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TOPOLOGY OPTIMIZATION BASED ON DEFORMATION ENERGY

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Abstract: *This paper deals with algorithms for topology optimization based on deformation energy. Proper distribution of the mass leads to an increase in the stiffness of the part while maintaining the volume. This algorithm aims to reduce the deformation energy in the component. It is complicated to find a global minimum for the problem. However, the local minimum can help design the shape of the part, and complex shapes can be made by 3D printing.*

Keywords: Topology optimization, Finite element method, Deformation energy.

1. Introduction

Topology optimization based on deformation energy is an efficient and fast algorithm compared to other algorithms such as a genetic algorithms. Displacements of nodes control this algorithm. The first step in each iteration is to solve the equation:

$$\mathbf{U} = \mathbf{K}^{-1}.\mathbf{F}, \tag{1}$$

where $\mathbf{U}$ is the column matrix of nodal displacements, $\mathbf{K}$ is global stiffness matrix and $\mathbf{F}$ is the column matrix of nodal forces. After solving the equation (1) we can get the prepared score for each element. The prepared score is derived from the equation for deformation energy. The deformation energy is multiplied by the volume fraction of the element raised to the power of the parameter p. So $\mathbf{S}^{pre}$ is calculated for all elements from the following equation (2) (Andreassen etal, 2011):

$$S_i^{pre} = w_i^p.\mathbf{U}_i^T.\mathbf{K}_i.\mathbf{U}_i, \tag{2}$$

where w_i is weight of the i-th element, $\mathbf{U}_i$ is vector displacement of i-th element, $\mathbf{K}_i$ is stiffness matrix of i-th element. Mass distribution is controlled by $\mathbf{S}^{pre}$. The following examples highlight the procedures for the 2D case, where $\mathbf{w}$ is a vector of volume fraction, and the 3D case, where $\mathbf{w}$ represents the coefficients by which the stiffness matrix of the element is multiplied.

2. Beam in 2D

The first example is a beam in 2D. In this example, the mass distribution is described by the thickness of each element. The thickness values are continuous in interval ranging from $T_{min} = 0.01mm$ to $T_{max} = 100mm$. And material volume is prescribed to 25% of $V_{max} = T_{max}S$, where S denotes the area of a beam. In the first iteration, the thickness of all elements is equal to $25mm$.
Volume fraction w_i of i-th element is gained from:

$$w_i = \frac{T_i - T_{min}}{T_{max} - T_{min}}. \tag{3}$$

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After $\mathbf{w}$ is gained, $\mathbf{S}^{pre}$ is calculated from equation (2). The next modification is the distribution of scores to neighboring elements. Scores from the element are distributed evenly through each of its nodes to elements that are formed by at least one common node. In the original, element the score is reduced by the corresponding distributed amount. In this case weight of distribution was set to 50 %. This distribution is made for all elements by a matrix $\mathbf{M}$. Matrix $\mathbf{M_{n\times n}}$ is sparse symmetric, n is number of elements. The Matrix $\mathbf{M}$ depends only on the mesh and weight of distribution. Values of the matrix are constant during iteration.

$$\mathbf{S}^{filtered} = \mathbf{M}.\mathbf{S}^{pre} \tag{4}$$

The next step is computation of normalized score denoted $\mathbf{S}^{pren}$ from filtered score $S^{filtered}$. It is normalized in the same manner as thicknesses. Then we modify the normalized score by a parameter k:

$$S_i^{mod} = \begin{cases} (S_i^{pren})^k & S_i^{pren} \leq 0.5 \\ \frac{1}{2}(2 - 2S_i^{pren})^k + 1 & S_i^{pren} > 0.5 \end{cases} \tag{5}$$

Then the new thicknesses $\mathbf{T}$ are created from scores S_i^{mod} by transformation from range $\langle \mathbf{S}_{min}^{mod}, \mathbf{S}_{max}^{mod} \rangle$ to range $\langle T_{min}, T_{max} \rangle$. After this transformation, the volume of used material may differ from volume corresponding to chosen 25% of material. Therefore thicknesses are modified in iterations. The iteration process consists of computing the deviation of material used and material at our disposal. Based on this deviation value, it is decided that material will be added or removed. Elements whose values are on the edge of the interval $\langle T_{min}, T_{max} \rangle$ and can not be increased or decreased are excluded from this loop. From volume deviation through element areas, the thickness change is computed. Moreover, thickness change is distributed on elements based on element score S_i^{mod}. That means elements with a high score undergo greater change than elements with a small score. After the redistribution is performed the values outside the interval $\langle T_{min}, T_{max} \rangle$ are set to limiting values T_{min} and T_{max} correspondingly. The thickness modification procedure is called until the deviation of the used material is smaller than the acceptable value. In this example, the acceptable deviation was 0.5% of material volume.

