27/28th International Conference
ENGINEERING MECHANICS 2022
Milovy, Czech Republic, May 9 –12, 2022
Paper **#13,** pp. 289–292, doi: 10.21495/51-2-289

SEMI-PROBABILISTIC ASSESSMENT OF CONCRETE BRIDGE USING POLYNOMIAL CHAOS AND GRAM-CHARLIER EXPANSIONS

Novák L.*, **Novák D.****

Abstract: *The paper presents an application of novel methodology for design and assessment of structures using semi-probabilistic approach. The significant limitation of existing semi-probabilistic methods is an assumption of Lognormal probability distribution of structural resistance described by the first two central statistical moments. In this study, we investigate the possibility of Gram-Charlier expansion described by the first four central moments, which are efficiently obtained directly from Polynomial Chaos Expansion. The proposed methodology is applied for determination of load-bearing capacity of existing structure – the post-tensioned concrete bridge. The obtained results show the importance of the higher statistical moments and their influence on design value of resistance.*

Keywords: Semi-probabilistic Approach, Gram-Charlier Expansion, Polynomial Chaos Expansion, Concrete Bridge.

1. Introduction

The paper is focused on semi-probabilistic assessment of a concrete bridge using advanced probabilistic techniques. The selected existing bridge is represented by a highly computationally demanding non-linear finite element model (NLFEM), which reflects non-linearity of concrete (fracture mechanics) as well as construction process (three construction phases). Due to extreme computational burden of each numerical simulation, it is not feasible to perform fully probabilistic assessment of the bridge by standard Monte Carlo simulation technique and semi-probabilistic approach was adopted in this study.

In the semi-probabilistic approach for NLFEM (Val et al., 1997; Novák and Novák, 2021), the resistance of structure R is separated and the design value R_d that satisfies safety requirements is evaluated, instead of the direct calculation of failure probability $p_f = P(Z(\mathbf{X}) < 0)$. The typical formula for the estimation of R_d, assuming a Lognormal distribution of R, is

$$R_d = \mu_R \cdot \exp(-\alpha_R\, \beta\, v_R)\,, \tag{1}$$

where μ_R is the mean value, v_R is the coefficient of variation (CoV) and α_R represents sensitivity factor derived from First Order Reliability Method (FORM); the recommended value is $\alpha_R = 0.8$ according to Eurocode 1990. The target reliability index is dependent on consequence classes (dependent on a type of structure), e.g., β for the ultimate limit state, moderate consequences of failure and a reference period of 50 years is set at $\beta = 3.8$ according to the Eurocode.

Note that from a pure probabilistic point of view, the whole process represents the estimation of a quantile satisfying the given safety requirements under the prescribed assumption of Lognormal distribution. In this paper, we investigate novel methodology for semi-probabilistic approach based on approximation of cumulative distribution function (CDF) of R by Gram-Charlier expansion (G-C). The G-C is completely determined by the first four statistical moments obtained here efficiently from Polynomial Chaos Expansion (PCE) approximation of the original quantity of interest (QoI) – typically resistance of structure.

* Ing. Lukáš Novák, Ph.D.: Institute of Structural Mechanics, Brno University of Technology, Veveří 331/95; 602 00, Brno; CZ, Novak.L@fce.vutbr.cz

** Prof. Ing. Drahomír Novák, DrSc.: Institute of Structural Mechanics, Brno University of Technology, Veveří 331/95, 602 00, Brno, CZ, Novak.D@fce.vutbr.cz

2. Polynomial Chaos and Gram-Charlier Expansions

It is important to remember that PCE is an expansion to construct a random variable with a probability distribution identical to the quantity of interest Y. It is not an expansion of its probability density function f_Y (PDF) or CDF F_Y. If one is interested in the distribution of Y^{PCE}, it is possible to utilize one of the well-known classical distribution expansions such as Gram-Charlier expansion (G-C). Let us assume that it is possible to write probability distribution of Y as a perturbation of Gaussian PDF ϕ. Once the QoI is normalized to be zero-mean and unit-variance, it is possible to write the Gram-Charlier approximation of CDF in the terms of its higher central moments (skewness γ_Y and kurtosis κ_Y:

$$F_Y = \Phi(y) - \left[\frac{\gamma_Y}{3\sqrt{2!}} H_2(y) + \frac{\kappa_Y - 3}{4\sqrt{3!}} H_3(y)\right] \phi(y). \tag{2}$$

where $H_n(y)$ are probabilists' Hermite polynomials of nth order and $\Phi(y)$ represents standard Gaussian CDF. In practical computation, the moments are estimated from samples and the estimation is highly sensitive to outliers.

