belt is wrapped around a surface (wrap angle $\varphi \in (\varphi_1;\ \varphi_2)$ /rad/), the tension on one side F_2 /N/ is greater than the tension on the other F_1 /N/, see Fig. 1(a,b). This difference exists because the friction (described by coefficient of friction f /1/) between the belt and the surface "absorbs" or "resists" some of the pull.

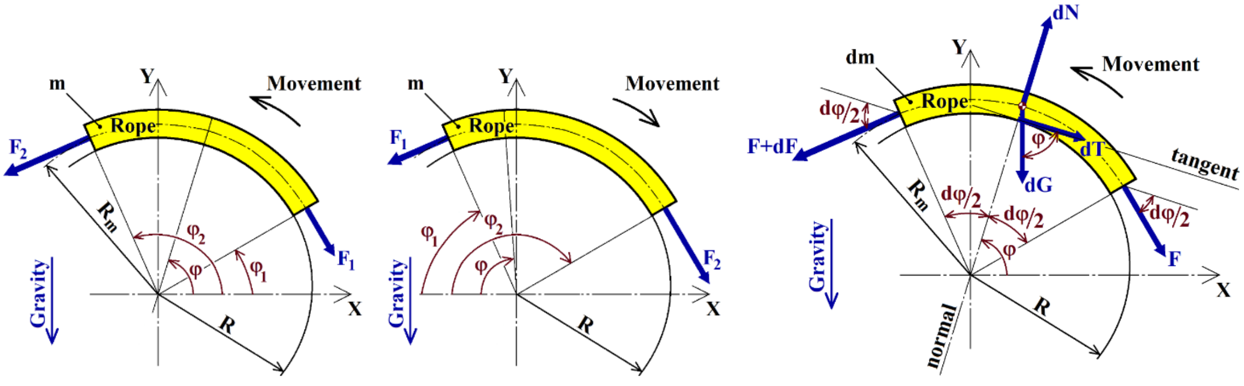

Fig. 1: Basic description of belt friction and force equilibrium on an elementary segment.

Reviewing belt friction involves developments from the classical "capstan" logic to modern dynamic models that account for material deformation and high-speed effects in connections with theories and experiments etc. The transition from simple static equilibrium to complex viscoelastic interactions defines the current state of the field. For example, see references (Bulín, 2019; Euler, 1762; Hrabovský, 2022;

[1] Prof. M.Sc. Karel Frydrýšek, Ph.D., ING-PAED IGIP, FEng.: Department of Applied Mechanics, Faculty of Mechanical Engineering, VSB – Technical University of Ostrava, 17. listopadu 2172/15, 708 00 Ostrava, CZ, karel.frydrysek@vsb.cz.

[2] Assoc. Prof. M.Sc. Leopold Hrabovský, Ph.D.: Department of Machine and Industrial Design, Faculty of Mechanical Engineering, VSB – Technical University of Ostrava, 17. listopadu 2172/15, 708 00 Ostrava, CZ, leopold.hrabovsky@vsb.cz.

[3] M.Sc. Daniel Čepica, Ph.D.: Department of Applied Mechanics, Faculty of Mechanical Engineering, VSB – Technical University of Ostrava, 17. listopadu 2172/15, 708 00 Ostrava, CZ, daniel.cepica@vsb.cz.

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Imado, 2008; Kong, 2005; Konyukhov, 2021; Kušnerová, 2020; Wasfy, 2016 etc.). However, the Euler-Eytelwein formula is insufficient in some cases and should be modified.

2. New "Simple" Belt Friction Model that Respects a Rope's Weight and its Position Relative to Gravitational Force

The derivation is based on the equilibrium in a cylindrical coordinate system (i.e., equilibrium in the direction of the normal and tangential forces) on an elementary segment of an ideal rope with mass m /kg/, see Fig. 1(c) and Tab. 1. The centroid of the wrapped rope's cross-sectional area is at the distance R_m /m/ and the rope is subjected to the tensile forces F and $F + dF$ /N/. The elementary gravitational force of the rope $dG = gdm = a_m R_m d\varphi$ /N/ acts in the direction opposite to the Y-axis, where $g = 9.807\ m s^{-2}$ is the gravitational acceleration and a_m $/kg m^{-1}/$ is the specific weight of the rope. Coulomb's law applies $dT = f\ dN$, where dT /N/ is friction force and dN /N/ is normal Force.

