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STABILITY OF BEAMS UNDER UNIFORM LOADING *q* COMBINED WITH END MOMENTS M_a AND M_b

Baláž I.[*], Koleková Y.[**], Partov D.[***]

Abstract: *Calculation of the critical moments M_{cr} of the beams under various combinations of the uniform loading q and the end moments. Solution of the differential equations is presented in the graphical form. Two diagrams enable to obtain the factors C_1 and C_2, which depend on the parameters $\overline{M}$ and ψ. Approximate formula for calculation of the critical moments depending on three factors C_1, C_2, C_3 and three dimensionless parameters κ_{wt}, ζ_g, ζ_j was derived in (2000a). The values of the factors C_1, C_2, C_3 for basic loadings of the beams with the various combinations of the boundary conditions in vertical and horizontal bending and torsion for the monosymmetric I-sections were published in (2000b). They were later accepted for Eurocode (2007) and many national annexes. The presented results are valid for the beams simply supported with double symmetric I-sections. They may be used also for the continuous beams being slightly on the safe side. Numerical examples show that our results equal to the exact results of commercial computer programs. The procedure gives quickly practically exact results and therefore it is efficient tool for designers*

Keywords: Stability of beam in bending, Critical moment, Uniform loading combined with end moments, Double symmetric I-section.

1. Application of proposed procedure

Numerical example 1:

Input values: beam with span L = 6m simply supported in vertical bending ($k_y = 1$), horizontal bending ($k_z = 1$) and torsion ($k_w = 1$) with rolled section IPE 200 is loaded by uniform loading q = 5 kN/m acting on the surface of the upper flange (z_g = + 100 mm). The values of the end moments are M_a = – 9,6 kNm and M_b = 0 kNm (Fig.1). E = 210 GPa, G = 81 GPa, S 235, f_y = 235 MPa, γ_{M0} = 1,0, γ_{M1} = 1,0. Cross-sectional properties of IPE 200 – DIN 1025, Part 5 (03/1994): h = 200 mm, b = 100 mm, t_f = 8,5 mm, t_w = 5,6 mm, buckling curve "*a*", cross-section Class 1, I_y = 1 943 cm^4, $W_{pl,y}$ = 220,64 cm^3, $M_{pl,y,Rk}$ = 51,85 kNm, I_z = 142,4 cm^4, I_w = 12 746 cm^6, I_t = 6,846 cm^4. For the double symmetric section, the parameter of section symmetry z_j = 0 mm, the factor C_3 is not necessary. $M_{Ed,max}$ = 17,955 kNm is in the section x = 3,3 m.

Numerical example 2:

Input values: the same as in the example 1, only difference is that point load of application of the uniform loading q is on the surface of the bottom flange of IPE 200 section (z_g = – 100 mm). The results of the example 2 are given below in the brackets. Dimensionless parameters of the beam:

$$\kappa_{wt} = \frac{\pi}{k_w L}\sqrt{\frac{EI_w}{GI_t}} = 0{,}364\,,\ \zeta_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}} = 0{,}385\ (-0{,}385)\,,\ \zeta_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}} = 0 \qquad (1)$$

a) Calculation of the critical moment M_{cr} of the ideal beam:

Diagram parameters $\overline{M}$ and ψ : $|M_a = -9{,}6\,\text{kNm}| > |M_b = 0\ \text{kNm}| \rightarrow M = M_a$ = – 9,6 kNm, M_q = = $qL^2/8$ = 5 kN/m (6 m)^2/8 = 22,5 kNm. For parameters $\psi = M_b / M_a = 0$, $\overline{M} = M/(|M| + M_q) =$

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[*] Emeritus Prof. Ing. Ivan Baláž, PhD.: Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11; 810 05, Bratislava; Slovak Republic, ivan.balaz@stuba.sk

[**] Assoc. Prof. Yvona Koleková, PhD., Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11; 810 05, Bratislava; Slovak Republic, yvona.kolekova@stuba.sk

[***] Prof. Doncho Partov, PhD.: University of Structural Engineering and Architecture,VSU "Lyuben.Karavelov", Suchodolska 175; 1373 Sofia, Bulgaria, partov@vsu.bg

$=-9{,}6/\left(\left|-9{,}6\right|+22{,}5\right)=-0{,}299$ we obtain the factors $C_1 = 1{,}18$, $C_2 = 0{,}59$ from Figs.1, 2.