It is typically not feasible to get higher statistical moments by Monte Carlo simulation due to its computational demands. Fortunately, it is possible to get statistical moments analytically in case of PCE, which represents the output variable Y as a function g^{PCE} of an another random variable ξ called the germ with given distribution and representing the original model $g(X)$ via polynomial expansion. A set of polynomials, orthonormal with respect to the probability distribution of the germ, are used as a basis of the Hilbert space of all real-valued random variables of finite variance. In the case of $\boldsymbol{X}$ and $\boldsymbol{\xi}$ being vectors containing M random variables, the polynomial $\Psi(\boldsymbol{\xi})$ is multivariate and it is built up as a tensor product of univariate orthogonal polynomials:

$$Y = g(\boldsymbol{X}) = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^M} \beta_{\boldsymbol{\alpha}} \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}), \tag{3}$$

where $\boldsymbol{\alpha} \in \mathbb{N}^M$ is a set of integers called the *multi-index*, $\beta_{\boldsymbol{\alpha}}$ are deterministic coefficients and $\Psi_{\boldsymbol{\alpha}}$ are multivariate orthogonal polynomials. From a statistical point of view, truncated PCE is a simple linear regression model with intercept. Therefore, it is possible to use ordinary least square (OLS) regression in order to estimate $\beta_{\boldsymbol{\alpha}}$.

A specific form of PCE and orthogonality of polynomials allows for a powerful and efficient post-processing. Once a PCE approximation is created, it is possible to obtain statistical moments of QoI directly from. Specifically, the first statistical moment (mean value) is obtained as the first deterministic coefficient of the expansion

$$\mu_Y = \langle Y^1 \rangle = \beta_0. \tag{4}$$

Further it is possible to obtain the variance $\sigma_Y^2 = \langle Y^2 \rangle - \mu_Y^2$ as a sum of all squared deterministic coefficients except the intercept, which represents the mean value:

$$\sigma_Y^2 = \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \beta_\alpha^2. \tag{5}$$

Higher statistical central moments, skewness γ_Y (3^{rd} moment) and kurtosis κ_Y (4^{th} moment), are generally obtained as follows:

$$\gamma_Y := \frac{1}{\sigma^3}\mathbb{E}[(Y - \mu_Y)^3] = \frac{1}{\sigma^3} \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \sum_{\substack{\beta \in \mathcal{A} \\ \beta \neq 0}} \sum_{\substack{\gamma \in \mathcal{A} \\ \gamma \neq 0}} \langle \Psi_\alpha \Psi_\beta \Psi_\gamma \rangle \beta_\alpha \beta_\beta \beta_\gamma \tag{6}$$

$$\kappa_Y := \frac{1}{\sigma^4}\mathbb{E}[(Y - \mu_Y)^4] = \frac{1}{\sigma^4} \sum_{\substack{\alpha \in \mathcal{A} \\ \alpha \neq 0}} \sum_{\substack{\beta \in \mathcal{A} \\ \beta \neq 0}} \sum_{\substack{\gamma \in \mathcal{A} \\ \gamma \neq 0}} \sum_{\substack{\delta \in \mathcal{A} \\ \delta \neq 0}} \langle \Psi_\alpha \Psi_\beta \Psi_\gamma \Psi_\delta \rangle \beta_\alpha \beta_\beta \beta_\gamma \beta_\delta \tag{7}$$

3. Application: Concrete Bridge

The proposed methodology is applied for the existing post-tensioned concrete bridge with three spans. The super-structure of the mid-span is 19.98 m long with total width 16.60 m and it is crucial part of the bridge for assessment. In transverse direction, each span is constructed from 16 bridge girders KA-61 commonly used in Czech Republic. Load is applied according to national annex of Eurocode for load-bearing capacity of road bridges by exclusive loading (by six-axial truck).

