26th International Conference
ENGINEERING MECHANICS 2020
Brno, Czech Republic, November 24 – 25, 2020

STRUCTURAL MODAL MODIFICATION OF NON-PROPORTIONALLY DAMPED SYSTEM

Musil M.[*], Chlebo O.[**], Úradniček J.[***], Havelka F.[****] , Milata M.[*****]

Abstract: *Undesirable levels of vibration are an especially common occurrence in modern engineering. Being able to effectively isolate and solve the problem areas of a structure would prove to be a valuable tool in correcting these levels of vibration. This work presents a method for modal synthesis that enables the structural properties to be purposefully changed and that can be applied to systems with non-proportional damping. In order to present the methodology in detail, the case where additional elements are made of aluminium foam are presented. These have very unique properties and broad applications. This methodology is demonstrated, for better understanding, on an example of a beam with added aluminium foam elements. A beam with aluminium foam was chosen since its connection with the base beam changes the inertial as well as stiffness and damping properties of the resulting system.*

Keywords: Modal synthesis, Model reduction, Aluminium foam layers, Dynamic parameters modification.

1. Introduction

A common occurrence, in practical engineering are undesirable levels of vibration in the machinery structures, which can influent their safety and reliability. The dynamic properties of the machines structure itself, to an extent, affects the level of vibration of each of its individual parts. In the research stage, it is now necessary to extensively analyze/synthesize the dynamic properties of the machine and its structure followed by the optimization of significant parameters.

In the design of a structure which must vibrate within acceptable levels during operation, a suitable concept must be chosen. For example, based on numeric analysis and optimization of individual structural components, it is possible to create a real structure which satisfies the operational conditions set upon it. But in general, the real structure partially exhibits differing properties than those predicted computationally, either due to inappropriate simplification or inaccurate physical or geometrical parameters. It is therefore necessary to modify critical structural elements in order to fulfil the desired properties.

This operation can be performed through modal synthesis, where the original structure is modified by an additional component. This approach combines the modal properties of the real structure obtained through measurements and the modal properties of additional components obtained computationally or through measurements. Through optimization of the additional components it is possible to obtain the desired properties of a modified structure while reducing the computational requirements and increasing accuracy of the results.

[*] Prof. Miloš Musil, PhD.: Slovak University of Technology in Bratislava, Nám. Slobody 17; 812 31, Bratislava; SK, milos.musil@stuba.sk

[**] MSc. Ondrej Chlebo, PhD.: Slovak University of Technology in Bratislava, Nám. Slobody 17; 812 31, Bratislava; SK, ondrej.chlebo@stuba.sk

[***] MSc. Juraj Úradniček, PhD.: Slovak University of Technology in Bratislava, Nám. Slobody 17; 812 31, Bratislava; SK, juraj.uradnicek@stuba.sk

[****] MSc. Ferdinand Havelka, PhD.: Slovak University of Technology in Bratislava, Nám. Slobody 17; 812 31, Bratislava; SK, ferdinand.havelka@stuba.sk

[*****] Ing. Michal Milata, PhD.: Slovak University of Technology in Bratislava, Nám. Slobody 17; 812 31, Bratislava; SK, michalxmilata@gmail.com

doi: 10.21495/5896-3-366

2. Procedure of structural modal modification

This chapter is aimed to the modal synthesis explanation of the original structure and a substructure to determine modal parameters of the resulting system. The original structure is represented by the proportionally damped system whose eigenvalue solution has the following form:

$$\left(\boldsymbol{K}_0-\boldsymbol{M}_0\omega_{0j}^2\right)\boldsymbol{v}_{0j}=\boldsymbol{0}, \tag{1}$$

where stiffness and mass parameters are represented by coefficient matrices $\boldsymbol{K}_0$ and $\boldsymbol{M}_0$ and jth eigen mode and natural frequency for this mode by symbols $\boldsymbol{v}_{0j}$ and ω_{0j}.

Following equations describe conditions for orthonormality.

$$\boldsymbol{V}_0^T\boldsymbol{K}_0\boldsymbol{V}_0=\boldsymbol{\Omega}_0^2,\ \boldsymbol{V}_0^T\boldsymbol{M}_0\boldsymbol{V}_0=\boldsymbol{I},\ \boldsymbol{V}_0^T\boldsymbol{B}_0\boldsymbol{V}_0=2\boldsymbol{\Delta}=2(\alpha\boldsymbol{I}+\beta\boldsymbol{\Omega}_0^2) \tag{2}$$

$$\delta_j=\xi\omega_{0j} \tag{3}$$

The symbol ξ introduces damping ratio of the structure.

