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MEASURED MET DATA FILTERING FOR CORRECTIONS OF VERY LONG PROJECTILE TRAJECTORIES

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Abstract: *If for any reason it is not possible to use an automated fire control system, then especially in combat conditions it is necessary to use an alternative method of calculating the mean point of impact position. To do this, tabular firing tables (TFT) are used together with the corresponding type of artillery meteorological message. This case is extremely common in practice. So far, we have demonstrated the effectiveness of filtering measured met data using generalized reference height of the projectile trajectory Y_{CR} (characteristics of the digital FIR filter) for effective estimation of ballistic values of met parameters, which are outputs from corresponding filters (wind vector, air virtual temperature, air pressure, air density). The relationships for Y_{CR} are derived assuming an approximation of the measured met data by a single linear function. This assumption is not suitable for very long artillery projectile trajectories, when their vertex (summit) is in the tropopause or even in the lower stratosphere. In this paper, I present a follow-up theory of filtering based on the assumption that the measured met data are approximated by a suitable linear spline function. Derived relationships are also suitable for use in field and combat conditions.*

Keywords: Exterior ballistic, Non-standard (perturbed) projectile trajectory, Green's function, Weighting factor function, Digital FIR filter.

1. Introduction

We have been studying the same theme since 2013 during which our motivation has not changed: It follows from the analysis of artillery fire errors, that approximately two-thirds of the inaccuracy of indirect artillery fire is caused by inaccuracies in the determination of met parameters included in the error budget model (STANAG 4635). Consequently, it is always important to pay close attention to the problems of including the actual met parameters in ballistic calculations. The following met parameters μ are primarily utilized: wind vector **w** (w_x – range wind, w_z – cross wind), air virtual temperature T_v, air pressure p, air density ρ and speed of sound a. In the following, we will denote their measured deviations from the corresponding standard courses (functions) as Δμ.

This paper only deals with problems relating to unguided projectiles (including Base Burn projectiles) without propulsion system for the sake of clarity of the solved problems. The proposed theory can be easily modified not only for Rocket Assisted Projectiles and, unguided rockets but also for illuminating and cargo projectiles and for the passive section of projectile trajectory with terminal guidance.

The relation (Kovalenko and Shevkunov, 1975, Molitz and Strobel 1963) holds for the calculation of ballistic values ($\Delta\mu = \{w_x, w_z, \Delta T_v, \Delta\rho\}$) – Fig. 1

$$\Delta\mu_B = \int_0^1 \Delta\mu(\eta) \cdot g(\eta) \cdot d\eta, \tag{1}$$

where
$\Delta\mu = \mu - \mu_{STD}$ – absolute deviation of met parameter μ
μ - measured or model values
μ_{STD} – standard values.
A similar relationship applies to relative deviations $\delta\mu = \Delta\mu/\mu_{STD}$ (Cech and Jevicky 2016 and 2019)).
g(η) – corresponding normed Green's function (normed impulse response function) for which is true

$$g(\eta) = \frac{dr(\eta)}{d\eta}, \tag{2}$$

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$r(\eta)$ – weighting factor function WFF (normed step response function), $r(\eta_{j=0}) = 0$, $r(\eta_{j=n}) = r(1)$.

This Duhamel's convolutional integral has three possible interpretations (Cech 2019, Cech and Rozehnal 2022):

- in a purely mathematical sense, it is a solution of a given differential equation at a given time $t = t_{PI}$, where t_{PI} is a time of projectile flight to the mean point of impact for standard trajectory.
- it is usually interpreted as a formula for calculating the weighted average. To calculate the arithmetic average, $g(\eta) = 1$ is chosen.
- formula describing the operation of a *Low-pass digital Finite Impulse Response (FIR) filter*. It is therefore an application of signal processing procedures in exterior ballistics. This interpretation makes it possible to correctly understand the true nature of the so-called *ballistic values* of individual met parameters μ. These are therefore the values at the output of the assigned digital FIR filter. Green's function is then referred to as the *impulse response function of the FIR filter*.