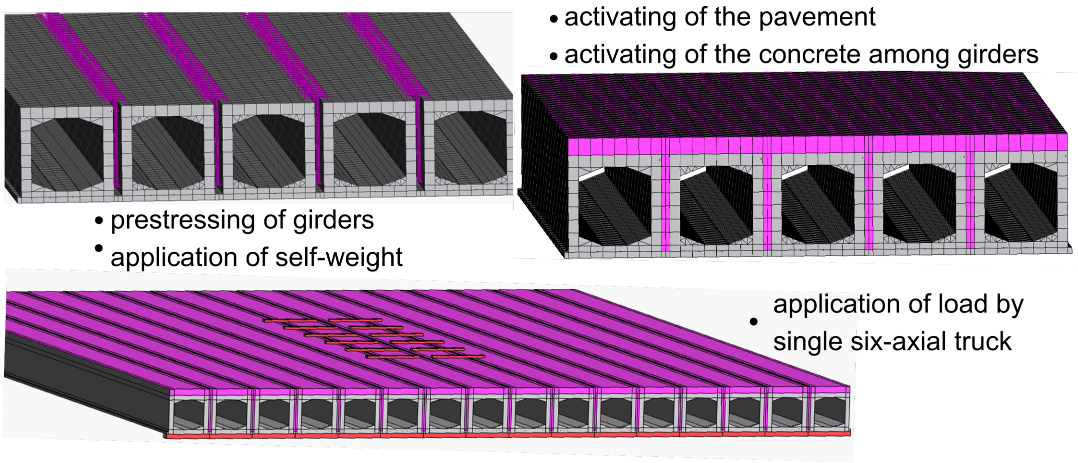

Fig. 1: Three construction phases of the bridge represented by NLFEM.

The NLFEM is created using software ATENA Science based on theory of non-linear fracture mechanics Červenka and Papanikolaou (2008). In order to reflect complex behavior of the bridge, the numerical model contains three construction phases as illustrated in Fig.1. The NLFEM consists of 13,000 elements of hexahedra type in the major part of the volume and triangular 'PRISM' elements in the part with complicated geometry. Reinforcement and tendons are represented by discrete 1D elements with geometry according to original documentation. The numerical model is further analysed in order to investigate the ultimate limit state (ULS) (peak of a load-deflection diagram) in order to determined the load-bearing capacity of the bridge.

The stochastic model contains 4 random material parameters of a concrete C50/60: Young's modulus E ; compressive strength of concrete f_c; tensile strength of concrete f_{ct} and fracture energy G_f. Characteristic values of E, f_{ct}, G_f were determined from f_c according to formulas implemented in the fib Model Code 2010 (fib federation internationale du beton, 2013) (G_f, E) and prEN 1992-1-1: 2021 (f_{ct}). The last random variable P represents prestressing losses with CoV according to JCSS: Probabilistic Model Code (JCSS, 2001). The stochastic model is summarized in Tab. 1. Mean values and coefficients of variation were obtained according to prEN 1992-1-1: 2021 (Annex A) for adjustment of partial factors for materials.

Tab. 1: Stochastic model of the numerical example.

Var.	Mean	CoV [%]	Distrib.	Units
f_c	56	16	Lognormal	[MPa]
f_{ct}	3.64	22	Lognormal	[MPa]
E	36	16	Lognormal	[GPa]
G_f	195	22	Lognormal	[Jm2]
P	20	30	Normal	[%]

The experimental design (ED) contains 30 numerical simulations generated by Latin Hypercube Sampling (LHS). Note that each simulation takes approximately 24 hours and construction of the whole ED took approx. 1 week of computational time. The PCE is created with maximum polynomial order $p = 5$. The whole algorithm of adaptive construction of PCE connects state of art techniques into stand-alone software tool (Novák and Novák, 2018). The design values of resistance R_d are determined as a quantile of distribution of QoI with identified statistical moments and target reliability indices $\beta_{ULS} = 3.8$ according to EN 1990. Additionally, design values are reduced by global safety factor reflecting model uncertainties $\gamma_{R_d} = 1.06$ introduced originally in fib Model Code 2010 (fib federation internationale du beton, 2013).