After new thicknesses $\mathbf{T}$ were assigned, new global stiffness matrix $\mathbf{K}$ is recalculated and another iteration of topology optimization is started from Eq. (1).

The iteration process is stopped after either 250 iterations or if the norm of thicknesses changes between iterations is less than one.

Figures 2a) and 2b) show the effect of the Matrix $\mathbf{M}$. The matrix $\mathbf{M}$ is used to prevent the formation of a checkerboard pattern. With the matrix $\mathbf{M}$, the results are smoother and more refined.

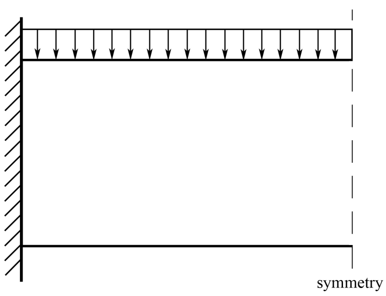

Fig. 1: Beam fixed at both ends, continuous load.

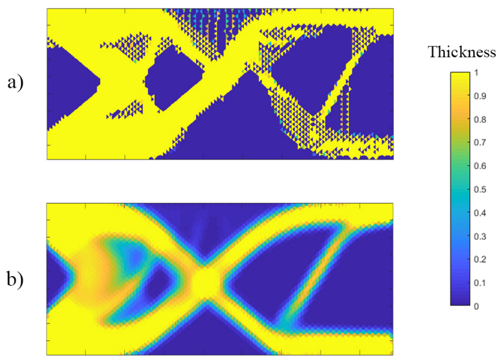

Fig. 2: Mass distributions over elements a) without use of $\mathbf{M}$ *b) with use of* $\mathbf{M}$.

3. Comparison of results with respect to parameters p and k

Although the parameters p and k are constant across the iteration process, they significantly affect the result and speed of convergence. Figure (3) shows converged mass distributions for different combinations of parameter values. For some combinations of parameters p and k the resulting mass distribution in a beam improved the sum of absolute values of nodal displacements. In Figure (4) are these sums divided by the sum of absolute values of nodal displacements for uniform mass distribution (reference beam). Not all combinations of parameters p and k improved beam deflections. These combinations are left blank. Especially combination of parameters $p > 1.5$ with $k > 2$ showed none improvement at all.

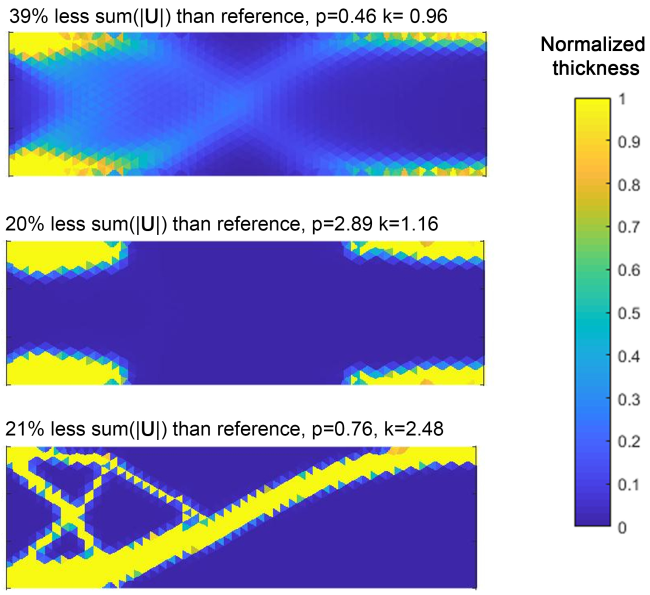

Fig. 3: Thickness distribution for different parameters p and k. The examples shown in this figure are marked in red in Fig. (4).

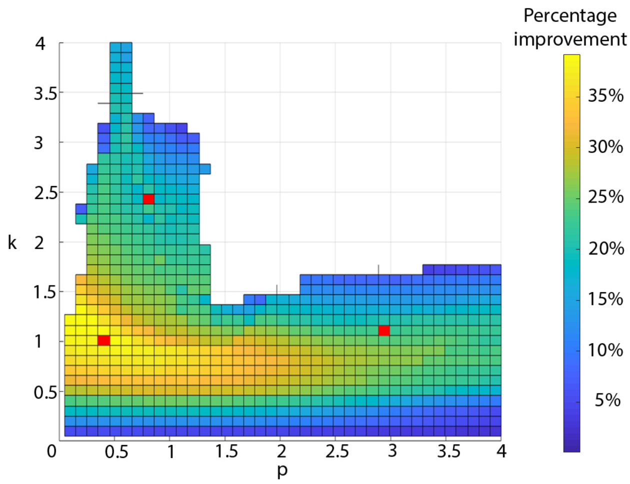

Fig. 4: Percentage improvement in the sum of absolute values of nodal displacements compared to reference beam for different parameters p and k.

