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PID CONTROLLER FAST TUNING METHOD

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Abstract: *The tuning method presented in the article belongs to the group of frequency - response methods. Only simple explicit formulas are used in a single tuning cycle. The input quantities are r_0 – P - controller gain and two arbitrary frequencies out of three (f_1, f_2, f_0), where f_1 is the lower and f_2 is the upper frequency of the P - controller interval (3 or 1 dB) and f_0 is the geometric mean (scale, norm) frequency. The calculated quantities are r_{-1}, r_1, where r_{-1} is I - controller gain and r_1 is D - controller gain. The formulas used provide accurate output values, unlike the relationships valid for asymptotic estimates. Asymptotic estimates are very inaccurate for relatively narrow (f_2 - f_1) intervals of the P - controller. The published method is valid for an ideal PID controller. Using a similar procedure, analogous relations can be derived even if a simple filter is used in the D - controller branch.*

Keywords: PID controller, Fast tuning method, Frequency – response methods, Damping ratio, Nondimensional frequency

1. Introduction

The presented article was created as the author's response to recommended algorithms for tuning systems with PID controllers, for example (Kubík et al., 1974, 1982; Raven, 1995) and extensive company literature.

The main drawback of the usual instructions is the lack of clarity when implementing them on real mechatronic systems. I believe that the main advantage of the proposed tuning method is its clarity. In each tuning step, the operator always has an overview of which $\Delta f_p = (f_2 - f_1)$ frequency interval is covered by the P-controller interval and therefore where the I and D-controller intervals are located.

2. Proposed tuning method

The transfer function of the ideal PID controller is given by the well-known relationship

$$G_{PID}(s) = \frac{Y}{E} = r_0 \cdot G_r(s) \text{ and} \tag{1}$$

$$G_r(s) = \frac{Y_r}{E} = 1 + \frac{\omega_i}{s} + \frac{s}{\omega_d} = \frac{\Omega_0}{2 \cdot \xi \cdot s} \cdot \left[\left(\frac{s}{\Omega_0}\right)^2 + 2 \cdot \xi \cdot \left(\frac{s}{\Omega_0}\right) + 1 \right], \tag{2}$$

where

r_0 – P – controller gain

$\omega_i = \frac{1}{T_i} = \frac{r_{-1}}{r_0} = 2\pi \cdot f_i$ [rad/s] - angular frequency of the I – controller,

$\omega_d = \frac{1}{T_d} = \frac{r_0}{r_1} = 2\pi \cdot f_d$ [rad/s] - angular frequency of the D – controller,

r_{-1} – I – controller gain,

r_1 - D – controller gain,

f_i [Hz] – the break (corner) and cutoff (half - power) frequency of the separate PI – controller,

f_d [Hz] – the break (corner) and cutoff (half - power) frequency of the separate PD – controller,

$$\Omega_0 = 2\pi \cdot f_0 = \sqrt{\omega_i \cdot \omega_d}\ , \tag{3}$$

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$f_0 = \sqrt{f_i \cdot f_d}$ [Hz] – geometric mean (scale, norm) frequency,

$$\frac{2 \cdot \xi}{\Omega_0} = \frac{1}{\omega_i} \, . \tag{4}$$

By adjusting relations (3) and (4) we obtain the formula for the damping ratio [-]

$$\xi = \frac{1}{2}\sqrt{\frac{\omega_d}{\omega_i}} = \frac{1}{2}\sqrt{\frac{f_d}{f_i}} \tag{5}$$

and two other relationships

$$f_i = \frac{f_0}{2 \cdot \xi} \quad \text{and} \quad f_d = 2 \cdot \xi \cdot f_0 \, . \tag{6}$$

For the limiting case, the following applies: $f_i = f_d = f_0$ and $\xi = 1/2$.

For $\xi \in \langle 0; 0.5)$ it applies $f_i > f_d$ and for $\xi > 0.5$ it applies $f_i < f_d$.