Obtained probability distribution and corresponding Rd obtained from G-C expansion are compared to standard approach assuming Lognormal distribution in Fig 2. Note that Lognormal distribution in contrast to Gram-Charlier expansion takes only the first two central moments into account, which is beneficial for existing ECoV methods but rather limiting for PCE. As can be seen from the obtained results, G-C lead to the $R_d = 384$ tons and standard assumption of Lognormal distribution to $R_d = 370$ tons. The difference between two design values (corresponding to the identical percentile) is caused by higher statistical moments obtained from PCE.

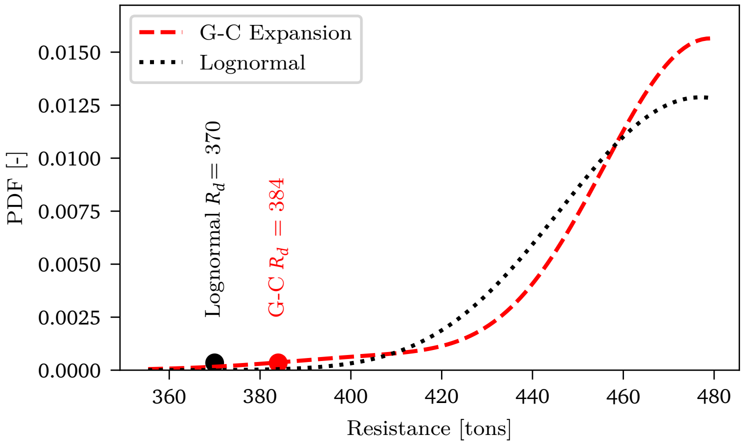

Fig. 2: Three construction phases of the bridge represented by NLFEM.

4. Conclusions

The paper presents pilot numerical results of the novel semi-probabilistic approach based on Gram-Charlier and Polynomial Chaos Expansion. It was shown that higher statistical moments could play significant role in identification of design values of resistance. Therefore, it is beneficial to use the estimation of four statistical moments obtained directly from PCE and use them in G-C expansion in order to construct artificial probability distribution. Such approach represents improvement of the standard methods typically assuming Lognormal distribution of resistance, widely accepted nowadays in codes and recommendations. However, estimation of higher statistical moments is highly sensitive to an accuracy of PCE and thus advanced sampling algorithms should be employed, e.g. recently proposed algorithm focused on accurate estimation of variance (Novák et al., 2021), which will be investigated during the further research.

Acknowledgments

This research was funded by Czech Science Foundation under project No. 22-00774S. Numerical results were partially obtained by the software SEMIP developed with support of Technology Agency of the Czech Republic under project No. TH04010138.

References

Červenka, J. and Papanikolaou, V.K. (2008). Three dimensional combined fracture-plastic material model for concrete. *International Journal of Plasticity*, 24(12), pp. 2192–2220.

fib federation internationale du beton (2013) *fib Model Code for Concrete Structures 2010*. John Wiley & Sons, Berlin, Heidelberg.

JCSS (2001) *JCSS Probabilistic Model Code*. Joint Committee on Structural Safety. ISBN 978-3-909386-79-6.

Novák, L., Vořechovský, M., Sadílek, V. and Shields, M. D. (2021). Variance-based adaptive sequential sampling for polynomial chaos expansion. *Computer Methods in Applied Mechanics and Engineering*, 386, 114105.

Novák, L. and Novák, D. (2018). Polynomial chaos expansion for surrogate modelling: Theory and software. *Beton- und Stahlbetonbau*, 113, pp. 27–32.

Novák, L. and Novák, D. (2021). Estimation of coefficient of variation for structural analysis: The correlation interval approach. *Structural Safety*, 92, 102101.

Val, D., Bljuger, F., and Yankelevsky, D. (1997). Reliability evaluation in nonlinear analysis of reinforced concrete structures. *Structural Safety*, 19(2), pp. 203–217.