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COMPRESSOR CASCADE POSITIVE AND NEGATIVE STALL INCIDENCE ANGLE CORRELATION MODELLING USING ARTIFICIAL NEURAL NETWORKS

Kovář P. [*], **Fürst J.** [**]

Abstract: *The analysis of the flows by computational fluid dynamics becomes useful design and optimization method during recent years. Despite the advances in the computational power but it could be still very demanding. Therefore empirical models are commonly used as a main tool for design and prediction of basic performance of axial compressor cascades. The empirical correlations are derived from experimental data obtained from two-dimensional measurements. Unfortunately, sufficient amount of data is available only in cases of well-known airfoils as e.g. NACA 65-series or C.4 profiles. Thus, there is en effort to find a similar relation which will serve in the same manner for another family of the airfoils. The construction of such correlations using artificial neural networks is proposed in this work. In contrast to standard deep neural network, the proposed neural network is built using higher order neural units.*

Keywords: Compressor cascade, CFD, loss correlations, neural networks, machine learning.

1. Introduction

When low Mach number flow occur, C.4 circular-arc profiles or NACA 65-series are sufficient. According to Aungier (2003), when the flow is accelerated to high subsonic, transonic even to low supersonic velocities, well-known profiles as DCA (Double-circular arc) and MCA (Multi-circular arc) perform better. CD (Controlled diffusion) airfoils introduce suitable family of the airfoils for subsonic and transonic cascade applications. Their power lies in their construction and optimization for these applications. The shape construction employs the concept of shaping the blade beyond the point of peak suction of the surface velocity such that the diffusion rate and associated suction boundary layer results in minimum loss for the airfoil section Salunke and Channiwala (2010). On the other hand they provide relatively tight range of acceptable incidence angles Aungier (2003).

In the real operation, it can be very tricky to reach stable design conditions that can have fatal consequences, especially in some complex engineering applications, e.g., nuclear reactor cooling by an axial compressor as a part of the secondary system. Thus it is necessary to ensure reliable operation of the device when off-design conditions occur. Klesa (2021) introduced new family of the airfoils working in wide range of acceptable incidence angles which should outperform well-known NACA 65-series and even their performance should be comparable with CD airfoils.

Flow analysis by means of computational fluid dynamics (CFD) could be still very demanding, thus empirical correlations are commonly used as a tool for design and prediction of axial compressor cascade performance. Our previous research Kovář and Fürst (2022a) was aimed to total pressure loss correlation modelling at the design point of the cascade for the new airfoils family in order to accelerate the design of compressor cascade. When off-design points have to be investigated, applicable incidence angles bounds knowledge which allows us more comprehensive study is necessary. Present contribution deals with positive and negative stall incidence angle correlation modelling for the new family of airfoils using artificial neural network (ANN).

[*] Ing. Patrik Kovář: Center of Aviation and Space Research, Czech Technical University in Prague, Jugoslávských partyzánů 1580/30; 160 00, Prague; CZ, Patrik.Kovar@fs.cvut.cz

[**] Prof. Ing. Jiří Fürst, Ph.D.: Department of Technical Mathematics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Karlovo náměstí 13; 121 35, Prague; CZ, Jiri.Furst@fs.cvut.cz

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2. Objective statement

The loss coefficient is fairly constant over a range of incidence angles near the design incidence angle i^*, but increases rapidly when the cascade is operated too far from the design incidence angle. It is conventional practice to define the limits of low-loss operation by the positive and negative stall incidence angles, i_s and i_c, where the loss coefficient becomes twice the minimum loss coefficient PL. According to Aungier (2003), low-loss working ranges for low-speed cascades can be designated as R_c and R_s

$$R_c = \alpha^* - \alpha_c = i^* - i_c; \quad R_s = \alpha_s - \alpha^* = i_s - i^*. \tag{1}$$

Since α is a function of β_1, these equations are not directly usable in a performance analysis. But since $\beta_1 = \alpha + \gamma$, they can be applied by a simple iterative solution as described in Aungier (2003)

$$\alpha_c = \alpha^* - 9 + \left(1 - \left(\frac{30}{\beta_{1,c}}\right)^{0.48}\right)\frac{\theta}{4.176}; \quad \alpha_s = \alpha^* + 10.3 + \left(2.92 - \frac{\beta_{1,s}}{15.6}\right)\frac{\theta}{8.2}. \tag{2}$$

According to Johnsen and Bullock (1965) effect of Mach number on positive and negative stall incidence angles can be involved as

$$i_c = i^* - \frac{R_c}{1 + 0.5M_1'^3}; \quad i_s = i^* + \frac{R_s}{1 + 0.5(K_{sh}M_1')^3}; \quad K_{sh} \leq 1. \tag{3}$$

As it can be seen in equations above, the dependence between stall incidence angles i_c and i_s, camber angle Θ and parameters of the flow β_1, α^*, α_c, α_s, i^*, M_1' is strongly non-linear that is a suitable task for ANN.

