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COMPARISON BETWEEN THE CURRENT MODELS OF THE THEORY OF CREEP IN THE LIGHT OF NUMERICAL RESULTS OBTAINED BY TIME HISTORY ANALYSIS OF COMPOSITE SECTIONS, USING INTEGRAL EQUATION OF VOLTERRA

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Abstract: *Abstract: The paper make attempt to give some answers about the differences between the current models of the theory of creep of concrete: Eurocode 2, ACI 209-R2 and Gardener&Lockman models, using the results obtained by time dependent analysis of composite steel-concrete beams. On the basis of the theory of the viscoelastic body of Arutyunian–Trost-Bažant for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time "t", two independent Volterra integral equations of the second kind have been derived. Numerical method based on linear approximation of the singular kernel function in the integral equation is presented. The elastic modulus of concrete $E_c(t)$ is assumed to be constant in time "t".*

Keywords: Creep, Current models, Steel-concrete section, Integral equations of Volterra, Rheology.

1. Introduction

The time-varying behavior of composite steel-concrete members under sustained service loads drawn the attention of engineers who were dealing with the problems of their design more than 75 years (ACI, 2004). Creep and shrinkage have a considerable impact upon the performance of composite beams, causing increased deflection as well as affecting stress distribution. In general, time-dependent deformation of concrete may severely affect the serviceability, durability and stability of structures.

2. Basic equations for determining the creep coefficient according current provisions

2.1. Eurocode 2

The creep (compliance) function proposed by the 1990 CEB Model Code ("CEB-FIP"1991) is given by the relationship (ACI, 2004): $J(t,t_0) = \frac{1}{E_c(t_0)} + \frac{\phi(t,t_0)}{E_c}$, where $\phi(t,t_0) =$ the creep coefficient; and $E_c(t_0)$ and $E_c =$ modulus of elasticity at the age of t_0 and 28 days, respectively. The creep coefficient is evaluated with the following formula: $\phi(t,t_0) = \phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0)$ where: $\phi_{RH} = 1 + \frac{1-RH/100}{0.46(RH/100)^{0.33}}$ - is a factor to allow for the effect of relative humidity on the notional creep coefficient. RH is the relative humidity of the ambient environment in % $\beta(f_{cm}) = \frac{5.3}{(f_{cm}/10)^{0.5}}$ - is a factor to allow for the effect of concrete strength on the notional creep coefficient. $\beta(t_0) = \frac{1}{0.1+(t_0)^{0.2}}$ - is a factor to allow for the effect of concrete age at loading on the notional creep coefficient (for continuous process we consider the function): $\beta(\tau) = \frac{1}{0.1+(\tau)^{0.2}}$. $\beta(t-t_0) = \left[\frac{t-t_0}{\beta_H+(t-t_0)}\right]^{0.3}$ is a function to describe the development of creep with time

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after loading. $\beta_H = 150\left[1+\left(1.2\frac{RH}{100}\right)^{18}\right]\frac{h_0}{100}+250 < 1{,}500$, coefficient depending on the relative humidity (RH in %) and notional member size (h_0 in mm) (ACI, 2004).

2.2. ACI 209R-92

The creep (compliance) function proposed by the ACI 209R-92 model, that presents the total stress dependent strain by unit stress is given by the relationship (ACI, 2004): $J(t,t_0) = \frac{1}{E_{cmt_0}} + \frac{\phi(t,t_0)}{E_{cmt_0}} = \frac{1+\phi(t,t_0)}{E_{cmt_0}}$ where - $\phi(t,t_0)$ is the creep coefficient as the ratio of the creep strain to the elastic strain at the start of loading at the age t_0 (days) and E_{cmt0} is the modulus of elasticity at the time of loading t_0 (MPa), respectively. The creep coefficient is evaluated with the following formula: $\phi(t,t_0) = \phi_u\beta_c(t-t_0)$, where: $\phi(t,t_0)$ is the creep coefficient at the concrete age t due to a load applied at the age t_0; $(t-t_0)$ is the time since application of load; ϕ_u is the ultimate creep coefficient. For the standard conditions in the absence of specific creep date for local aggregates and conditions, the average value proposed for the ultimate creep coefficient ϕ_u is equal to 2,35. For conditions other than standard conditions the value of the ultimate creep coefficient $\phi_u = 2.35$ needs to be modified by six correction factors, depending on particular conditions: where: $\phi_u = 2.35\gamma_c$; and: $\gamma_c = \gamma_{c,t_0}\gamma_{c,RH}\gamma_{c,vs}\gamma_{c,s}\gamma_{c,\psi}\gamma_{c,\alpha}$ $\beta_c(t-t_0) = \left[\frac{(t-t_0)^{0.6}}{10+(t-t_0)^{0.6}}\right]$ - is a function to describe the development of creep with time after loading (ACI, 2004).

