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WINKLER AND HETENYI ELASTIC FOUNDATION APPLIED IN BELT CONVEYORS FOR WHEAT TRANSPORT

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Abstract: *This article deals with experimental and subsequent deterministic and stochastic applications of the Hetenyi and Winkler model of elastic foundation. In practice, it is focused on the task of transporting wheat grains, i.e., loose (particular) material, using belt conveyors with rubber high angle sandwich belts. From the repeated experimental measurements of forces and deflections on the equipment that we developed and designed ourselves, the relationships for the moduli of foundation are compiled.*

Keywords: Belt conveyors, Winkler and Hetényi Elastic foundation, Wheat seed, Measurements, Deterministic and stochastic evaluation, Loose (particular) material.

1. Introduction

Sandwich belt high angle conveyors are used to transport loose (particular) materials on inclined tracks, see Fig. 1. Between the covering (upper) conveyor belt and the lower conveyor belt, loose material is transported thanks to the movement of these two belts and the possible action of additional downforce acting on the gap between. The utmost case of an inclined system suitable for loose material, transported using sandwich belt high angle conveyors, is vertical transport. The amount of loose material transported is defined primarily by the loading width of the conveyor belt B /m/, gap width and lateral compressing force F /N/, caused by the pressure device in the transport rollers and also the type and quality of loose material transported. More detail can be found in (Hrabovský et al., 2021) and (Pozynich, 2019).

For the research of deformation and force dependencies in the transport of loose material, a test device has been designed, see Fig. 1. The interaction of the belt and the transported grain can be conceived as an issue of having a beam rested on elastic foundation, in accordance with the theory of the 2nd order applicable in mechanics. The obtained dependencies can be used for a simpler deterministic solution or a more complex and accurate stochastic solution, e.g., in the design of transport equipment, see (Frydrýšek, 2006) and (Marek et al., 2003).

2. Methods

For the laboratory strain measurement (i.e., height/deflection changes) of the loose material layer compressed by the applied compression force F /N/ caused by the conveyor roller acting on the pressure belt, original experimental testing equipment was designed and developed, see Fig. 1(a). The testing device consists of a steel frame with a conveyor belt track fitted with conveyor rollers. According to the theoretical assumption, the task is axially symmetric. Pressure rollers can slide in a vertical plane. In order to obtain moduli of foundation K /Nm^{-3}/ for simpler single-parameter Winkler model and K_1 /Nm^{-3}/, K_4 /Nm/ for a more complex two-parameter Hetenyi model for elastic foundations, it was necessary to measure the deflection v /m/ on the test equipment used for the conveyor belt, depending on force F acting on the rollers and tensional axial force F_2 /N/ of the belt, whose size was detected by the sensor. When measured, the

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spacing for rollers or forces F was L=0.5 m and the belt width was B=0.4 m. For the application of beams on an elastic foundation, the bending stiffness of the belt has been determined as EJ_{ZT}= 6.318 Nm^2. Due to the complexity of the solved issue and the high novelty of the design, a static solution of the task was chosen for the initial approach. However, the results can also be used for a dynamic task, where the values K, K_1, K_4 should be approximately the same or slightly smaller.

