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ADVANCED STATISTICAL EVALUATION OF FATIGUE DATA OBTAINED DURING THE MEASUREMENT OF CONCRETE MIXTURES WITH VARIOUS WATER-CEMENT RATIO

Seitl S.[*], Benešová A.[**], Blasón S.[***], Miarka P.[†], Klusák J.[‡], Bílek V.[§]

Abstract: *The Basquin's law is usually applied for the evaluation of fatigue properties of civil engineering materials. For materials like concrete, some researchers recommended applying the Weibull model. In this contribution, Basquin's law, Castillo-Canteli model and Kohout-Věchet model were applied for the advanced statistical description of S−N curves (Wöhler curve). For the application of the models, the experimental data measured on reference concrete mixtures used for the development of self-healing concrete at the Faculty of Civil Engineering, TU Ostrava. Fitting data of applied models were compared and analyzed.*

Keywords: Concrete, fatigue, S-N curve, Kohout-Věchet model, Castillo-Canteli model, Basquin model.

1. Introduction

Fatigue failure in materials, structural components, and structures is a factor that should be considered in engineering design to ensure safety and reliability during service life. This is even more critical in structures that are cyclically loaded (Lee & Barr 2004). It has been found that the cyclic stress causes failure of the materials before the strength limit is reached, that is why this failure is such dangerous and many studies deal with this issue (Seitl et al. 2019). Fatigue performance of materials is described by fatigue curves, which represent the dependence of fatigue lifetime on the number of cycles to failure, also known as *S-N* (Wöhler 1860) curves. The fib Model Code 2010 proposes an empirical approximation of the Wöhler curve for concrete under tensile stresses by defining the number of cycles to failure as

$$\log(N_f) = 12(1 - S_{max}) \tag{1}$$

However, this approximation does not consider the influence of the lower load level. Another approximation for characterizing that behavior is proposed in Cornelissen & Reinhardt (1984) based on a large number of fatigue tests, given as

$$\log(N_f) = 14.81 - 14.52 S_{max} + 2.79 S_{min} \tag{2}$$

Nevertheless, the fatigue response of individual materials like concrete is very different. Therefore, other models that describe the fatigue of the material more accurately have been postulated (Basquin 1910, Kohout and Věchet 2001, Castillo et al. 2008, Castillo and Canteli 2009) over the time.

The aim of this contribution is to calculate and discuss the appropriate model suitable for reference structural concrete with focus on the self-healing one. The experimental results obtained from concrete notched beams under fatigue flexural loading are compared and analyzed.

[*] doc. Ing. Stanislav Seitl, Ph.D.: Institute of Physics of Materials of the Czech Academy of Sciences, ŽIŽKOVA 513/22, 616 00 Brno, CZ, seitl@ipm.cz

[**] Anna Benešová, Institute of Physics of Materials of the Czech Academy of Sciences, ŽIŽKOVA 513/22, 616 00 Brno, CZ, and Faculty of Civil Engineering, Brno University of Technology, Veveří 331/95, 602 00 Brno, CZ, 211599@vutbr.cz

[***] MSc. Sergio Blasón, Ph.D., The Bundesanstalt für Materialforschung und -prüfung, Unter den Eichen 87 12205 Berlin, GE, sergio.blason-gonzalez@bam.de

[†] Ing. Petr Miarka, Ph.D.: Institute of Physics of Materials of the Czech Academy of Sciences, ŽIŽKOVA 513/22, 616 00 Brno, CZ, miarka@ipm.cz

[‡] doc. Ing. Jan Klusák, Ph.D.: Institute of Physics of Materials of the Czech Academy of Sciences, ŽIŽKOVA 513/22, 616 00 Brno, CZ, klusak@ipm.cz

[§] doc. Ing. Vlastimil Bílek, Ph.D., Faculty of Civil Engineering, VŠB Technical University of Ostrava, Ludvíka Podéště 1875/17, 708 33 Ostrava-Poruba CZ, vlastimil.bilek@vsb.cz