$$\mu_{cr} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}^2+\left(C_2\xi_g+C_3\zeta_j\right)^2}-\left(C_2\xi_g+C_3\zeta_j\right)\right]=1{,}016,\ (1{,}552) \qquad (2)$$

Critical moment: $$M_{cr}=\mu_{cr}\frac{\pi\sqrt{EI_zGI_t}}{L}=21{,}667\,\text{kNm},\ (33{,}082\ \text{kNm}) \qquad (3)$$

Critical moment calculated by LTBeam: $M_{cr,LTBeam}=21{,}568\,\text{kNm},\ (32{,}799\ \text{kNm})$ (4)

b) Verification of the real beam:

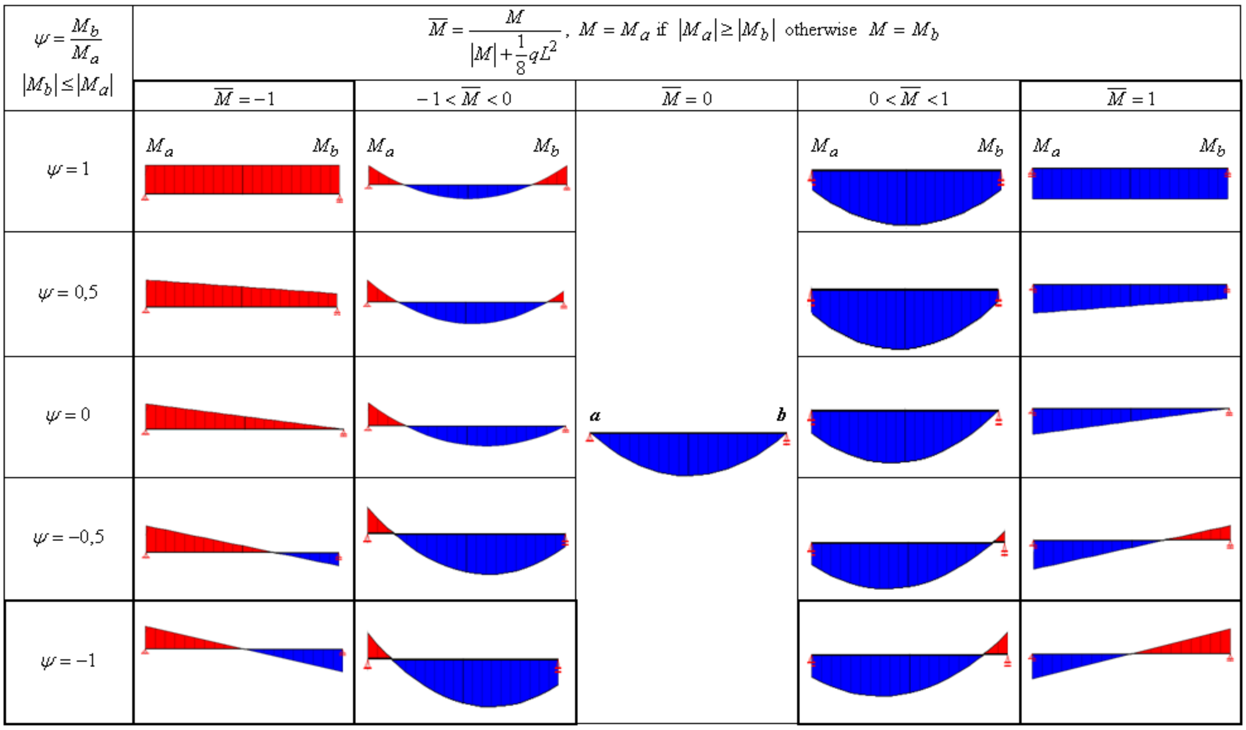
relative slenderness $\bar{\lambda}_{LT}=\sqrt{\dfrac{W_{pl,y}f_y}{M_{cr}}}=1{,}547,\ (1{,}252)$, imperfection factor $\alpha_{LT}=0{,}21$, (5)

$$\Phi_{LT}=0{,}5\left[1+\alpha_{LT}\left(\bar{\lambda}_{LT}-0{,}2\right)+\bar{\lambda}_{LT}^2\right]=1{,}838,\ (1{,}394),\quad \chi_{LT}=\frac{1}{\Phi_{LT}+\sqrt{\Phi_{LT}^2-\bar{\lambda}_{LT}^2}}=0{,}353,\ (0{,}498). \qquad (6)$$