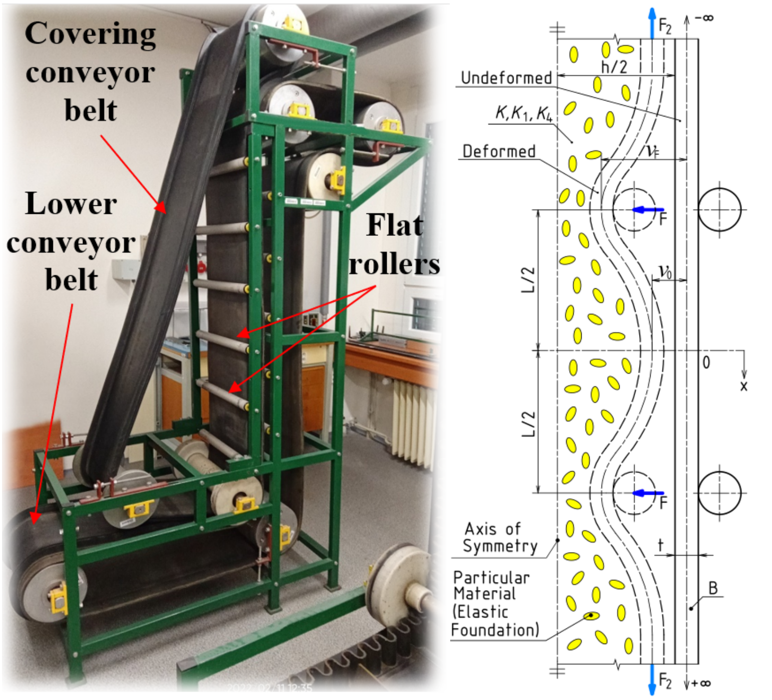

Fig. 1: (a) Sandwich belt high (upper) angle conveyor, (b) Schematic diagram of the experimental compression of the conveyor belt as a beam on an elastic foundation.

Loose material, in this case wheat grain (Triticum aestivum), has a bulk density ρ = 767 kg m^{-3} and granularity (maximum grain size) a_{max} = 6.9 ± 0.4 mm. In the experiment, the wheat was evenly distributed in the gap between the belts up to the height h/2 = 36 mm, see Fig. 1(b). In accordance with Frydrýšek (2006), the relationships listed in Tab. 1 and illustrated in Fig. 1(b) are valid.

Tab. 1: Beam models used on an elastic foundation.

	Winkler	**Hetényi**
Differential equation	$EJ_{\mathrm{ZT}}\frac{d^4v}{dx^4} - \mathrm{F}_2\frac{d^2v}{dx^2} + K\mathrm{B}v = 0$	$(EJ_{\mathrm{ZT}} + \mathrm{B}K_4)\frac{d^4v}{dx^4} - \mathrm{F}_2\frac{d^2v}{dx^2} + \mathrm{B}K_1 v = 0$
Deflection v_{F} /m/ at the point of force F	$\dfrac{\mathrm{F}(\omega_{\mathrm{I}} + e^{-\omega_{\mathrm{R}}\mathrm{L}}f)}{8EJ_{\mathrm{ZT}}\omega_{\mathrm{R}}\sqrt{\frac{K\mathrm{B}}{4EJ_{\mathrm{ZT}}}}}$	$\dfrac{\mathrm{F}(\omega_{\mathrm{IH}} + e^{-\omega_{\mathrm{RH}}\mathrm{L}}f_{\mathrm{H}})}{8EJ_{\mathrm{ZT}}\omega_{\mathrm{RH}}\sqrt{\frac{K_1\mathrm{B}}{4(EJ_{\mathrm{ZT}} + K_4\mathrm{B})}}}$
Deflection v_0 /m/ in the middle between the forces F	$\dfrac{\mathrm{F}e^{-\omega_{\mathrm{R}}\mathrm{L}/2}g}{4EJ_{\mathrm{ZT}}\omega_{\mathrm{R}}\sqrt{\frac{K\mathrm{B}}{4EJ_{\mathrm{ZT}}}}}$	$\dfrac{\mathrm{F}e^{-\omega_{\mathrm{RH}}\mathrm{L}/2}g_{\mathrm{H}}}{4EJ_{\mathrm{ZT}}\omega_{\mathrm{RH}}\sqrt{\frac{K_1\mathrm{B}}{4(EJ_{\mathrm{ZT}} + K_4\mathrm{B})}}}$
Other parameters	$\omega_{\mathrm{R}} = \sqrt{\sqrt{\frac{K\mathrm{B}}{4EJ_{\mathrm{ZT}}}} + \frac{\mathrm{F}_2}{4EJ_{\mathrm{ZT}}}}$ $\omega_{\mathrm{I}} = \sqrt{\sqrt{\frac{K\mathrm{B}}{4EJ_{\mathrm{ZT}}}} - \frac{\mathrm{F}_2}{4EJ_{\mathrm{ZT}}}}$ $f = \omega_{\mathrm{I}}cos(\omega_{\mathrm{I}}\mathrm{L}) + \omega_{\mathrm{R}}sin(\omega_{\mathrm{I}}\mathrm{L})$ $g = \omega_{\mathrm{I}}cos\left(\omega_{\mathrm{I}}\frac{\mathrm{L}}{2}\right) + \omega_{\mathrm{R}}sin\left(\omega_{\mathrm{I}}\frac{\mathrm{L}}{2}\right)$	$\omega_{\mathrm{RH}} = \sqrt{\sqrt{\frac{K_1\mathrm{B}}{4(EJ_{\mathrm{ZT}}+K_4\mathrm{B})}} + \frac{\mathrm{F}_2}{4(EJ_{\mathrm{ZT}}+K_4\mathrm{B})}}$ $\omega_{\mathrm{IH}} = \sqrt{\sqrt{\frac{K_1\mathrm{B}}{4(EJ_{\mathrm{ZT}}+K_4\mathrm{B})}} - \frac{\mathrm{F}_2}{4(EJ_{\mathrm{ZT}}+K_4\mathrm{B})}}$ $f_{\mathrm{H}} = \omega_{\mathrm{IH}}cos(\omega_{\mathrm{IH}}\mathrm{L}) + \omega_{\mathrm{RH}}sin(\omega_{\mathrm{IH}}\mathrm{L})$ $g_{\mathrm{H}} = \omega_{\mathrm{IH}}cos\left(\omega_{\mathrm{IH}}\frac{\mathrm{L}}{2}\right) + \omega_{\mathrm{RH}}sin\left(\omega_{\mathrm{IH}}\frac{\mathrm{L}}{2}\right)$