2. Methods

The most traditional and simplistic function for describing the dependency of fatigue lifetime on stress level $\Delta\sigma(N)$ is the Basquin's function (Basquin 1910):

$$\Delta\sigma\,(N) = aN^b \tag{3}$$

Kohout-Věchet (2001) modified the Basquin's model to be able to describe the whole range of the fatigue lifetime, including the low-cycle and high-cycle fatigue regions. Parameters B, C are calculated from σ_1 (value of stress for the number of cycles $N = 1$) and σ_∞ (permanent fatigue limit).

$$\Delta\sigma(N) = a\left(\frac{(N+B)C}{N+C}\right)^b \tag{4}$$

Castillo and Canteli (2009) developed the model based on the compatibility between the probability distributions describing the scatter in terms of the lifetime and the loading conditions, that is $F(N;\Delta\sigma)$ and $F(\Delta\sigma;N)$, respectively. They achieved a general solution, only satisfied by the Weibull and Gumbel extreme value distributions for minima, for describing the *S-N* field. These distributions are defined by the location- (λ) and scale-parameter (δ), while the Weibull distribution additional requires the shape-parameter (β).

Castillo-Canteli Weibull model:

$$Pf(N,\Delta\sigma) = 1 - exp\left\{-\left[\frac{(logN - B)(log\Delta\sigma - C) - \lambda}{\delta}\right]^\beta\right\} \tag{5}$$

Castillo-Canteli Gumbel model:

$$Pf(N,\Delta\sigma) = 1 - exp\left\{-\left[\frac{(logN - B)(log\Delta\sigma - C) - \lambda}{\delta}\right]\right\} \tag{6}$$

3. Materials

Three different concrete mixtures were prepared for these fatigue tests. The first is a common reference concrete with water-cement ratio w/c = 0.50 and a cement dosage of 450 kg/m^3. At the age of 28 days, this concrete shows cube compressive strength of 45.0 ± 0.4 MPa and flexural strength of 7.1 ± 0.2 MPa. The second concrete contains also 450 kg cement but w/c= 0.40, which means this concrete represents the boundary between usual and high-performance concrete (HPC). Its compressive strength was 69.1 ± 1.0 MPa and its flexural strength 8.2 ± 0.5 MPa. The third concrete is HPC with w/c = 0.30 and cement dosage 450 kg/m^3. In this case, compressive and flexural strengths were 78.6 ± 0.7 MPa and 9.6 ± 0.5 MPa, respectively.

Fatigue tests were performed by using three-point bending load on concrete beams (80 × 40 × 240 mm) with notch (*a/W*=0.1). Stress ratio $R = 0.1$ and 10 Hz frequency were applied along the tests. The number of cycles corresponding to the fatigue limit of cement composites was set to 2×10^6 cycles.

Particular concrete composites differ in the ratio of water and cement, marked as w/c.

Reference common concrete w/c = 0.3

Tab. 1: Model parameters of reference common concrete w/c = 0.3

Basquin $1 - 2\cdot10^6$		Basquin $10^3 - 2\cdot10^6$		Kohout-Věchet				Castillo-Canteli model (Weibull solution)				Castillo-Canteli model (Gumbel solution)			
a	10.096	a	4.427	a	4.427	σ_∞ [MPa]	1.88	σ_∞ [MPa]	1.86	δ	0.40	σ_∞ [MPa]	1.95	δ	0.26
b	-0.054	b	-0.052	b	-0.052	σ_1 [MPa]	3.60	B	5.25	λ	1.36	B	6.11	λ	1.28
						σ_c [MPa]	1.97	C	0.62	β	4.12	C	0.67		

Tab. 2: Coefficients of determination of reference common concrete w/c = 0.3

	Basquin $1 – 2·10^6$	Basquin $10^3 – 2·10^6$	Kohout- Věchet	Castillo-Canteli model (Weibull solution)	Castillo-Canteli model (Gumbel solution)
R^2 (N ≥ 1)	0.874	0.485	0.933	-	-
R^2 (N ≥ 10^3)	0.693	0.731	0.729	0.067	-0.62

Concrete with w/c = 0.4

Tab. 3: Model parameters of concrete with w/c = 0.4

Basquin $1 – 2·10^6$		Basquin $10^3 – 2·10^6$		Kohout- Věchet				Castillo-Canteli model (Weibull solution)				Castillo-Canteli model (Gumbel solution)			
a	3.378	*a*	5.245	*a*	5.245	σ_∞ [MPa]	1.84	σ_∞ [MPa]	0.79	δ	2.04	σ_∞ [MPa]	0.79	δ	1.00
b	-0.038	*b*	-0.074	*b*	-0.074	σ_1 [MPa]	3.13	*B*	0.00	λ	10.09	*B*	0.00	λ	12.58
						σ_c [MPa]	1.86	*C*	-0.23	β	1.64	*C*	-0.24		