Because modifying substructures are not located evenly along the original structure, the resulting structure can be interpreted as a disproportionally damped system of 2n dimensional space represented by the coefficient matrices N and P. The vibration of this system can be described by the following second order differential equation.

$$\boldsymbol{N}\dot{\boldsymbol{x}}-\boldsymbol{P}\boldsymbol{x}=\boldsymbol{r}, \tag{4}$$

where

$$\boldsymbol{P}=\begin{bmatrix}-\boldsymbol{K} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}\end{bmatrix},\ \boldsymbol{N}=\begin{bmatrix}\boldsymbol{B} & \boldsymbol{M}\\ \boldsymbol{M} & \boldsymbol{0}\end{bmatrix},\ \boldsymbol{x}=\begin{bmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\end{bmatrix},\ \boldsymbol{r}=\begin{bmatrix}\boldsymbol{f}\\ \boldsymbol{0}\end{bmatrix}, \tag{5}$$

where coefficient matrices are represented by $\boldsymbol{K},\boldsymbol{M}$ and $\boldsymbol{B}$ and the variables $\boldsymbol{q},\boldsymbol{f}$ express generalised displacement vector and the excitation force.

Eigenvalue problem solution for this system:

$$\left(\boldsymbol{P}-s_j\boldsymbol{N}\right)\boldsymbol{w}_j=\boldsymbol{0},\ \left(\boldsymbol{K}+s_j\boldsymbol{B}+s_j^2\boldsymbol{M}\right)\boldsymbol{v}_j=\boldsymbol{0} \tag{7}$$

Where $\boldsymbol{v}_j$ interpret jth eigen mode and imaginary part of its eigenvalue s_j express the angular frequency of damped system ω_{Dj}.

The following equations represents conditions for orthonormality:

$$\boldsymbol{W^T P\, W}=\boldsymbol{S},\qquad \boldsymbol{W^T N\, W}=\boldsymbol{I},$$

$$\boldsymbol{S\, V^T M\, V\, S}-\boldsymbol{V^T K\, V}=\boldsymbol{S},\qquad \boldsymbol{V^T B\, V}+\boldsymbol{V^T M\, V\, S}+\boldsymbol{S\, V^T\, M\, V}=\boldsymbol{I}, \tag{8}$$

where $\boldsymbol{W}$ and $\boldsymbol{V}$ are modal matrices and $\boldsymbol{S}$ represents the spectral matrix. These matrices can be expressed by relations:

$$\boldsymbol{W}=\left\{\boldsymbol{w}_j\right\}=\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{V\,S}\end{bmatrix},\ \boldsymbol{V}=\left\{\boldsymbol{v}_j\right\},\ \boldsymbol{S}=diag\left(s_j\right) \tag{9}$$

Relations (8) and (9) can be used to determine modal-spectral parameters of the resulting structure using original structure modal matrices $\boldsymbol{V}_0$, $\boldsymbol{\Omega}_O$, $2\boldsymbol{\Delta}$ and coefficient matrices of the modifying structure $\boldsymbol{M}_N$, $\boldsymbol{B}_N$, $\boldsymbol{K}_N$. Because dimensions of original structure matrices are usually different then coefficient matrices of the modifying structure needs reduction methods. This process represents the modification of added substructure matrices $\boldsymbol{M}_N$, $\boldsymbol{B}_N$, $\boldsymbol{K}_N$ through $\boldsymbol{M}_R$, $\boldsymbol{B}_R$, $\boldsymbol{K}_R$ to $\boldsymbol{M}_A$, $\boldsymbol{B}_A$, $\boldsymbol{K}_A$ as:

Condition for orthogonality then can be formed as:

$$[\boldsymbol{T}_L^T\ \boldsymbol{S}\ \boldsymbol{T}_L^T]\begin{bmatrix}-(\boldsymbol{\Omega}_{0O}^2+\boldsymbol{V}_0^T\boldsymbol{K}_A\boldsymbol{V}_0) & 0\\ 0 & \boldsymbol{I}+\boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0\end{bmatrix}\begin{bmatrix}\boldsymbol{T_L}\\ \boldsymbol{T_L\, S}\end{bmatrix}=\boldsymbol{S}$$

$$[\boldsymbol{T}_L^T\ \boldsymbol{S}\ \boldsymbol{T}_L^T]\begin{bmatrix}2\Delta_P+\boldsymbol{V}_0^T\boldsymbol{B}_A\boldsymbol{V}_0 & \boldsymbol{I}+\boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0\\ \boldsymbol{I}+\boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{T_L}\\ \boldsymbol{T_L\, S}\end{bmatrix}=\boldsymbol{I} \tag{10}$$

Matrix $\boldsymbol{S}$ can be determined by the solution of following eigenvalue problem:

$$\left(\boldsymbol{P}_T-s_j\boldsymbol{N}_T\right)\boldsymbol{w}_{Tj}=\boldsymbol{0} \tag{11}$$

The transformation matrix and then the modal matrix can be figured out from equations (10) and (11).