From the 75 deflection measurements v_F and v_0 for different values of forces F and F_2 and spacing L, it is possible to determine K, K_1 and K_4 using regression nonlinear analysis.

3. Results

In accordance with the previous text, the values of the moduli of foundation according to Winkler and Hetenyi were calculated. The results show a relatively large variance, which is given by the statistical variabilities of random interaction and the wheat grain configuration (i.e. loose material) and loaded rubber belt, see, e.g., Fig. 2. Loose materials generally show a considerable variance of behaviour, see, e.g., (Gelnar and Zegzulka, 2019).

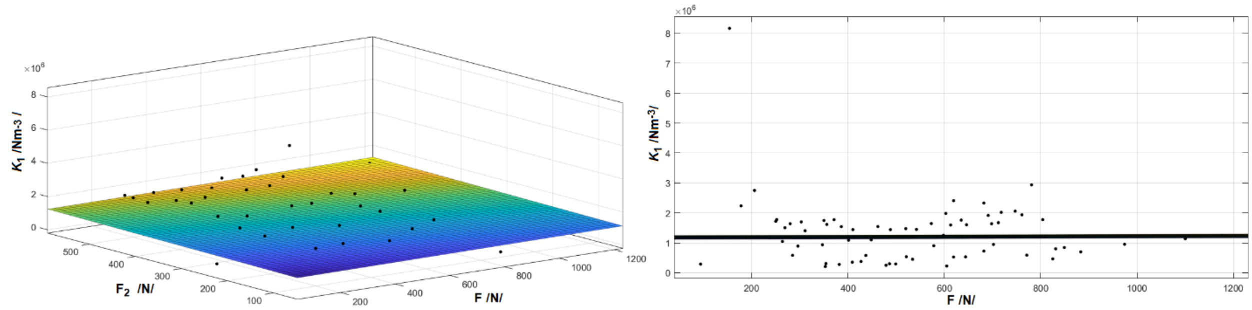

Fig. 2: Dependence of modulus of foundation K_1 on forces F and F_2.

From the statistical evaluation, linear dependence on forces F and F_2 appears to be the most significant, i.e. $K_{(\mathrm{F},\mathrm{F}_2)}$, $K_{1(\mathrm{F},\mathrm{F}_2)}$ and $K_{4(\mathrm{F},\mathrm{F}_2)}$ for the deterministic solution or $K_{stoch(\mathrm{F},\mathrm{F}_2)}$, $K_{1stoch(\mathrm{F},\mathrm{F}_2)}$ and $K_{4stoch(\mathrm{F},\mathrm{F}_2)}$ for the stochastic solution, see Tab. 2.

