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ON UNCERTAIN PARAMETERS IN A MODEL OF HYPOPLASTICITY

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Abstract: *The contribution deals with the mathematical modelling of hypoplasticity of granular materials. The problem of stress-controlled hypoplasticity with a cyclic proportional loading and unloading of drained granular material under different anisotropic settings is considered, and a gradual compaction of the material, called ratchetting, is investigated together with the influence of uncertain input parameters on the model outputs, namely the strain components in the material in its limit (i.e. compacted) state. The uncertainty quantification is presented through 3D graphs. The depicted data also allows for a fuzzy set interpretation.*

Keywords: Granular material, Hypoplasticity, Ratchetting, Uncertain parameters.

1. Introduction

This contribution is focused on the impact of uncertain input parameters in a mathematical model of hypoplasticity of granular materials such as soil, sand, or gravel.

A new hypoplastic model introduced by (Bauer et al., 2022) is used. It describes stress-strain relation for drained granular materials. A problem of stress-controlled hypoplasticity is analysed. The behaviour of strain paths (i.e. unknown deformations) under cyclic loading and unloading is found out. Unlike its original simplified form (Bauer et al., 2020), where the theoretical ratchetting would be infinite, the new model is in a better agreement with experimental observations.

In a general case, the model is able to accommodate a general stress-controlled loading and unloading process that leads to a nonlinear system of the first order differential equations. The key features of the model behaviour can, however, be described by a simplified loading setting that results in an initial value problem for one ordinary differential equation that will be used here. The coefficients appearing in the model are constants, functions of a prescribed type, or depend on the porosity of the material, which is influenced by the deformations.

The gradual compaction of the material subjected to cycles of loading and unloading, called ratchetting, is observed especially in the stress-strain graphs.

This contribution is a sort of continuation of (Chleboun et al., 2023), where the anisotropy of the cyclic loading and unloading was considered only in one component of the principal stresses. In the current contribution, all three principal loading stresses can be mutually different and, moreover, more general and advanced uncertainty analysis is presented through 3D graphs.

2. Methods

First, the model is described. Then, the set of uncertain parameters is established, and numerical simulations are performed.

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2.1. Hypoplastic model

The investigated hypoplastic model is described in detail in (Bauer et al., 2022). Let us mention only its main features which closely concern our simulations. We study a stress-controlled problem (hypoplasticity) with a known stress tensor $\boldsymbol{\sigma}(t)$ which is composed of the product of a given continuous and alternately increasing (loading period) and decreasing (unloading period) positive $2T$ periodic function $\sigma(t)$ and a chosen fixed symmetric tensor (matrix) $\mathbf{S}$, the trace of which is negative $(\mathrm{tr}(\mathbf{S}) < 0)$. A specific loading-unloading function which periodically oscillates between fixed values $0 < \sigma_1 < \sigma_2$ is considered as follows:

$$\sigma(t) = \sigma_1 e^{t-t_{2j}} \text{ for } t \in (t_{2j}, t_{2j+1}), \qquad \sigma(t) = \sigma_2 e^{t_{2j+1}-t} \text{ for } t \in (t_{2j+1}, t_{2j+2}), \tag{1}$$

where $j = 0,1,2, \ldots$ and $t_k = kT$ for $k = 0,1,2, \ldots$ with $T = \ln(\sigma_2) - \ln(\sigma_1)$.

The original governing equation, i.e. the relation between the stress tensor $\boldsymbol{\sigma}(t)$ and the strain tensor $\boldsymbol{\varepsilon}(t)$, takes the following form

$$\dot{\sigma}(t)\,\mathbf{S} = c_1(t)\sigma(t)\left(a^2\,\langle\mathbf{S},\mathbf{I}\rangle\dot{\boldsymbol{\varepsilon}}(t) + \frac{1}{\langle\mathbf{S},\mathbf{I}\rangle}\langle\mathbf{S},\dot{\boldsymbol{\varepsilon}}(t)\rangle\mathbf{S} + af(t)\|\dot{\boldsymbol{\varepsilon}}(t)\|\left(2\mathbf{S} - \frac{1}{3}\langle\mathbf{S},\mathbf{I}\rangle\mathbf{I}\right)\right), \tag{2}$$

where $\langle\cdot,\cdot\rangle$ denotes the canonical scalar product in $\mathbb{R}^{3\times3}$, that is, for matrices $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{3\times3}$,

$$\langle\mathbf{A},\mathbf{B}\rangle = \mathbf{A} : \mathbf{B} = \mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{A}) = \sum_{1\le j,k\le 3} a_{jk} b_{jk}, \tag{3}$$