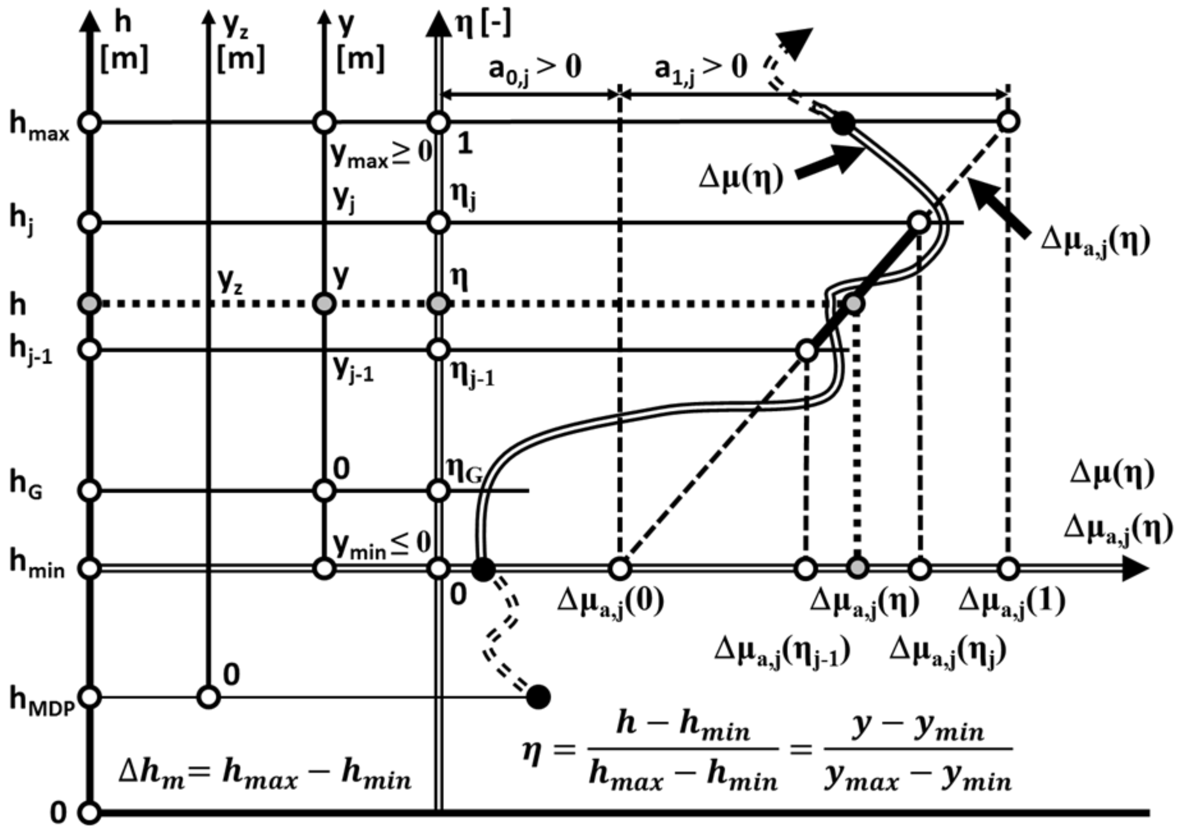

Fig. 1: Approximation of the measured course $\Delta\mu(\eta)$ of the met parameter μ using a linear function on interval $\eta \in \langle \eta_{j-1}, \eta_j \rangle$, $j = 1, 2, \ldots, n$, $\eta_{j=0} = 0$, $\eta_{j=n} = 1$.

2. Mathematical model

The basis of other considerations is the approximation of the measured course $\Delta\mu(\eta)$ using a linear spline – Fig. 1:

$$\Delta\mu_{a.j}(\eta) = a_{0,j} + a_{1,j} \cdot \eta; \quad j = 1,2,\ldots,n; \; n \geq 1 \,. \tag{3}$$

The continuity conditions of a linear spline have a shape, Fig. 1

$$\Delta\mu_{a.j-1}(\eta_{j-1}) = \Delta\mu_{a.j}(\eta_{j-1}); \quad j = 2,3,\ldots,n \,. \tag{4}$$