3. Modal synthesis of the non-proportionately damped layered beam

The above mentioned modal synthesis method can be used also in determining the modal and spectral properties of beam structures with added layers of vibroisolation at specific points. A very simple illustration of such a system can be explained on an existing cantilever beam with a connected (added) beam which creates the modified beam seen in Fig. 1. also shows the schematic illustration of the above mentioned modal synthesis methodology.

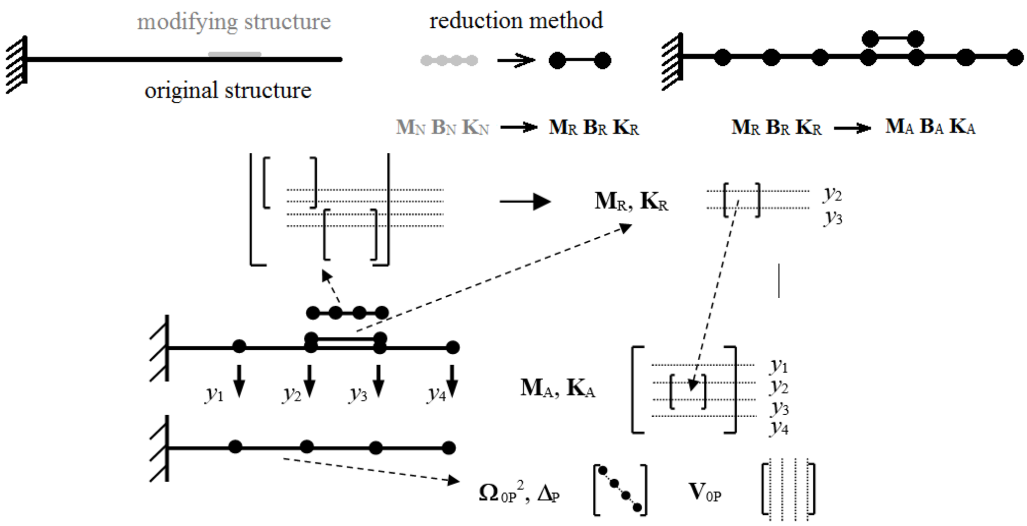

Fig. 1: Principle of modal synthesis of an original and modifying structure.

In conclusion, the method of modal synthesis provides modal and spectral parameters determination of the resulting structure that consist of the original structure interpreted as a proportionally damped system and the modifying substructure described by its modal parameters.

The above mentioned modal synthesis method can be used to modify modal and spectral properties of beam structures with added vibroisolating layers to appropriate locations. This situation can be explained on a simple steel beam with an added beam with aluminium foam properties. The presented method can be automated and used for the parameter optimization of vibroisolating layers (orientation, geometry, material properties, etc.). The schematic representation of the optimizing position *a* and thickness *h* of the vibroisolating layer with respect to the maximum damping ratio ξ in the second Eigen mode is shown in Fig. 2.

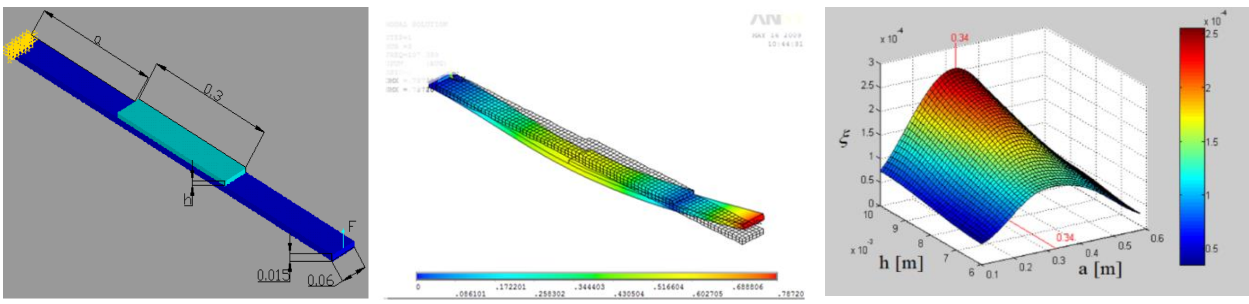

Fig. 2: Optimisation of position "a" and thickness "h" of the vibro-isolating layer with respect to the maximum ratio of damping "ξ".

It is possible to achieve the desired damping of some Eigen modes by choosing the appropriate position and thickness of the vibroisolating layer (Kirch, 1993). Therefore, based on modal synthesis it is possible, without any time-consuming calculations, to determine how effective the chosen parameters of the vibroisolating layer.