$\mathbf{I}$ denotes the identity matrix, a represents a material constant, and the dot denotes the derivative with respect to t. The symbols $c_1(t) < 0$ and $f(t) \geq 0$ denote scalar functions where their dependence on t is mediated by the void ratio $e(t)$, that is,

$$f(t) = \hat{f}(e(t)) = f_0\left(\frac{e(t)-e_d}{e_c-e_d}\right)^{\alpha}, \tag{4}$$

$$c_1(t) = \hat{c}_1(e(t)) = -\bar{c}(e(t) - e_d)^{-\beta}, \tag{5}$$

where $e_c > e_d > 0$, $\alpha > 0$, $\bar{c} > 0$, $\beta > 1$, $f_0 = 1$ are constants.

3. Numerical simulations with uncertain parameters

The problem (1), (2), (4), (5) can be reformulated into a sequence of alternating first order nonlinear ordinary differential equations for the void ratio $e(t)$, see (Bauer et al., 2022), which is solved in Matlab. A gradual compaction of material during 100 loading-unloading cycles is observed. If $e(t)$ is known, then a direct integration gives the strain tensor $\boldsymbol{\varepsilon}(t)$, see (Bauer et al., 2022). To assess the main features of the model response, it suffices to reduce $\mathbf{S}$ to a diagonal matrix. Then the numerically approximated limit values of the strain tensor also form a diagonal matrix. A ratchetting curve is depicted in Fig. 5, where the lines **A** and **B** indicate the theoretical bounds for the ratchetting cycles.

For each investigated parameter, a set of values is established which differ from the nominal value of the parameter up to ± 25 % in 5% steps, except for the parameter beta whose perturbation scheme is slightly different. The nominal values are $a = 0.4$, $e_c = 0.8$, $e_d = 0.4$, $\alpha = 0.1$, $\bar{c} = 2$, $\beta = 1.03$, $\sigma_1 = 10$, $\sigma_2 = 12$. The nominal values of the initial conditions are $e_0 = e(0) = 0.7$ and $\boldsymbol{\varepsilon}(0) = 0 \cdot \mathbf{I}$.

The case of isotropic loading and unloading stress cycles is defined by $\mathbf{S} = -\mathbf{I}$ and is not as interesting as anisotropic stresses. Let the matrix $\mathbf{S}$ be diagonal with $s_{11} = -1$, $s_{22} = -0.5$, $s_{33} = -1$. In s_{11}, the nominal value -1 will be perturbed. That is, we will consider $s_{11} = -0.75 + 0.05m$, where $m = 0,1,\ldots,10$, and for each m, an uncertainty quantification with respect to a parameter will be performed and depicted. In other words, the uncertainty quantification will depend on the anisotropy of the stress matrix $\mathbf{S}$.

In the figures, the model response (the limit strain) to negative perturbations of the investigated uncertain parameter is denoted by o; the response to positive perturbations by $+$. The color of the marks indicates the magnitude of perturbations. The size of the marks increases with the increasing rate of anisotropy s_{11}/s_{22}.

The responses to the uncertainty in the parameters a^{unc}, e_d^{unc}, $e^{unc}(0)$ are shown in Fig. 1, 2, 3, respectively. The changes in this input parameters have a significant impact on the limit state of ratchetting. The dependencies are almost linear, small nonlinearities appear.

