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COMPARISON OF ANALYTICAL AND EXPERIMENTAL APPROACHES FOR MODIFIED VON MISES TRUSS BEHAVIOUR

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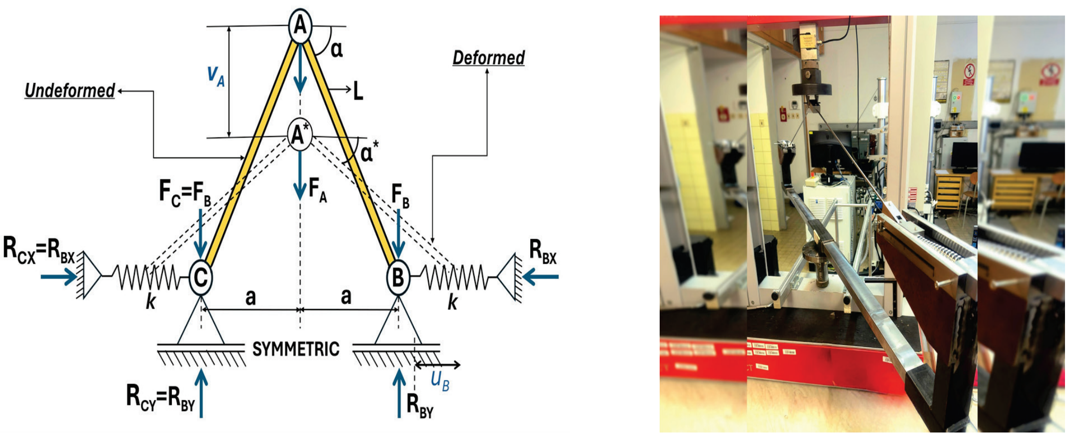
Abstract: *This work examines a simple planar symmetric truss design (modified von Mises truss) which involves two members subjected to deformation by an applied force, with supporting springs at the ends. The analytical numerical model of the truss structure is formulated using first- and second-order theories of deformation, and the ensuing nonlinear model is solved by applying established techniques. The study subsequently correlates the solution using the results derived from the experimental setup. The two approaches are compared, buckling, snap-through effect and error estimation are evaluated.*

Keywords: Planar truss, von Mises truss, theories of first and second order, numerical methods, experimental methods, nonlinearities, buckling, snap-through

1. Introduction

Nonlinear mechanics is a significant field of study that addresses difficulties that have application in real life. One proven approach to understanding and addressing nonlinearities in deformable bodies is the utilization of planar truss structures. In mechanics, minor deformation problems can be addressed using either straightforward first-order (linear) theory or more accurate yet more complex second-order (nonlinear) theory. If the impact of structural deformations under load is insignificant in relation to the equilibrium of external and internal forces, first-order analysis may be used. The second-order theory typically results in a non-linear equation that can be resolved using several numerical approaches. For more information see (Eremeyev 2024; Bažant et al. 1991; Frydrýšek et al. 2016, 2019, 2021 and 2023; Galishnikova et al. 2009; Klučka et al. 2016) and (Pelliciari et al. 2022).



(a) Deformed and Undeformed structure (b) Experimental Setup

Fig. 1: Simple Symmetric Modified von Mises Planar Truss

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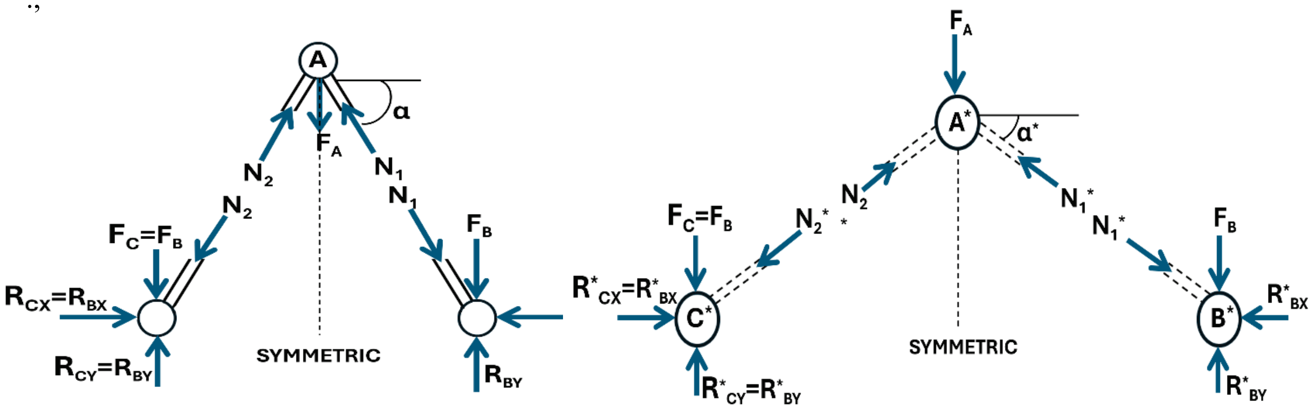
The study focuses on numerical and experimental solutions for a simple modified two-dimensional planar von Mises truss with two elements and two springs; see Fig.1. The truss experiences a vertical force F. The lower sections of the trusses can only move horizontally and are fastened with springs at the ends to restrict the range of motion of the truss members. Different von Mises truss structures are solved by Eremeyev (2024), Bažant et al. (1991), Frydrýšek (2016), and Pelliciari et al. (2022).