Design resistance of the beam: $$M_{b,Rd}=\chi_{LT}W_{pl,y}\frac{f_y}{\gamma_{M1}}=18{,}319\ \text{kNm},\ (25{,}828\ \text{kNm}). \qquad (7)$$

Utilization factor: $$U=\frac{M_{Ed}}{M_{b,Rd}}=\frac{17{,}955}{18{,}319}=0{,}98<1{,}0,\ (0{,}695<1{,}0). \qquad (8)$$

Tab.1: Bending moment distributions as function of the parameters $\overline{M}$ and ψ



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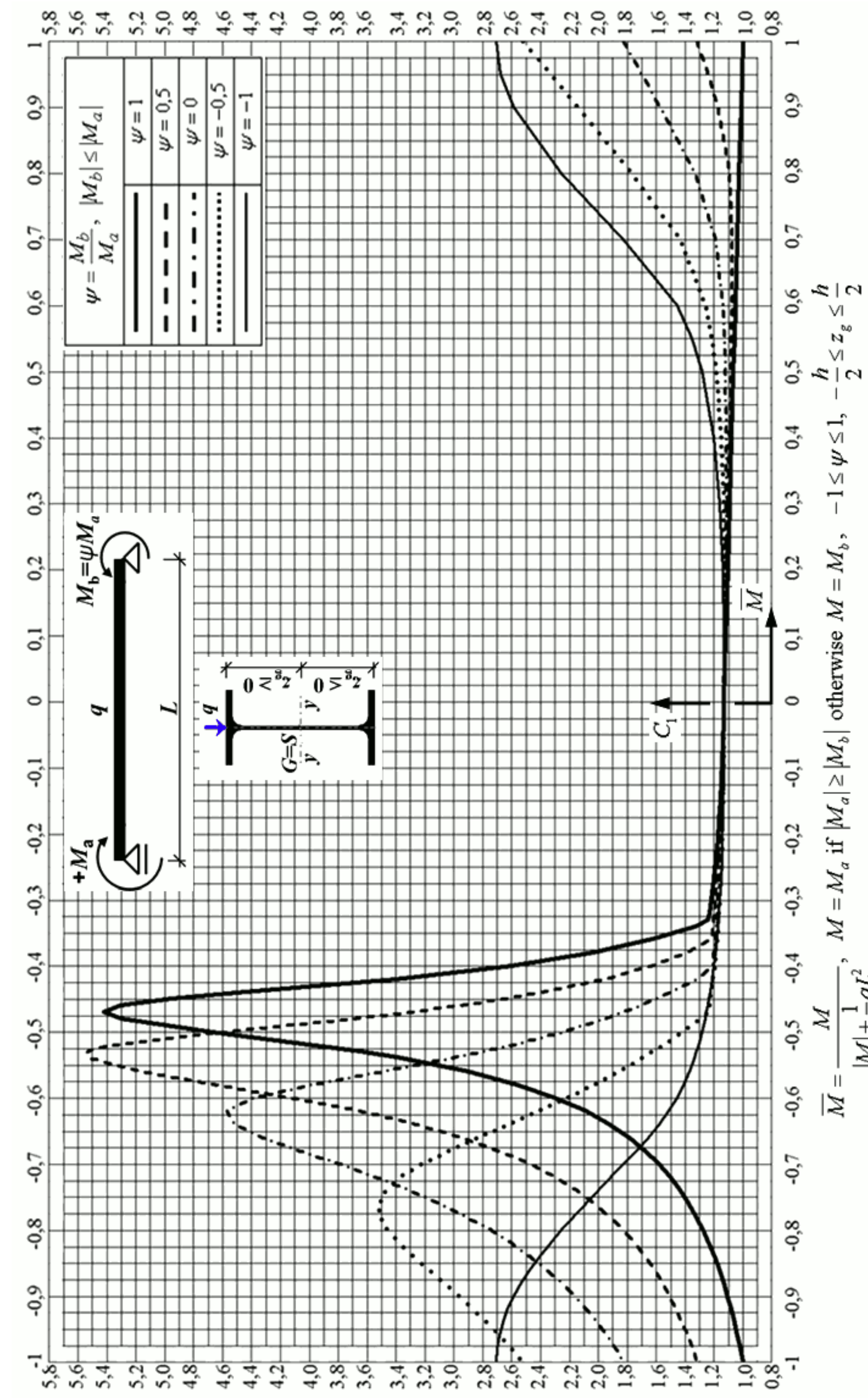
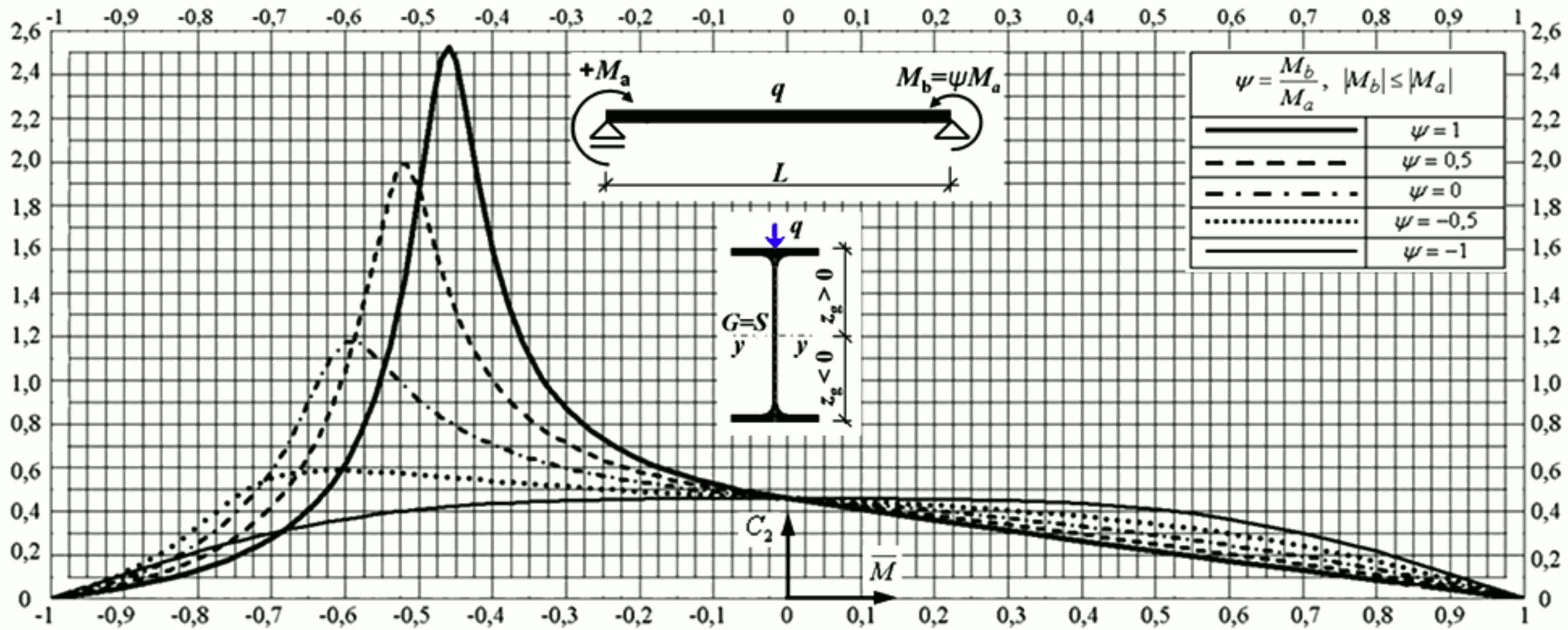


$\psi = \frac{M_b}{M_a}$, $|M_b| \le |M_a|$
$\psi = 1$
$\psi = 0,5$
$\psi = 0$
$\psi = -0,5$
$\psi = -1$
$+M_a$
$M_b = \psi M_a$
q
L
q
G=S
y
$z_g > 0$
$z_g < 0$
C_1
$\overline{M}$
$\overline{M} = \frac{M}{|M| + \frac{1}{8}qL^2}$, $M = M_a$ if $|M_a| \ge |M_b|$ otherwise $M = M_b$, $-1 \le \psi \le 1$, $-\frac{h}{2} \le z_g \le \frac{h}{2}$



$$\overline{M} = \frac{M}{|M| + \frac{1}{8}qL^2}, \quad M = M_a \text{ if } |M_a| \ge |M_b| \text{ otherwise } M = M_b, \quad -1 \le \psi \le 1, \quad -\frac{h}{2} \le z_g \le \frac{h}{2}$$

Fig.2: Factor C_2 for calculation of critical moment M_{cr} of simply supported beam with double symmetric I-section as function of parameters $\overline{M}$ and ψ