Tab. 1: New belt friction model that respects a rope's weight and its position relative to gravity.

New solution considering the rope's weight and wrapping geometry (mass $m \neq 0$, more complex and precise):	
Inhomogeneous differential equation $\frac{dF}{d\varphi} - fF = b_m(cos\varphi + f sin\varphi)$	General solution of the differential equation $F_{(\varphi)} = Be^{f\varphi} + c_m[(1 - f^2)sin\varphi - 2f\ cos\varphi]$, where B /N/ is an integration constant, and constant $c_m = \frac{g\, a_m R_m}{f^2+1}$
The particular solution according to Fig. 1 $F_{(\varphi)} = F_1 e^{f(\varphi-\varphi_1)} + c_m\left(e^{f(\varphi-\varphi_1)}[2f\ cos\varphi_1 + (f^2 - 1)sin\varphi_1] + (1 - f^2)sin\varphi - 2f\ cos\varphi\right)$ or more precisely for determining the force $F_2 = F_1 e^{f(\varphi_2-\varphi_1)} + c_m\left(e^{f(\varphi_2-\varphi_1)}[2f\ cos\varphi_1 + (f^2 - 1)sin\varphi_1] + (1 - f^2)sin\varphi_2 - 2f\ cos\varphi_2\right)$	
Nonlinear formula for calculating the coefficient of friction f using a homogeneous equation according to Fig. 1(a,b) $F_1 e^{f\pi} + 2c_m f\left(e^{f\pi} + 1\right) - F_2 = 0$	

If the weight of the rope itself is neglected during the derivation of the rope element's equilibrium (i.e., $a_m = 0$), equations in Tab. 1 reduce to the well-known simple form of the Euler-Eytelwein (Capstan) formula, see Tab. 2.

Tab. 2: Classical belt friction model that do not respects a rope's weight and its position relative to gravity.

The classical solution, neglecting the weight of the rope itself (mass $m = 0$, simpler model)	
Homogeneous differential equation $\frac{dF}{d\varphi} - fF = 0$	General solution of the differential equation $F_{(\varphi)} = Ae^{f\varphi}$, where A /N/ is an integration constant.
The particular solution according to Fig. 1, the Euler-Eytelwein (Capstan) formula $F_{(\varphi)} = F_1 e^{f(\varphi-\varphi_1)}$, or more precisely for determining the force $F_2 = F_1 e^{f(\varphi_2-\varphi_1)}$	
Simple formula for calculating the coefficient of friction $f = \frac{\ln\left(\frac{F_2}{F_1}\right)}{\pi}$	

From the initial comparison of the approaches mentioned in Tab. 1 and 2, it is evident that the forces F_1 or F_2 can be characterized as functions of these parameters:

- The classical solution (the Euler-Eytelwein formula, see Tab. 2) without consideration for the weight of the rope itself $\frac{F_1}{F_2} = function(f, \varphi_2 - \varphi_1) = e^{f(\varphi_2-\varphi_1)}$.
- New solution taking the weight of the rope itself into consideration, see Tab. 1, $\frac{F_1}{F_2} = function(f, \varphi_1, \varphi_2, c_m)$.

The formulas in Tab. 1 and 2 are used for comparison with the experiments in the following text.

3. Experiments

The experiment and rope are explained by Hrabovský (2022), in ČSN (2004) and shown in Fig. 2. Hence, $F_1 = F_M + a_m L_1 g$, $F_2 = F_G + a_m L_2 g$, $a_m = 0.246\ \text{kg}\,\text{m}^{-1}$, $R_m = R + \frac{h}{2} = 0.164$ m and F_M.and F_G /N/ are forces measured via strain gauges and gravity.