2. Analytical - Numerical Solution

The expressions for the analytical solution are derived using the joint method for each joint to obtain a relation between the force applied to Joint A and the displacement resulting from this force in Joints B and C; see Fig.2. Since the truss structure follows symmetry, we can safely assume that the displacements at points B and C will be similar in nature. The main focus will be on determining the normal forces on the members of the truss($|N_1|$=N_2) and the reaction forces (R_X and R_Y); see Fig.3. The inputs given are the forces (including the forces due to the weight of the joints, the truss members, and the springs), the length of the members L[m], the modulus of elasticity E[Pa] of the material of the members, and the area of cross sections A[m^2] of the members. The results of the second-order theory are depicted using the superscript "*".,



(a) Theory of first order (no changes of angle) (b) Theory of second order (changes of angle)

Fig. 2: Loading and Reactions of von Mises Truss

The forces acting on the truss structures could be defined as

$$F_A = F + G_A + G_{ROD},$$

$$F_B = G_B + \frac{1}{2} G_{ROD}, \text{ and}$$

$$F_C = F_B \text{ (symmetry)}.$$

The values F_A, F_B, and F_C [N] reflect the total forces acting on the respective joints. G_A and G_B [N] represent the force as a result of the design of the mass of joints A and B, and the value of G_{ROD} [N] indicates the force due to the weight of the members of the truss. The value F represents the external force applied to the truss, which results in non-linear deformation of the members.

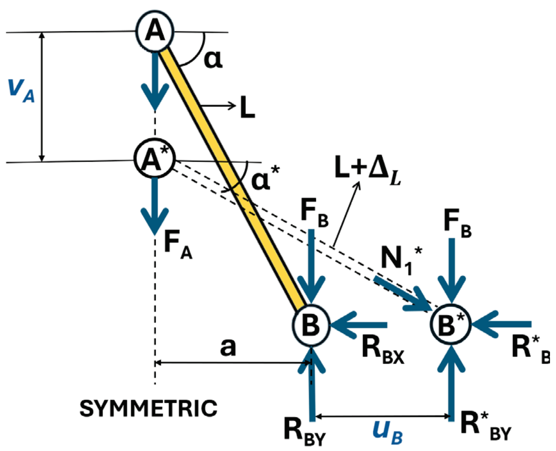


Fig. 3: Method of Joints

Using the theory of second order, the normal forces in the members are derived via the method of joints. The equations of equilibrium of forces in the x-direction ($\sum F_X$) and the forces in the y-direction ($\sum F_Y$) are given at point A* as:

$$\sum F_X \text{ at point } A^* = 0 \Rightarrow \qquad N_1^*\cos(\alpha^*) - N_2^*\cos(\alpha^*)=0 \;\Rightarrow\; N^* = N_1^* = N_2 \qquad (1)$$

$$\sum F_Y \text{ at point } A^* = 0 \Rightarrow \qquad N_1^* sin(\alpha) + N_2^* sin(\alpha) - F = 0 \;\Rightarrow\; N^* = \frac{F_A}{2 sin\alpha^*} \qquad (2)$$

For Joint B, there are additional reaction forces present on top of the normal force (N^*) and the force as a result of the masses (F_B). The horizontal reaction force (R_{BX}) is a combination of the spring force and the force due to friction between the translational surfaces

$$\Sigma F_X \text{ at point } B^* = 0 \Rightarrow \qquad N^* \cos\alpha^* - k\,u_B - f R_{BY} = 0 \tag{3}$$

$$\Sigma F_Y \text{ at point } B^* = 0 \Rightarrow \qquad N^* \sin\alpha^* + F_B - R_{BY} = 0 \tag{4}$$

The values of the frictional property of the spring k (N/mm) and the Coloumb friction f are already known. According to Fig. 3, the deformation condition can be derived:

$$L\cos(\alpha^*) + u_B = (L + \Delta_L)\cos(\alpha^*) \tag{5}$$

With the help of Eq. (3), (4) and (5), we arrive at the relation

$$N^* = \frac{kL(\cos\alpha^* - \cos\alpha) + fF_B}{\cos\alpha^* - f\sin\alpha^* - \left(\frac{kL}{EA}\right)\cos\alpha^*} \tag{6}$$

By substitution of Eq. (2) into Eq. (6), we obtain the load force value (F) as a function of angle (α^*), by which we can obtain the relation between the force applied (F) and the displacement (v_A). The solution of the system of equations is obtained by using the Newton-Raphson method.