2.3. Gardner&Lockman 2000

The creep coefficient $\phi_{28}(t,t_0)$ is the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days (ACI, 2004). So: $J(t,t_0) = \frac{1}{E_{cmt_0}(t_0)} + \frac{\phi_{28}(t,t_0)}{E_{cm28}}$ where: E_{cmt_0} is the modulus of elasticity of concrete at the time of loading t_0; E_{cm28} - is the mean modulus of elasticity concrete at 28 days (MPa); $\frac{1}{E_{cmt_0}(t_0)}$ - represents the initial strain per unit stress at loading. $\phi_{28}(t,t_0)$ gives the ratio of the creep strain since the start of loading at the age t_0 to the elastic strain due to a constant stress applied at a concrete age of 28 days. The 28-day creep coefficient $\phi_{28}(t,t_0)$ calculated using the next formula:

$$\phi_{28}(t,t_0) = \Phi(t_c)\left[2\frac{(t-t_0)^{0.3}}{(t-t_0)^{0.3}+14} + \left(\frac{7}{t_0}\right)^{0.5}\left(\frac{(t-t_0)}{(t-t_0)+7}\right)^{0.5} + 2.5(1-1.086h^2)\left(\frac{(t-t_0)}{(t-t_0)+0.12(V/S)^2}\right)^{0.5}\right].$$

The creep coefficient includes three terms. The first two terms are required to calculate the basic creep, and the third term is for the drying creep. At a relative humidity of 0,96 there is only basic creep .There is no drying creep. $\Phi(t_c)$ is the correction term for the effect of drying before loading. If $t_0 = t_c$; $\Phi(t_c) = 1$. When: $t_0 > T_c$, $\Phi(t_c) = \left[1-\left(\frac{(t-t_0)}{(t-t_0)+0.12(V/S)^2}\right)^{0.5}\right]^{0.5}$; $t_0 =$ age of concrete at loading, (days); $t_c =$ age of concrete when drying starts at the end of moist curing, (days).