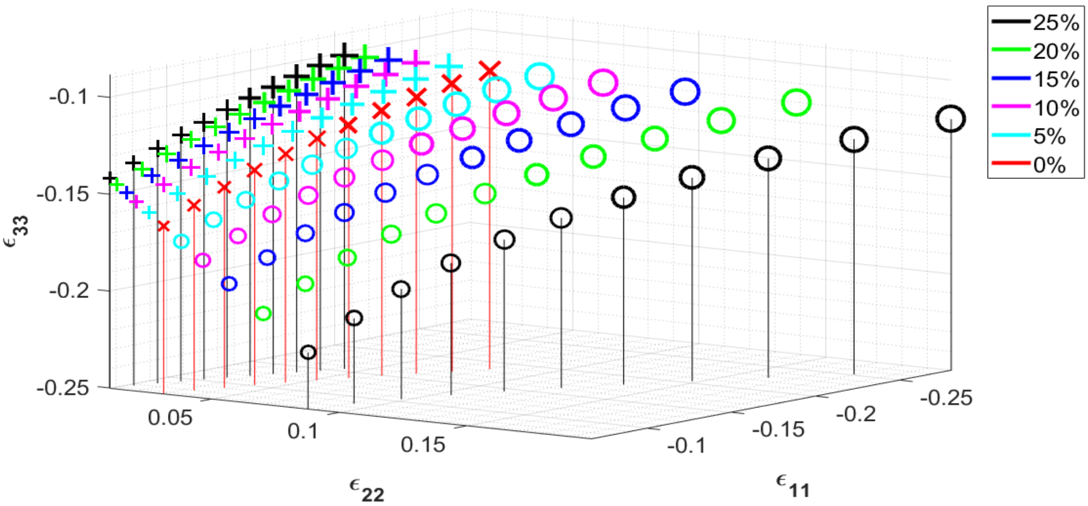

Fig. 1: Response to the uncertainty in the parameter a; for clarity, only selected stems are depicted.

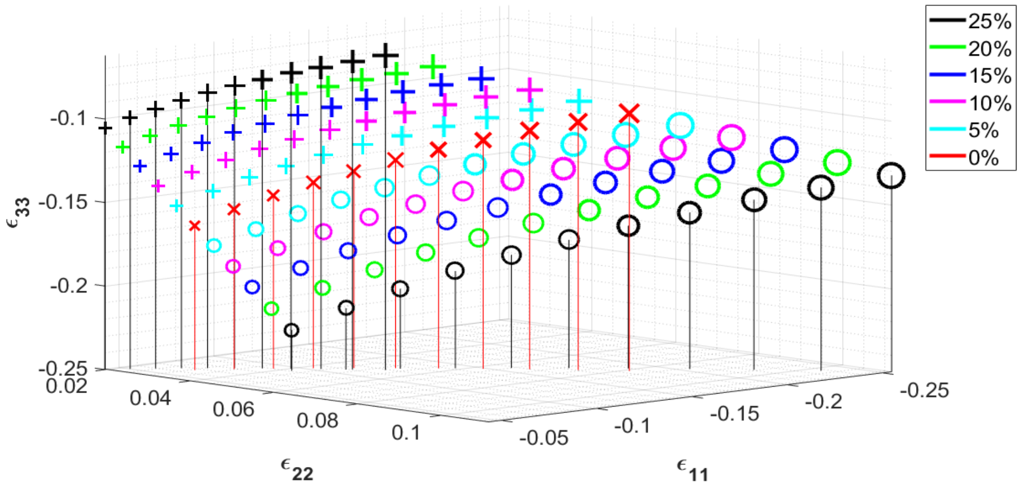

Fig. 2: Response to the uncertainty in the parameter e_d; for clarity, only selected stems are depicted.

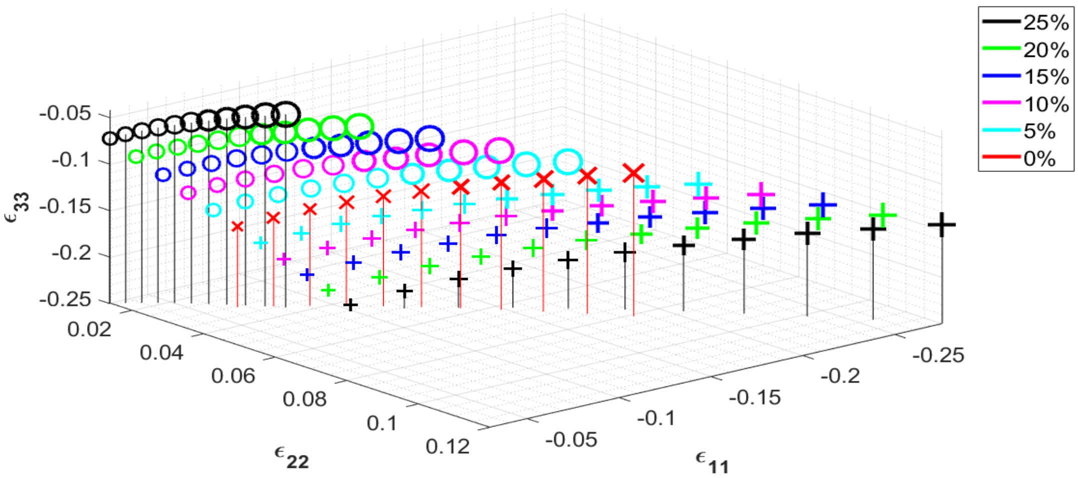

Fig. 3: Response to the uncertainty in the parameter e(0); for clarity, only selected stems are depicted.