3. Methodology

The information in the individual neurons is processed in two different ways Gupta et al. (2013). Synaptic operation, the first, contains weights of the synapse which represents storage of knowledge and thus the memory for previous knowledge. Somatic operation is the second and provides various mathematical operations such as thresholding, non-linear activation, aggregation, etc. Neural output of the unit $\widetilde{y}$ is scalar as it is indicated in Figure 1 (left). Let us assume N-th order neural unit, then neural output can be written as Gupta et al. (2004).

$$\widetilde{y} = \sigma(s); \quad s = w_0x_0 + \sum_{i=1}^{n} w_ix_i + \sum_{i=1}^{n}\sum_{j=i}^{n} w_{ij}x_ix_j + \cdots + \sum_{i_1=1}^{n}\cdots\sum_{i_N=i_{N-1}}^{n} w_{i_1i_2\ldots i_n}x_{i_1}x_{i_2}\ldots x_{i_n}, \tag{4}$$

where $x_0 = 1$ denotes threshold and n stands for length of input feature vector.

Shallow neural network is consisted of three neural units with quadratic polynomial synaptic operation. Two neurons are in the first layer and bipolar sigmoid activation function $\sigma(\cdot)$ is prescribed. The third neuron is in the output layer with linear activation. Neural network architecture is schematically shown in the Figure 1 (right). For the error propagation, multilayer backpropagation algorithm, described in Gupta et al. (2004), is employed in this work.

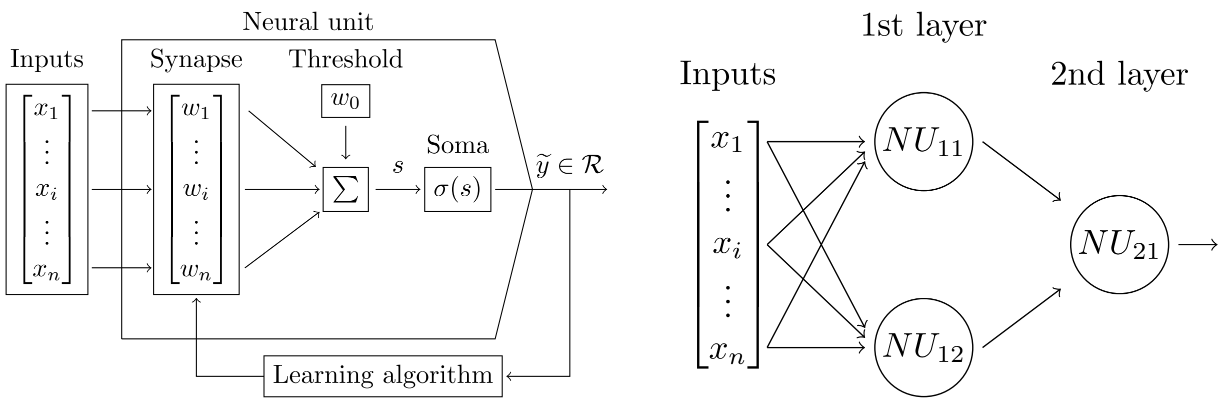

Fig. 1: Neural network: single neural unit (left); shallow neural network (right).

Since desired outputs are known, machine learning is called as supervised learning which is the task of learning a function that maps input to an output represented with cost function $\vec{e}$. As we could see, the neural output is strongly dependent on the neural memories represented by vector of the weights $\overrightarrow{W}$. Batch Levenberg-Marquardt algorithm for weights updating is employed in this work. See Kovář and Fürst (2022a), Kovář and Fürst (2022b) for more details of the learning methodology.

In order to obtain training data set for neural network and replace experimental measurement, various numerical simulations with different geometrical setups and inlet boundary conditions were performed as e.g. in Bublík et al. (2023). Design incidence angle was found through number of simulations as the flow angle with minimum pressure loss as described in Aungier (2003). Positive and negative stall incidence angles i_c, i_s was found as angles where pressure loss PL reaches twofold.

4. Results

In order to avoid network overtraining, the data set was divided into three parts. 80% of samples belongs to training subset and the rest was equally distributed to validating and testing subsets. Learning rate μ was set to $\mu = 0.4$. Following figures present modelling of the positive stall incidence angle i_s. Training processes for i and i_c incidence angles correlations modelling were performed similarly. Referring to Fig. 2 (left), 85 epochs was sufficient to neural network got learned with testing error $2.49e{-}3$. A comparison of the function learned by ANN and estimation performed by equations (1 - 3) is shown in the Fig. 2 (right).