Tab. 2: Deterministic and stochastic evaluation of moduli of foundation.

	Load-dependent moduli of foundation	
	Winkler (K)	Hetényi (K_1, K_4)
Deterministic approach (mode relationship)	$K_{(\mathrm{F},\mathrm{F}_2)} = a + b\mathrm{F} + c\mathrm{F}_2$	$K_{1(\mathrm{F},\mathrm{F}_2)} = a_1 + b_1\mathrm{F} + c_1\mathrm{F}_2$ $K_{4(\mathrm{F},\mathrm{F}_2)} = a_4 + b_4\mathrm{F} + c_4\mathrm{F}_2$
Constants	$a_1 = 808400$ Nm^{-3} , $b_1 = 372.4$ m^{-3} m^{-3} , $c_1 = 714.8$ m^{-3}	$a_1 = 1105000$ Nm^{-3} , $b_1 = 41.4$ m^{-3} m^{-3} , $c_1 = 235$ m^{-3}, $a_4 = -11.28$ Nm, $b_4 = 0.008022$ m, $c_4 = 0.006284$ m
Stochastic approach	$K_{stoch(\mathrm{F},\mathrm{F}_2)}$ $= hist_{\pm 47.31\%} K_{(\mathrm{F},\mathrm{F}_2)}$	$K_{1stoch(\mathrm{F},\mathrm{F}_2)} = hist_{\pm 34.03\%} K_{1(\mathrm{F},\mathrm{F}_2)}$ $K_{4stoch(\mathrm{F},\mathrm{F}_2)} = hist_{\pm 87.8\%} K_{4(\mathrm{F},\mathrm{F}_2)}$

Thus, a stochastic solution arises from multiplying the mode relationship by the relevant random simulations from bounded normal distributions given by histograms $hist_{\pm 47.31\%}$, $hist_{\pm 34.03\%}$ a $hist_{\pm 87.8\%}$, see, e.g., Fig. 3. The principle of creating these histograms is described in (Marek et al., 2003) and, e.g., $hist_{\pm 47.31\%}$ is given by the bounded normal distribution $\pm 47.31\%$, i.e., multiplier in the interval $\langle 0.5269; 1.4731 \rangle$.

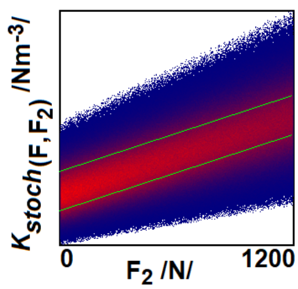

Fig. 3: Stochastic solution according to Winkler.

4. Conclusions

A new experimental device was created to study the properties of loose (particular) materials and their transportation using sandwich belt high angle conveyors. The initial application was focused on the transport of wheat grain. Simple models with beams on elastic foundations according to Winkler and Hetenyi enabled us to determine the stiffness of the moduli of foundation for loose material. Common nonlinear regression methods and statistical methods were used for processing the results of the experiments. The obtained models can be used for the application of traditional older deterministic approaches or younger stochastic solution approaches. The application of an elastic foundation offers us an elegant solution without the need to use the finite element method. The model by Hetenyi appears to be slightly more accurate than that by Winkler.

According to the literature, the application of Hetenyi's model has never been used in any practical solution to issues in mechanics. Therefore, this paper describes its first practical application. Another primacy is the use of an elastic foundation for the transport of wheat grain, which was never used before.

Due to the good results, the application of the Hetenyi and Winkler models appears to be suitable also for other types of loose materials with other grain sizes. Another advantage is the use of dynamic task solutions, where due to the nature of loose materials, the compression modules should be slightly less than the static values obtained by us. Dynamic tasks are solved, for example, by Fedorko et al., (2014), Fries and Hapla (2018) and Hrabovský et al. (2019).

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