4. Structural dynamic modification of the beam by several aluminium foam layers substructures

This chapter introduces changes of natural frequencies of a cantilever beam by adding the aluminium foam layers to appropriate locations. In this chapter there are included modal analysis results of the final element beam model modified by models of substructures. Mass properties of the substructure do not markedly influent the resulting mass matrix while its stiffness parameters significantly changed the stiffness matrix of the resulting system. Therefore, natural frequencies values which depends mainly on mass and stiffness matrices, can be increased by adding these substructures to anti-node locations. The following picture illustrates the increasing of the second and the third natural frequency of the cantilever beam considering bending oscillation.

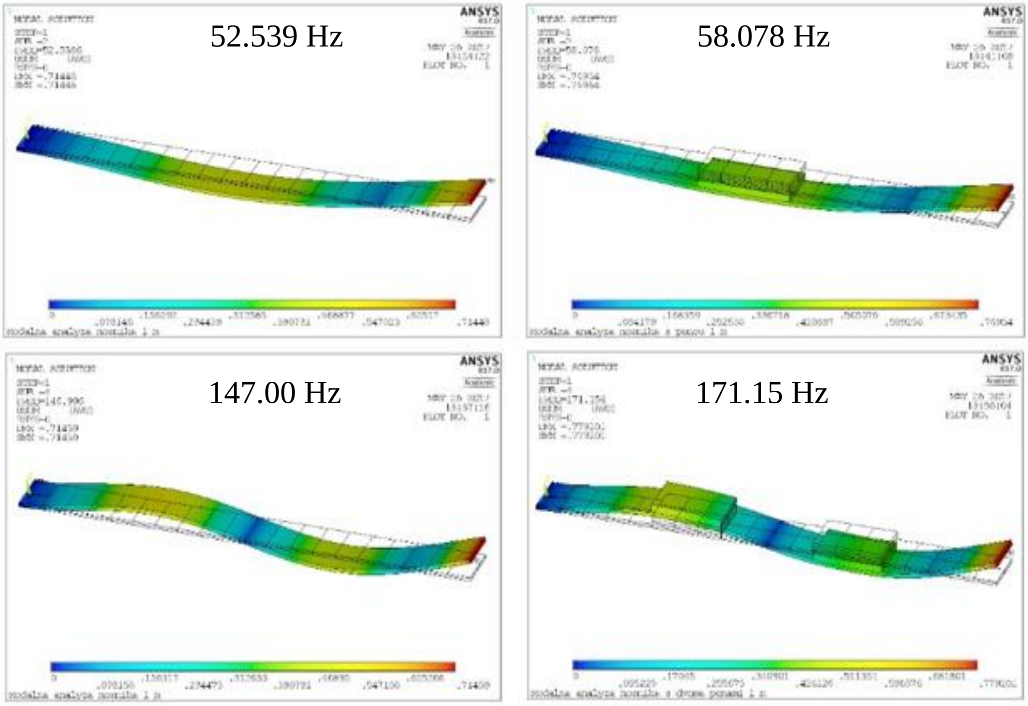

Fig. 3: Modal analysis of cantilever beam and its aluminium layer modifications.

It can be seen from the Fig. 3, that it is possible to effectively modify natural frequencies using relatively small, cheap and light additional components.

5. Conclusion

This work presented a method for a desired modification of dynamic parameters of a structure by additional components using the modal synthesis. This methodology combines the structural and modal parameters of joined structures to determine dynamic parameters of the resulting system. This approach can be automated and used to design suitable additional components and to determine their locations with respect to the damping ratio of competent modes as was shown in the third chapter on the modified beam example. This work also included the example of the effective changing of natural frequencies values of the structure by adding stiffening substructures with appropriate properties to specific locations.

Acknowledgement

The research in this paper was supported by the grant agency VEGA 1/0430/20 of Ministry of Education, Science and Sport of the Slovak Republic.

References

Avitabile, P. (2002) Twenty Years of Structural Dynamic Modification – A Review. University of Massachusetts Lowell, Lowell, Massachusetts, pp. 14-25.

Braun, S., Ewins, D., RAO, S. S. (2002) Encyclopedia of Vibration. New York: Academic Press.

Curnier, A. (1983) On Three Modal Synthesis Variants. Journal of Sound and Vibration, pp. 90-92

Kirch, U. (1993) Structural Optimization. Berlin: Springer-Verlag.

Musil, M., Chlebo, O. (2014) Use of Modal Synthesis for Composite Structures. In: Scientific Proceedings, Bratislava.

Ziaran S. Effects of low-frequency noise in closed space on the human. In: InterNoise 2011 Osaka/Japan.