Tab. 4: Coefficients of determination of concrete with w/c = 0.4

	Basquin $1 – 2·10^6$	Basquin $10^3 – 2·10^6$	Kohout-Věchet	Castillo-Canteli model (Weibull solution)	Castillo-Canteli model (Gumbel solution)
R^2 (N ≥ 1)	0.773	-1.812	0.871	-	-
R^2 (N ≥ 10^3)	0.601	0.778	0.764	0.533	0.354

Concrete with w/c = 0.5

Tab. 5: Model parameters of concrete with w/c = 0.5

Basquin $1 – 2·10^6$		Basquin $10^3 – 2·10^6$		Kohout – Věchet				Castillo-Canteli model (Weibull solution)				Castillo-Canteli model (Gumbel solution)			
a	3.025	*a*	3.768	*a*	3.768	σ_∞ [MPa]	1.31	σ_∞ [MPa]	0.78	δ	2.59	σ_∞ [MPa]	1.30	δ	0.46
b	-0.047	*b*	-0.065	*b*	-0.065	σ_1 [MPa]	2.84	*B*	0.00	λ	7.01	*B*	5.59	λ	1.88
						σ_c [MPa]	1.34	*C*	-0.25	β	2.51	*C*	0.26		

Tab. 6: Coefficients of determination of concrete with w/c = 0.5

	Basquin $1 – 2·10^6$	Basquin $10^3 – 2·10^6$	Kohout – Věchet	Castillo-Canteli model (Weibull solution)	Castillo-Canteli model (Gumbel solution)
R^2 (N ≥ 1)	0.859	0.500	0.880	-	-
R^2 (N ≥ 10^3)	0.737	0.810	0.806	0.713	0.460

When comparing the coefficients of determination among the models describing the fatigue performance of the analyzed materials, the fittings postulated by Castillo-Canteli are not suitable for the description when the variable $\Delta\sigma$ is taken as the reference driving force, see Tab 2, 4 and 6. In most cases, the index of dispersion R^2 is below 0.5 (only one case shows 0.713). Therefore, an accurate description of the fatigue life of the material cannot be assumed. The best fit for civil engineering materials like concrete is shown by the Kohout-Věchet model.

4. Conclusions

From the presented pilot experimental fatigue research work in the field of self-healing materials the following conclusions can be drawn:

The coefficient of determination of the Kohout–Věchet model gives rise high values in comparison to the Basquin's law (10^3–$2{\cdot}10^6$), which is the most accurate for this region. Additionally, the Kohout-Věchet model has the advantage of describing the low-cycle regime and sets the fatigue limit of the material. Therefore, it is a suitable model for materials like concrete.

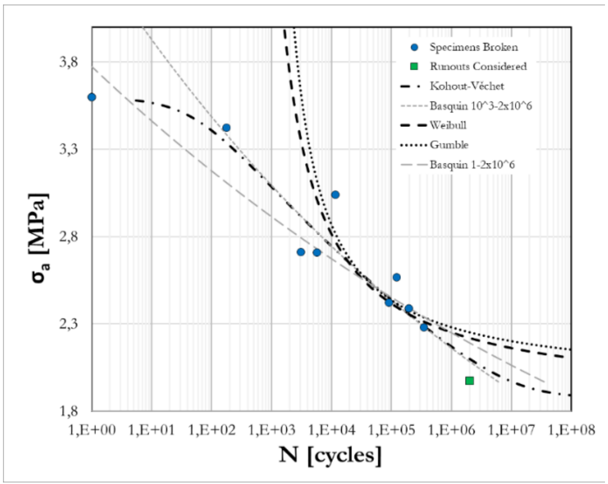

Fig. 1:S-N curves and fit for percentile curves for p=50% by Basquin, Kohout-Věchet and Castillo-Canteli models for reference common concrete w/c = 0.3.