In the following we will use the frequency - response method, so

$$s = j \cdot \omega, \quad j = \sqrt{-1} \tag{7}$$

and in addition, we will introduce nondimensional frequency [-]

$$\lambda = \frac{\omega}{\Omega_0} = \frac{f}{f_0} \, . \tag{8}$$

After modifying relation (2) using relations (7) and (8), we obtain

$$G_r(j \cdot \lambda) = 1 - \frac{1-\lambda^2}{2 \cdot \xi \cdot \lambda} \cdot j \, . \tag{9}$$

So, for the magnitude (gain) of the transfer function we have

$$G_r(\lambda) = \sqrt{1 + \left(\frac{1-\lambda^2}{2 \cdot \xi \cdot \lambda}\right)^2} \tag{10}$$

and for the tangent of the phase angle, we have

$$\tan\big(\varphi(\lambda)\big) = -\frac{1-\lambda^2}{2 \cdot \xi \cdot \lambda} \, . \tag{11}$$

Now we will look for such λ for which the gain G_r (formula (10)) is equal to $G_{r,0}$, for example +3 dB, i.e. √(2)

$$G_{r,0} = \sqrt{1 + \left(\frac{1-\lambda^2}{2 \cdot \xi \cdot \lambda}\right)^2} \, . \tag{12}$$

By modifying equation (12), we obtain a quartic equation in the usual form

$$\lambda^4 - 2 \cdot [1 + a] \cdot \lambda^2 + 1 = 0 \, , \tag{13}$$

where $a = 2 \cdot K_0^2 \cdot \xi^2$,

$K_0 = \sqrt{G_{r,0}^2 - 1}$.

For example, for $G_{r,0}$, which corresponds to +3 dB, +1 dB and +0.5 dB, the value of K_0 will be 1.0; 0.50885 and 0.34931, respectively.

The solution to equation (13) has the form

$$\lambda_{1,2}^2 = 1 + a \cdot \left[1 \mp \sqrt{D}\right] , \tag{14}$$

where $D = \frac{a+2}{a}$.

Using relation (14), two working relations can be derived ($\lambda_1 \leq \lambda_2$)

$$\lambda_1 \cdot \lambda_2 = 1 \, , \tag{15}$$

$$\bar{\lambda}^2 = \frac{\lambda_1^2 + \lambda_2^2}{2} = \frac{\lambda_2^4 + 1}{2 \cdot \lambda_2^2} = 1 + a \, . \tag{16}$$

We will also introduce two more relations

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_2^2-1}{\lambda_2}\ , \qquad \delta\lambda = \frac{\Delta\lambda}{\lambda_2}\ . \tag{17}$$

By modifying relation (16) using relation (13) for "*a*", we obtain *the key relation for the proposed tuning method*

$$\xi = \frac{\lambda_2^2-1}{2\cdot K_0\cdot\lambda_2} = \frac{\Delta\lambda}{2\cdot K_0}\,. \tag{18}$$

By gradually modifying relation (11) using relation (18), we obtain two relations for phase angels

$$\tan(\varphi(\lambda_2)) = K_0 \tag{19}$$

and

$$\varphi(\lambda_1) = -\varphi(\lambda_2)\ , \tag{20}$$

For example, for $G_{r,0}$, which corresponds to +3 dB, +1 dB and +0.5 dB, the value of $\varphi(\lambda_2)$ will be +45°; +26.97°; and +19.25°, respectively.

3. Illustrative example

An illustrative example serves to make the proposed tuning method easier to understand.

Selected input values:

r_0 = 2.6 – P – controller gain, f_0 = 2 Hz – geometric mean frequency of the P – controller interval (+3 dB, K_0 = 1.0, $\varphi(\lambda_2)$ = +45°) and f_2 = 3 Hz – upper frequency of the P – controller interval.