The following then applies to the approximation of the relation (1) using (4)

$$\Delta\mu_B \approx \sum_{j=1}^{n} \Delta\mu_{B,j} \ , \tag{5}$$

where

$$\Delta\mu_{B,j} = \int_{\eta_{j-1}}^{\eta_j} \Delta\mu_{a,j}(\eta) \cdot g(\eta) \cdot d\eta = r(\eta_j) \cdot \Delta\mu_{a,j}(\eta_j) - r(\eta_{j-1}) \cdot \Delta\mu_{a,j}(\eta_{j-1}) - a_{1,j} \cdot \Delta S_j \ , \tag{6}$$

$$\Delta S_j = S(\eta_j) - S(\eta_{j-1}); \qquad S(\eta) = \int_0^{\eta} r(\eta') \cdot d\eta' \ , \tag{7}$$

$S(\eta_{j=0}) = 0$ and $S(\eta_{j=n}) = S(1)$.

Substitution was used to derive the relations (6) and (7) (Cech V. and Jevicky J., 2019)

$$\eta \cdot g(\eta) = \frac{d(\eta \cdot r(\eta))}{d\eta} - r(\eta) \ . \tag{8}$$

After substituting the relation (6) into the relation (5) and using the relations (4) and after the adjustment, we obtain the following relation

$$\Delta\mu_B \approx r(1) \cdot \Delta\mu_{a,n}(1) - \sum_{j=1}^{n} a_{1,j} \cdot \Delta S_j \ . \tag{9}$$

We adjust the relation (9) using the relation (7) into the *final form*

$$\Delta\mu_B \approx \Delta\mu_{B,n}(1) + \Delta(\Delta\mu_{B,n}) \ , \tag{10}$$

where

$$\Delta\mu_{B,n}(1) = r(1) \cdot \Delta\mu_{a,n}(1) - a_{1,n} \cdot S(1) = a_{0,n} \cdot r(1) + a_{1,n} \cdot [r(1) - S(1)] \ , \tag{11}$$

$$\Delta(\Delta\mu_{B,n}) = \sum_{j=1}^{n-1} (a_{1,j+1} - a_{1,j}) \cdot S(\eta_j) \ . \tag{12}$$

The relation (10) changes to the traditional form (Cech and Jevicky 2019; Cech and Rozehnal 2022) when choosing n = 1, then $\Delta(\Delta\mu_{B,n}) = 0$. From the relationship thus modified, the relationship for the *generalized* reference height of the projectile trajectory Y_{CR} (Cech and Jevicky 2016 and 2019) is derived.

In the case of *traditional standardization* (Kovalenko and Shevkunov, 1975; Cech and Jevicky 2016, 2017 and 2019) will apply r(1) = 1 and then Y_{CR} goes into a *traditional relationship* Y_R (Kovalenko and Shevkunov 1975; Cech and Jevicky 2014, 2017 and 2019).

In the case of *exact norm effect* (r(1) = 0) (Cech and Jevicky 2016, 2017 and 2019; Cech and Rozehnal 2022), the relation (11) degenerates into a shape $\Delta\mu_{B,n}(1) = - a_{1,n} \cdot S(1)$.

For practical use of the relation (10), in most cases n = 2 or 3 will suffice.

For each firing table altitude $h_G = h_{FT}$, Fig. 1, projectile and fuze type, initial (muzzle) velocity v_0, angle of departure Θ_0 or vertex (summit) of trajectory y_s and met parameter μ, it will be necessary to tabulate the value of r(1) and the S (η), $\eta \in \langle 0, 1 \rangle$, function, or its approximation - see more (Cech and Jevicky 2014, 2017 and 2019)).

The relation (10) can be used without modifications, if the met data will be specified in the met messages METCMQ and METGM - for more details (Cech and Jevicky 2019; Cech and Rozehnal 2022).

If met data from met messages of the type of a Meteo - 11 (meteo - average) (Kovalenko and Shevkunov 1975) will be used, relation (10) must be modified. The adjustment procedure is evident from the derivations given in (Cech and Jevicky 2017 and 2019).

Met data in met messages are given in the y_z coordinate above the met station, which is at the altitude h_{MDP}, Fig.1. For relevant transformations, see (Cech and Jevicky 2017).

3. Conclusions

In the following period, a methodology for using relationship (10) in practice will be developed. Subsequently, the relevant characteristics (r(1), S(η)) for the most important types of projectiles will be tabulated. The relevant tables and graphs will be part of the corresponding annexes to the tabular firing tables (STANAG 4119).

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