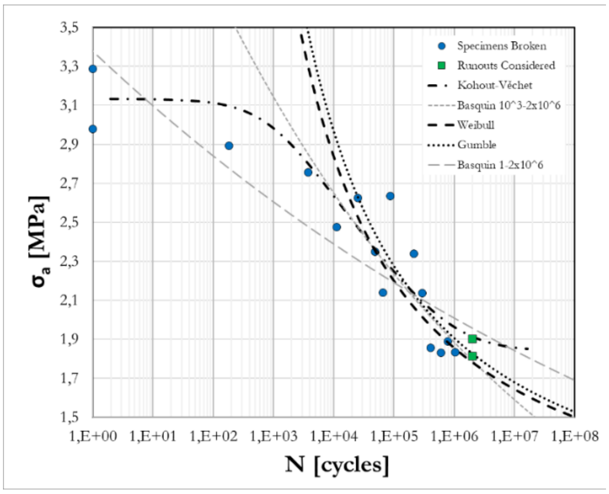

Fig. 2:S-N curves and fit for percentile curves for p=50% by Basquin, Kohout-Věchet and Castillo-Canteli models for concrete with w/c = 0.4.

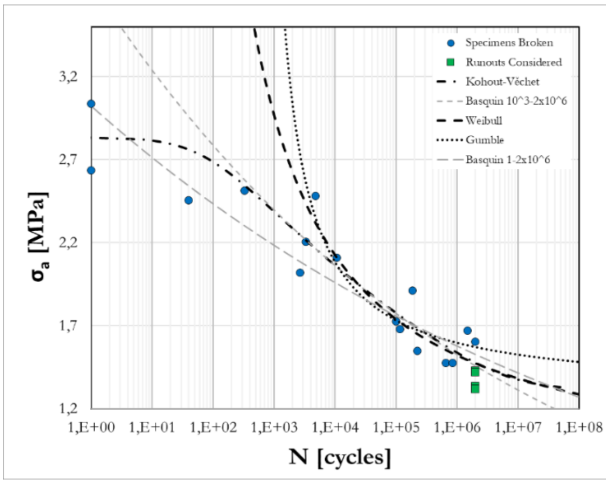

Fig. 3:S-N curves and fit for percentile curves for p=50% by Basquin, Kohout-Věchet and Castillo-Canteli models for concrete withw/c = 0.5.

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References

Barbosa, J.F., Correia, J.A., Freire Júnior, R., Zhu, S.-P. and De Jesus, A.M. (2019) Probabilistic S-N fields based on statistical distributions applied to metallic and composite materials: State of the art. *Advances in Mechanical Engineering*, 11(8), 168781401987039.

Basquin, O.H. (1910). The exponential law of endurance tests. *Proceedings of ASTM*. Vol. 10(II). pp. 625-630.

Castillo, E. and Fernández-Canteli, A. (2009) *A unified statistical methodology for modeling fatigue damage*. Springer.

Castillo, E., Ramos, A., Koller, R., López Aenlle, M. and Fernández-Canteli, A. (2008) A critical comparison of two models for assessment of fatigue data. *International Journal of Fatigue*, 30, pp. 45-57.

Cornelissen H. and Reinhardt H. (1984) Uniaxial tensile fatigue failure of concrete under constant-amplitude and programme loading. *Magazine of Concrete Research*, 36 (129), pp. 216-226.

fib Model Code 2010. Federation internationale du beton (2013) fib Model Code for Concrete Structures. Ernst & Sohn. Berlin.

Kohout. J. and Vechet. S. (2001) A new function for fatigue curves characterization and its multiple merits. *International Journal of Fatigue*. 23(2). pp. 175-183.

Lee. M.K. and Barr. B.I.G. (2004) An overview of the fatigue behavior of plain and fibre reinforced concrete. *Cement & Concrete Composites*, 26, pp. 299–305.

Seitl, S., Miarka, P. and Bílek, V. (2019) Fatigue and fracture mechanical properties of selected concrete for subtle precast structural elements. *MATEC Web of Conferences*, 310, p. 00033.

Wohler. A. (1860). Versuche über die Festigkeit von Eisenbahnwagen-Achsen, Zeitschrift Bauwesen, 10; English summary (1867), Engineering. Vol.4. pp.160-161.