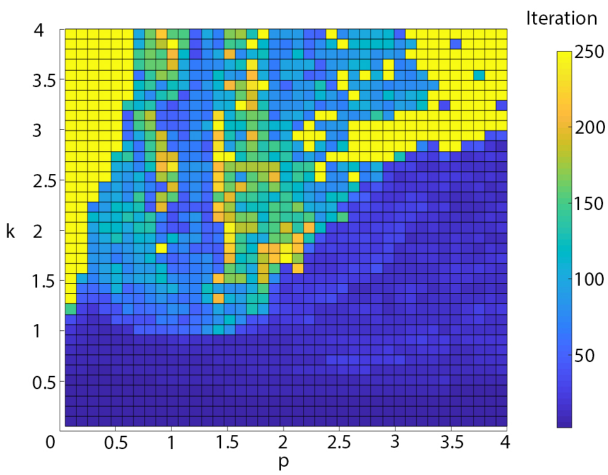

Fig. 5: Effect of parameters p and k on convergence.

4. Beam in 3D

This section shows the procedure for the algorithm in three dimensions. The parameter $p = 2$ in this case. $\mathbf{S}^{pre}$ is calculated from the equation:

$$S_i^{pre} = w_i^2.\mathbf{U}_i^T.\mathbf{K}_i.\mathbf{U}_i, \tag{6}$$

Vector of weights of the elements $\mathbf{w}$ has a length equal to the number of elements. Its components can have values of 0 or 1 only (not exactly 0, but limiting values tending towards zero to prevent matrix $\mathbf{K}$ being singular). The value 1 indicates the presence of mass against 0 which is devoid of any mass. The first iteration $\mathbf{w}$ contains a vector of 1s.

The score components only take values of 0 or 1, so the k parameter was not used in this case. In this case, the score was not modified any further but only filtered by a matrix $\mathbf{M}$:

$$\mathbf{S} = \mathbf{M}.\mathbf{S}^{pre} \,. \tag{7}$$

Then we sorted the vector of scores $\mathbf{S}$. After sorting, we obtained the position of the required number of elements with the highest score. On these positions we change the value in the vector $\mathbf{w}$ to 1. The remaining positions are equal to 0. Then a new iteration is initiated.

Results are shown on a 3D cantilever beam example fixed at one side.

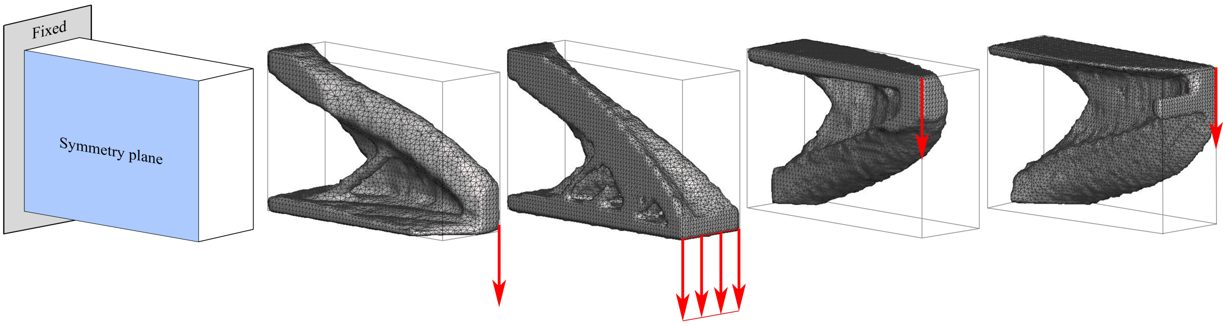

Fig. 6: The first image on the left shows the initial volume of the domain. The beam is fixed at one side. Its prescribed to use only 25% of the initial volume. The following images show results for different loadcases.

5. Conclusions

Results show the importance of the choice of parameters p and k. It is noticeable that there is not even an improvement for some combination of the parameters. Finding the local minimum of the problem can be helpful as a research objective. The shape calculated by the algorithm can inspire engineers. Results are converted to .STL file and can be further modified in CAD software or 3D printed.

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