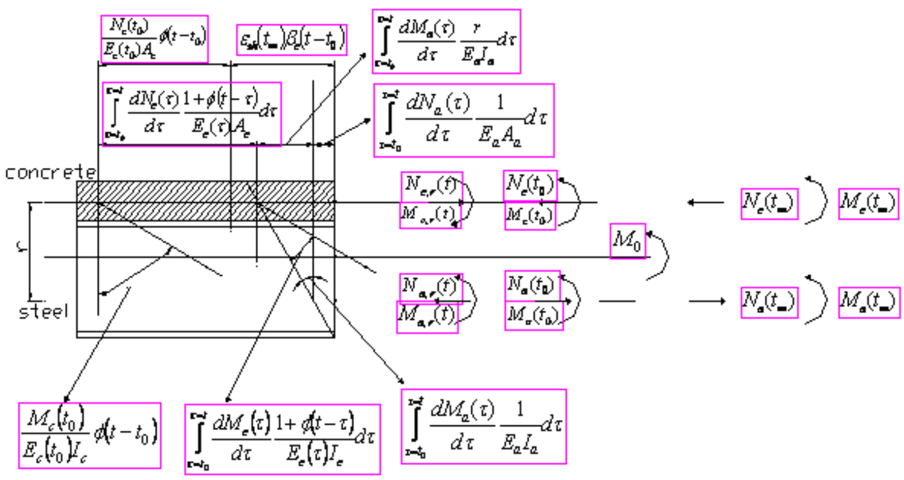

Fig. 1 Mathematical Model of deformations of cross-section

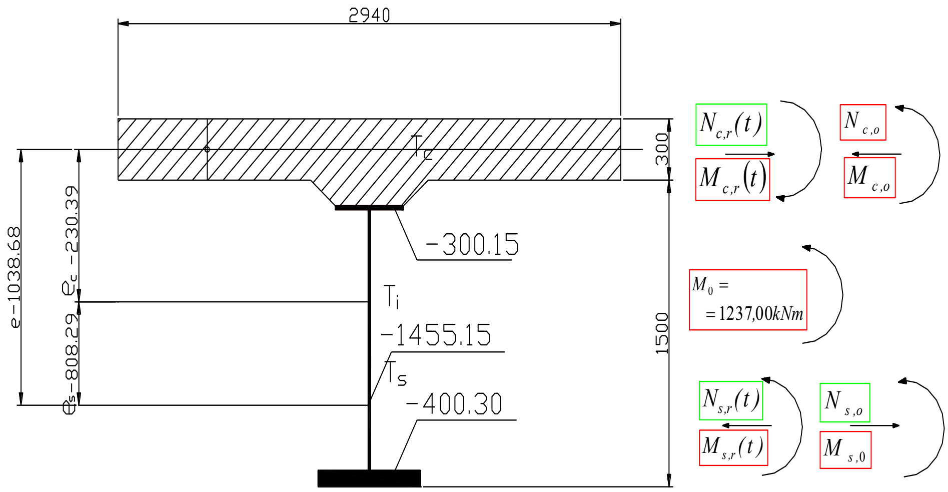

Fig.2. Composite beam cross-section

3. Basic Equations of Equilibrium

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time t = 0 with $N_{c,0}, M_{c,0}$, $N_{a,0}$, $M_{a,0}$ and with $N_{c,r}(t), M_{c,r}(t)$, $N_{a,r}(t)$, $M_{a,r}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete. For a composite bridge girder with: $J_c = \frac{A_c(n\, I_c)n}{A_s I_s} \leq 0.2$, we can write the equilibrium conditions in time t as follows (Fig. 1):

$$\sum N(t) = 0\,;\;\; N_{c,r}(t) = N_{a,r}(t);\;\; \sum M(t) = 0\,;\;\; M_{c,r}(t) + N_{c,r}(t)\, r = M_{a,r}(t) \tag{1}$$

4. Deriving of the generalized mechanic-mathematical models according existing provisions

The mathematical model for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time "t", involves the equation of: equilibrium, compatibility and constitutive relationship. On the basis of the theory of the visco-elastic body of Arutyunian–Trost-Bažant two independent linear Volterra integral equations of the second kind have been derived, for assessment of normal forces $N_{c,r}(t)$ and bending moment $M_{c,r}(t)$ (see equations: 2-3, 4-5, 6-7).

4.1. According EC 2

$$N_{c,r}(t)=\lambda_N\int_{t_0}^{t}N_{c,r}(\tau)\frac{d}{d\tau}\left[1+\varphi\phi_{RH}\beta(f_{cm})\beta(\tau)\beta(t-\tau)\right]d\tau+\lambda_N N_{c,0}\phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0)+\lambda_N N_{sh}\beta_c(t-t_0) \tag{2}$$

$$M_{c,r}(t)=\lambda_M\int_{t_0}^{t}M_{c,r}(\tau)\frac{d}{d\tau}\left[1+\phi_{RH}\beta(f_{cm})\beta(\tau)\beta_c(t-\tau)\right]d\tau+\lambda_M M_{c,0}\phi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t-t_0)-\lambda_M\frac{E_cI_c}{E_aI_a}N_{c,r}(t)r \tag{3}$$

4.2. According ACI 209R-92

$$N_{c,r}(t)=\lambda_N\int_{t_0}^{t}N_{c,r}(\tau)\frac{d}{d\tau}\left[1+2{,}35\gamma_{c5}\beta(\tau)\beta_c(t-\tau)\right]d\tau+\lambda_N N_{c,0}2{,}35\gamma_{c5}\beta(t_0)\beta_c(t-t_0)\,; \tag{4}$$

$$M_{c,r}(t)=\lambda_M\int_{t_0}^{t}M_{c,r}(\tau)\frac{d}{d\tau}\left[1+2{,}35\gamma_{c5}\beta(\tau)\beta_c(t-\tau)\right]d\tau+\lambda_M M_{c,0}2{,}35\gamma_c\beta(t_0)\beta_c(t-t_0)-\lambda_M\frac{E_cI_c}{E_aI_a}N_{c,r}(t)r, \tag{5}$$

4.3. According G &L model

$$N_{c,r}(t)=\lambda_N\int_{t_0}^{t}N_{c,r}(\tau)\frac{d}{d\tau}\left[1+\Phi(t_c)\beta_c(t-\tau)\right]d\tau+\lambda_N N_{c,0}\Phi(t_c)\beta_c(t-t_0) \tag{6}$$

$$M_{c,r}(t)=\lambda_M\int_{t_0}^{t}M_{c,r}(\tau)\frac{d}{d\tau}\left[1+\Phi(t_c)\beta_c(t-\tau)\right]d\tau+\lambda_M M_{c,0}\Phi(t_c)\beta_c(t-t_0)-\lambda_M\frac{E_cI_c}{E_aI_a}N_{c,r}(t)r, \tag{7}$$

5. Numerical example

Since the solutions of structural creep problems with realistic compliance function, mentioned above for creep of concrete provisions (ACI, 2004), cannot be performed analytically, for such type of integral equations of Volterra, numerical solutions, using formulae of quadratures - such as: trapezoidal rules is proposed. The method presented in the previous paragraph is now applied to a simply supported beam, whose cross section is shown in Fig.2, is subjected to a uniform load.