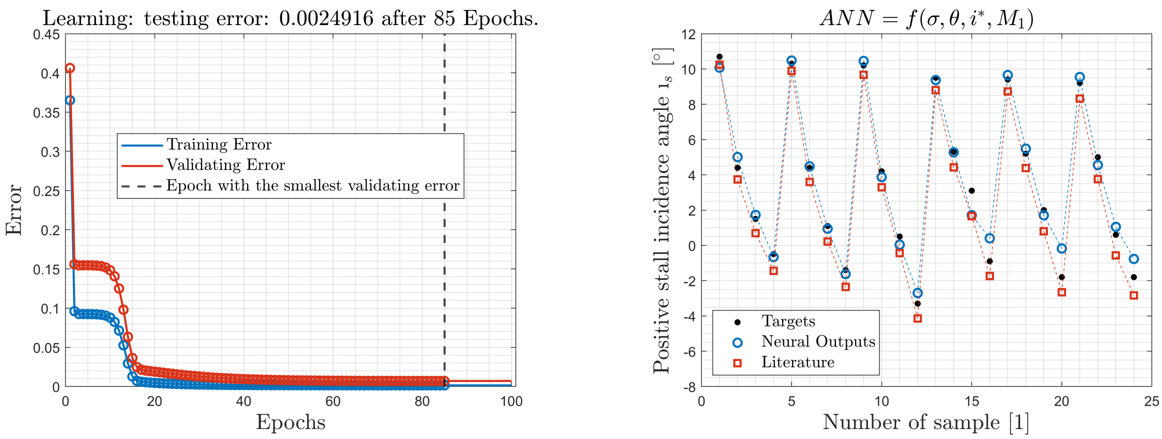

Fig. 2: Results: progress of the learning (left); ANN results compared to literature correlations (right).

Finally, there is a test of neural network predictions performed on the cascade geometry which was not included in the training data set, specifically the cascade with $\kappa_1 = 40°$ and $\theta = 30°$. A comparison between design, positive and negative stall incidence angles predictions obtained by aforementioned methods is shown in Fig. 3. As it can be seen at first sight, estimation of incidence angles by correlations from the literature is completely beyond the data from CFD unlike the approach by ANN.

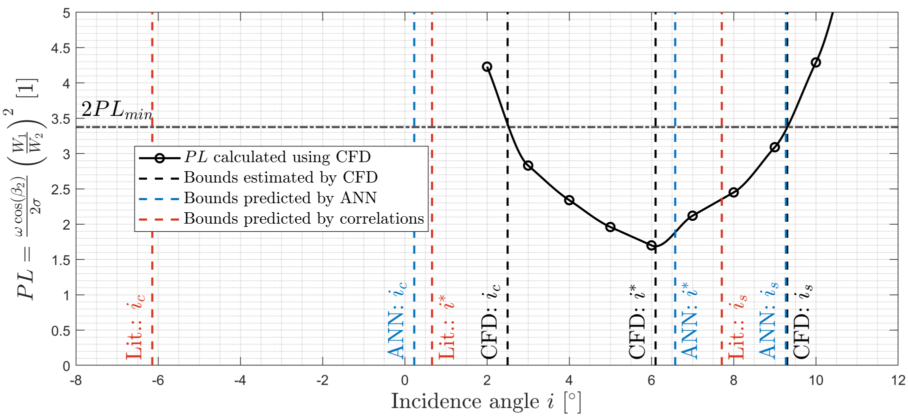

Fig. 3: Results: comparison of incidence angles obtained by literature correlations and ANN approach.

Deviations performed by artificial neural network and the correlations from the literature compared to data obtained by CFD are listed in the Table 1, both measured with relative error related to the value from CFD.

Tab. 1: Absolute and relative error comparison.

Method	CFD	ANN		Literature	
Quantity	Value $[^\circ]$	Value $[^\circ]$	E_R $[\%]$	Value $[^\circ]$	E_R $[\%]$
i_c	2.50	0.23	90.81	-6.14	345.75
i^*	6.10	6.58	7.82	0.66	89.12
i_s	9.30	9.27	0.32	7.71	17.09

5. Conclusions

An approach for correlation modelling was presented in this paper. Based on input data set obtained by CFD simulations, artificial neural network was learned to predict positive and negative stall incidence angle of axial compressor cascade designed with the new family of the airfoils. Results of the learning are compared against empirical model by Johnsen and Bullock (1965). Approximation using ANN outperformed available correlation model from the literature as it can be seen in the Table 1.

Further work should aim to axial compressor cascade performance predicting at off-design points which will require much larger training data set. Another way could be the inverse task, i.e., assemble the model estimating airfoil geometry based on desired flow parameters.

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