Comparison with Experimental Results

The results obtained from the numerical calculations are expressed in Fig. 4. (acquired for values $L = 0.533$ m, $a = 0.5$ m, $k = 4086$ Nm^{-1}, $E = 2.08\text{x}10^{11}$ Pa, $A = 1.963\ \text{x}10^{-5}$ m^2, $f = 0.07$) It shows the dependency between the force applied to the planar truss structure and the change in the angle of deflection due to the force applied.

The result shows a really good behavior and follows the curve in a predictable manner. A dependency behavior between the force (F) and the displacement (v_A) is derived and plotted as shown in Fig. 5. Furthermore, the result of the conducted test is also plotted in the same curve, and this clearly shows a good correlation between the results obtained from the numerical method and the experimental method.

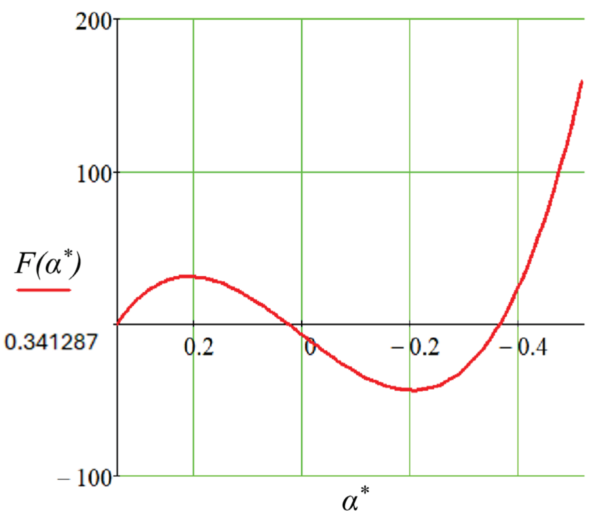


Fig. 4: Force (F) vs Angle (α)*

Modifications to the experimental results were required to ensure clarity and precision. The test exhibited an early displacement caused by the forces resulting from the weight of the structure. It generated an initial force that counteracts the reaction forces within the components of the beam.

An error estimation is conducted to assess the efficacy of the correlation and demonstrates exceptional accuracy, particularly when the force reaches its maximum amount. Evidence of a snap-through effect is seen.

Tab. 1: Error Determination

Error	% error of the peak Force (ΔF_{max})	% error of angles at peak Force ($\Delta\alpha^*$)
Values	0.6 %	3.5 %

Internal

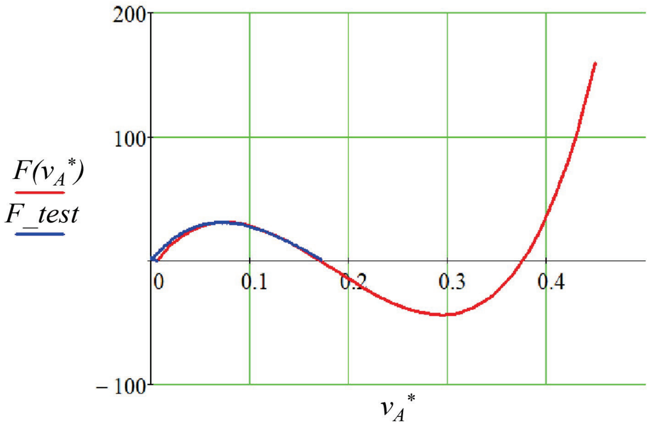


Fig. 5: Comparison of Numerical and Experimental Results

3. Conclusions

The study mainly focused on validating the results acquired from the numerical model using the experimental setting for the modified von Mises truss structure. We developed a basic planar truss structure that incorporates the nonlinearities present in the system. We solved the created model using numerical methods. We established an experimental apparatus to perform the model correlation analysis. We achieved encouraging results throughout the study, which motivated the continuation of future research. Future research may investigate various configurations of planar truss models, which can lead to practical applications. Hopefully, this study might lay the groundwork for future research in similar fields, where there is ample potential for significant improvements and opportunities.

Acknowledgement

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