5.1. The following parameters are according EUROCODE 2 model

$$E_c = 3,2.10^4 \ MPa, \ E_a = 2,1.10^5 \ MPa, \ A_c = 8820\,cm^2, \ A_a = 383,25\,cm^2, n = \frac{E_a}{E_c} = 6,56$$

$$I_c = 661500\,cm^4, \ I_a = 1217963,7\,cm^4, \ r_c = 23,039\,cm, \ r_a = 80,829\,cm, \ r = 103,868\,cm,$$

$$A_i = 2453,05\,cm^2, \ I_i = 4536360,758 \ cm^4, M_0 = 1237\,kNm, \ N_{c,o} = 846,60\,kN, \ M_{c,o} = 27,56\,kNm,$$

$$M_{a,o} = 330,13kNm, \ \lambda_N = \left[1+\frac{E_cA_c}{E_aA_a}\left(1+\frac{A_ar^2}{I_a}\right)\right]^{-1} = 0,060545358, \ \lambda_M = \left[1+\frac{E_cI_c}{E_aI_a}\right]^{-1} = 0,922950026$$

5.2 The following parameters are according ACE 209R-92 model

$$E_c = 2,8178.10^4 \ MPa, \ E_a = 2,1.10^5 \ MPa, \ A_c = 8820\,cm^2, \ A_a = 383,25\,cm^2, n = \frac{E_a}{E_c} = 7,452$$

$$I_c = 661500\,cm^4, \ I_a = 1207963,7\,cm^4, \ r_c = 25,407\,cm, \ r_a = 78,463\,cm, \ r = 103,870\,cm,$$

$$A_i = 1566,8248\,cm^2, \ I_i = 4420140,76 \ cm^4, M_0 = 1237\,kNm, \ N_{c,o} = 837,286\,kN, \ M_{c,o} = 24,716\,kNm; M_{a,o} = 338,05kNm, \ \lambda_N = 0,068220902, \ \lambda_M = 0.93155$$

5.3. The following parameters are according Gardner&Lockman model

$$E_c = 2,8014.10^4 \ MPa, \ E_a = 2,1.10^5 \ MPa, A_c = 8820\,cm^2, \ A_a = 383,25\,cm^2, n = \frac{E_a}{E_c} = 7,496$$

$$I_c = 661500\,cm^4, \ I_a = 1207963,7\,cm^4, \ r_c = 25,50\,cm, r_a = 78,37\,cm, \ r = 103,870\,cm,$$

$$A_i = 1560,82\,cm^2, \ I_i = 4415813,859 \ cm^4, M_0 = 1237\,kNm, \ N_{c,o} = 840,50kN, M_{c,o} = 24,7206\,kNm; \ M_{a,o} = 338,386kNm,$$

$$\lambda_N = 0,068593645, \ \lambda_M = 0,931921295$$

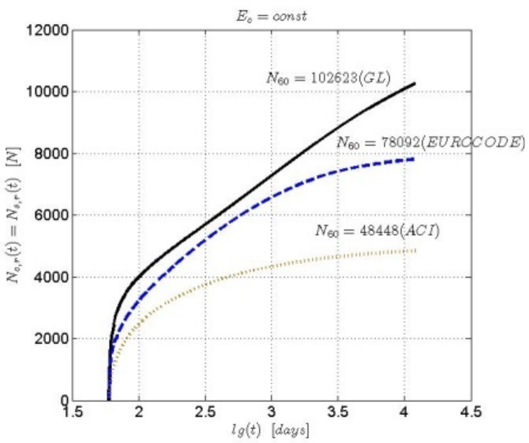

Fig. 3 Normal forces $N_{c,r}(t) = N_{a,r}(t)$

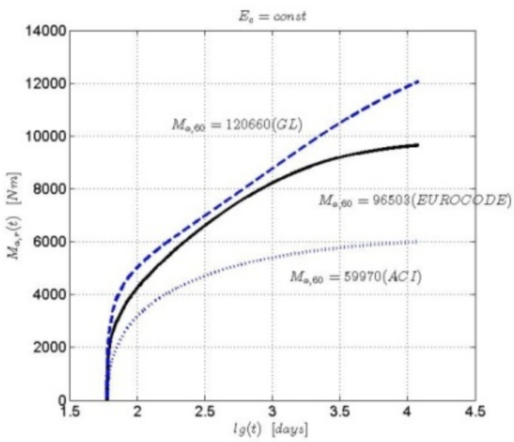

Fig. 4. Bending moments $M_{a,r}(t)$

6. Conclusion

A numerical method using Matlab software platform for time-dependent analysis of composite steel-concrete sections, according EUROCODE-2, ACI 209R-92 and G&L models are presented. In Fig. 3 and 4 it is shown the values of normal forces and bending moments in time $t_\propto = 12060$ days, when loading is applied in: $t_0 = 60$ *days and humidity 80%*. It is observed that GL2000 model in comparison with EC2 provision *overestimates*, and ACI 209 code provisions in comparison with EC2 model code *underestimates*, the influence of creep on time dependent behavior of composite steel-concrete beams.

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