As a representation of parameters which have only a small impact on the limit state of compaction after cycles of loading and unloading, the response to the perturbed parameter $\bar{c}$, see (5), is shown in Fig. 4. Although it is not illustrated here, a similar behaviour could also be observed for the parameters e_c, α, f_0, and β. We would observe that the range of the limit strain values is not significantly affected by the perturbation of these parameters.

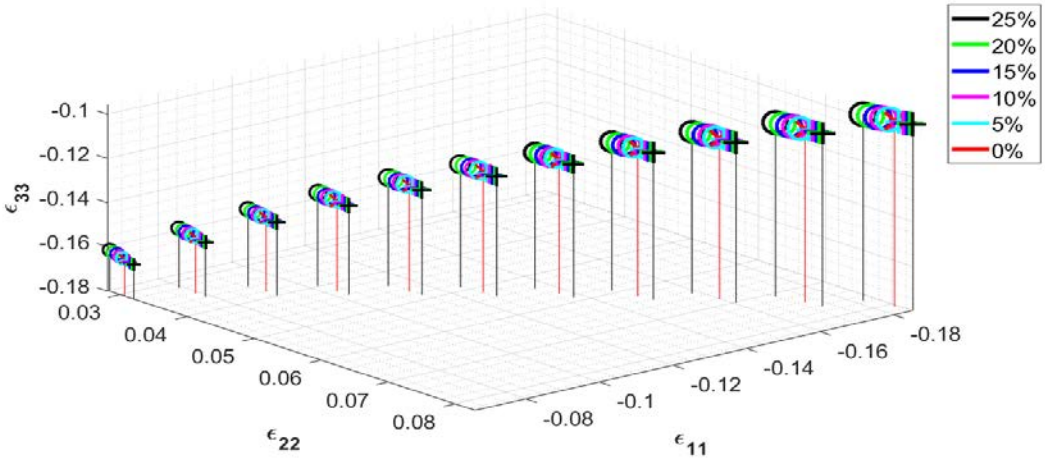

Fig. 4: Response to the uncertainty in the parameter $\bar{c}$; for clarity, only selected stems are depicted.

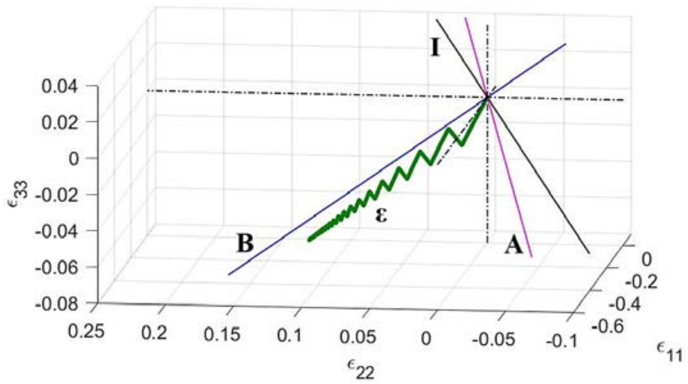

Fig. 5: Ratchetting (25 cycles) of $\boldsymbol{\varepsilon}(t)$ with the anisotropic settings $s_{11} = -1.5, s_{22} = -0.5, s_{33} = -1$.

3. Conclusions

The limit state of the strain ratchetting in the above-mentioned settings of anisotropic loading and unloading with different rates between the three principal stresses was calculated. Weak changes of the response to the parameters e_c, α, $\bar{c}$, f_0, β were detected in all the investigated settings of anisotropy. The uncertainty in the other parameters, i.e. a, e_d, $e(0)$, have a significant influence on the model response. The presented uncertainty analysis might help the experimenter to identify factors that deserve special attention.

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