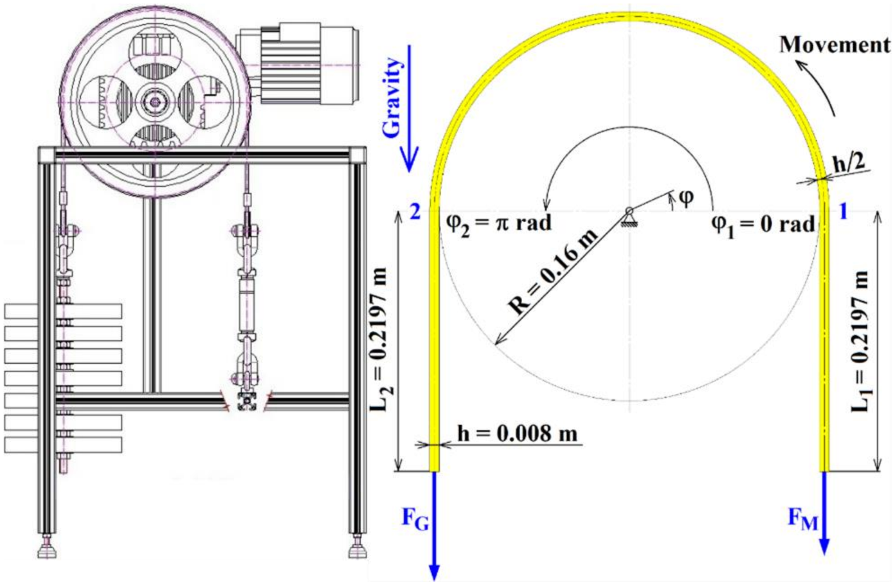

Fig. 2: Experiment set up for rope 6x7+WSC 1960 B sZ.

4. Results of Experiments

Five experiments and calculations were performed for models presented in Tab. 1 and 2. One set of measurements is presented in Tab. 3 together with relative errors.

Tab. 3: Classical belt friction model that do not respects a rope's weight and its position relative to gravity.

Experiment		Calculation without consideration of the rope weight and geometry (Euler-Eytelwein, see Tab. 2)			A new calculation with consideration of the rope weight and geometry (see Tab. 1)			Error of calculation without consideration of rope weight, i.e., error of the Euler-Eytelwein formula		
Given and measured forces		Forces taken from the experiment		Calculated coefficient of friction	Given and measured forces		Calculated coefficient of friction	Error f /%/	Error F_1 /%/	Error F_2/%/
F_G /N/	F_M /N/	F_1 /N/	F_2 /N/	f /1/	F_1 /N/	F_2 /N/	f /1/			
	484.6		484.6	0.7294		485.1	0.7237	-0.792		0.109
	564.7		564.7	0.7781		565.2	0.7723	-0.750		0.094
49.0	503.7	49.0	503.7	0.7417	49.5	504.2	0.7360	-0.781	1.070	0.105
	496.5		496.5	0.7371		497.0	0.7314	-0.785		0.107
	523.3		523.3	0.7539		523.8	0.7481	-0.770		0.101

From the results in Tab. 3, it is clear that when solving the belt friction problem in the experiment shown in Fig. 2 (i.e. when ignoring the rope weight), the maximal errors 1.070% occur for small tensile forces. Additionally, the error increases along with an increase in the rope weight or in gravitational acceleration too (e.g., on other planets). Our new model gives lower coefficient of friction.

5. Comparing with the Finite Element Method

An initial solution of the rope friction problem using the Finite Element Method (FEM), for Ansys Workbench 2022 R2 sw, is presented by Hrabovský (2022), see Fig. 3 and Tab. 4.

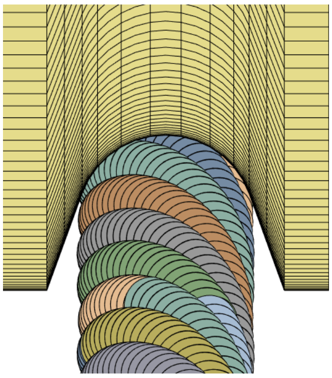

Fig. 3: FEM application.

	Force		Calculated coefficient of friction
	F_1 /N/	F_2 /N/	f /1/
Solution without gravity	49	484.66	0.6327
Solution with gravity	49.5	486.3	0.6238

Tab. 4: Results of FEM.

FEM solution with gravity application gives lower coefficient of friction, similarly to our new friction model. However, differences exist between FEM and the two presented analytical solutions, see Tab.3 and 4.

6. Conclusions

In our article, a new model for Coulomb belt friction that better accounts for belt weight and rope position was introduced. New model was then compared with the classical simple Euler-Eytelwein model. New model, which is more accurate and complex and demonstrated and supported by an experiments, better represents reality and opens up various new possibilities. The results were also compared with FEM solutions. Further possibilities for extending the model primarily lie in dynamics, other friction models, contact geometry description, wear and the probabilistic approach, see Čepica (2023), Lesňák (2020) etc.

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