Calculation procedure:

$\lambda_2 = f_2/f_0 = 1.5$,

$\lambda_1 = 1/\lambda_2 = 0.666$,

$f_1 = \lambda_1 \cdot f_0 = 1.333$ Hz – lower frequency of the P – controller interval,

$f \in \langle 1.333;\ 3.0\rangle$ Hz - +3 dB - frequency interval of the P – controller, $f \in \langle 0;\ 1.333\rangle$ Hz - +3 dB - frequency interval of the I – controller, $f > 3.0$ Hz - +3 dB - frequency interval of the D – controller,

$\xi = |\text{formula (18)}| = 0.4167$,

$f_i = f_0/(2\xi) = 2.4$ Hz, $\omega_i = 2\cdot\pi\cdot f_i = 15.08$ rad/s, $T_i = 1/\omega_i = 0.0663$ s,

$f_d = 2\cdot\xi\cdot f_0 = 1.667$ Hz, $\omega_d = 2\cdot\pi\cdot f_d = 10.47$ rad/s, $T_d = 1/\omega_d = 0.0955$ s,

$\xi < 0.5$ than $f_i > f_d$!!,

Output values:

$r_{-1} = \omega_i \cdot r_0 = 39.204$,

$r_1 = r_0/\omega_d = 0.2483$.

Conclusions

For quick reference, Table 1 lists the necessary values of the most important quantities.

The published method is valid for an ideal PID controller. Using a similar procedure, analogous relations can be derived even if a simple filter is used in the D - controller branch.

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Tab. 1: Selected values of the most important quantities

		$G_{r,0}$	dB	3	1	0,5
		$G_{r,0}$	-	1,414214	1,12202	1,05925
		K_0	-	1	0,509	0,349
λ_2	λ_1	$\Delta\lambda$	$\delta\lambda$	ξ	ξ	ξ
1,00	1,00	0,00	0,00	0	0	0
1,05	0,95	0,10	0,09	0,0488	0,0959	0,1397
1,10	0,91	0,19	0,17	0,0955	0,1876	0,2733
1,15	0,87	0,28	0,24	0,1402	0,2756	0,4014
1,1898	0,84	0,35	0,29	0,1747	0,3432	0,5000
1,2	0,83	0,37	0,31	0,1833	0,3603	0,5249
1,25	0,80	0,45	0,36	0,2250	0,4422	0,6441
1,2863	0,78	0,51	0,40	0,2544	0,5000	0,7284
1,30	0,77	0,53	0,41	0,2654	0,5215	0,7598
1,35	0,74	0,61	0,45	0,3046	0,5987	0,8721
1,40	0,71	0,69	0,49	0,3429	0,6738	0,9816
1,4086	0,71	0,70	0,50	0,3493	0,6864	1,0000
1,45	0,69	0,76	0,52	0,3802	0,7471	1,0884
1,50	0,67	0,83	0,56	0,4167	0,8188	1,1929
1,55	0,65	0,90	0,58	0,4524	0,8891	1,2952
1,60	0,63	0,98	0,61	0,4875	0,9580	1,3956
1,6180	0,62	1,00	0,62	0,5000	0,9826	1,4314
1,6309	0,61	1,02	0,62	0,5089	1,0000	1,4568
1,65	0,61	1,04	0,63	0,5220	1,0258	1,4943
1,70	0,59	1,11	0,65	0,5559	1,0924	1,5914
1,75	0,57	1,18	0,67	0,5893	1,1581	1,6870
1,80	0,56	1,24	0,69	0,6222	1,2228	1,7813
1,85	0,54	1,31	0,71	0,6547	1,2867	1,8744
1,90	0,53	1,37	0,72	0,6868	1,3498	1,9663
1,95	0,51	1,44	0,74	0,7186	1,4122	2,0572
2,00	0,50	1,50	0,75	0,7500	1,4739	2,1472
2,25	0,44	1,81	0,80	0,9028	1,7742	2,5845
2,4142	0,41	2,00	0,83	1,0000	1,9652	2,8628
2,50	0,40	2,10	0,84	1,0500	2,0635	3,0060

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