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EFFICIENT NUMERICAL COMPUTATION OF THE STEADY-STATE RESPONSE AND STABILITY ANALYSIS OF THE ROTOR SYSTEMS WITH SQUEEZE FILM DAMPERS

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Abstract: *The aim of this paper is to demonstrate capabilities of the created numerical procedure, which is based on harmonic balance method. Furthermore, the procedure incorporates the alternating frequency-time domain technique and the arc-length parameterization to solve the steady-state response of nonlinear systems in efficient manner, including unstable branches. The stability of the motion was assessed by two methods: the 2n-pass method and Hill's method. The procedure was verified on an example from literature to prove its sufficient accuracy and subsequently, the procedure was applied on the finite element model of the rotor system mounted on the squeeze film dampers. The carried out computational simulations confirmed that the created procedure is efficient for the strongly nonlinear response and it gives similar results as the time integration.*

Keywords: Harmonic balance method, Floquet theory, Hill's method, Nonlinear forces, Flexible rotor.

1. Introduction

The determination of the steady-state periodic response is the fundamental issue in the nonlinear rotating machinery design. Therefore, there is a current effort to develop a robust and efficient software for this very purpose. Particular examples of this effort can be seen in literature, where authors present procedures for response computation of e.g. a rotor-bearings systems (Villa et al., 2008) and a rotor-to-stator contact problem (Peletan et al., 2013).

The rotor systems of n degrees of freedom (DOF) can be described by the following motion equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{B}(\omega)\dot{\mathbf{x}}(t) + \mathbf{K}(\omega)\mathbf{x}(t) = \mathbf{f}_{\mathbf{NL}}(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}) + \mathbf{f}_{\mathbf{P}}(t, \omega) + \mathbf{f}_{\mathbf{S}}. \qquad (1)$$

$\mathbf{M}$, $\mathbf{B}(\omega)$, $\mathbf{K}(\omega)$ denote the $n \times n$ matrices of mass, damping, and stiffness, respectively, which depend on the excitation frequency ω due to the rotational effect, $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, and $\mathbf{x}$ are the vectors of generalized accelerations, velocities, and displacements, respectively, $\mathbf{f}_{\mathbf{NL}}$, $\mathbf{f}_{\mathbf{P}}$, and $\mathbf{f}_{\mathbf{S}}$ are the vectors of the nonlinear forces, periodic excitation forces, and static load, respectively. t is time and $\cdot$ denotes time derivation.

There are numerous methods for the response computation, e.g. the time integration, the shooting method, the trigonometric collocation method (Zapoměl, 2007) or the harmonic balance method (HBM). The main advantage of the HBM is its computational effort, which is often several orders of magnitude lower in comparison with the ordinarily used time integration (Krack, 2019). In the context of the steady-state response of the rotor systems, the stability analysis of motion is equally important characteristics, thus the created procedure evaluates the stability using the 2n-pass and Hill's method (Peletan et al., 2013).

The main goal of the presented work is to develop the effective procedure for computation of the steady-state response of nonlinear systems and its motion stability analysis. By utilization of the finite element method, engineering problems are often described by systems with large number of DOF.

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2. HBM and the Stability Analysis of Motion

The numerical procedure is based on HBM, which consists in approximation of the response by n_{H} Fourier terms, transformation of the equation of motion (1) to the following residual form

$$\mathbf{h}(\omega, \mathbf{q}) = \mathbf{P}(\omega)\mathbf{q} - \mathbf{T}\,\mathbf{f}_{\mathbf{NL}}(\mathbf{T}^{+}\mathbf{q}, \omega\mathbf{T}^{+}\nabla\mathbf{q}, \omega^{2}\mathbf{T}^{+}\nabla^{2}\mathbf{q}) - \mathbf{u}_{\mathbf{P}}(\omega) - \mathbf{g}_{\mathbf{S}} \qquad (2)$$

and subsequent solution by Newton-type method. The $\mathbf{P}(\omega) = \omega^{2}(\mathbf{I} \otimes \mathbf{M})\nabla^{2} + \omega(\mathbf{I} \otimes \mathbf{B})\nabla + \mathbf{I} \otimes \mathbf{K}$ is the dynamical stiffness matrix, $\mathbf{I}$ is the identity matrix of order $2n_{\mathrm{H}} + 1$, and the vectors $\mathbf{q}$, $\mathbf{u}_{\mathbf{P}}$, $\mathbf{g}_{\mathbf{S}}$ contain Fourier coefficients of the response, amplitudes of the periodic unbalance excitation forces, and the static forces, respectively. The ∇ is differential operator in the same form as in (Detroux et al., 2015) and $\otimes$ is Kronecker product. The $\mathbf{T}\,\mathbf{f}_{\mathbf{NL}}(\mathbf{T}^{+}\mathbf{q}, \omega\mathbf{T}^{+}\nabla\mathbf{q}, \omega^{2}\mathbf{T}^{+}\nabla^{2}\mathbf{q})$ term represents the so-called alternating frequency-time domain technique, where $\mathbf{T}$ denotes Fourier transformation matrix and $\mathbf{T}^{+}$ its Moore-Penrose pseudoinverse, therefore inverse Fourier transformation matrix.

For the efficient computation and ability to obtain unstable branches of the response curve, the arc-length parameterization was employed. The additional equation of its corrector step has the following form

$$\mathbf{p}(\omega, \mathbf{q}) = \left(\mathbf{q} - \mathbf{q}_{\mathrm{prev}}\right)^{\mathrm{T}}\left(\mathbf{q} - \mathbf{q}_{\mathrm{prev}}\right) + \left(\omega - \omega_{\mathrm{prev}}\right)^{2} - s^{2}, \qquad (3)$$

where $\mathbf{q}_{\mathrm{prev}}$ and ω_{prev} denote values acquired at the previous continuation step, $\mathbf{q}$ and ω the current step values, and s the arc-length parameter. The predictor step was based on secant predictor, and thus the successive step guess values were extrapolated from the two previous steps.

The stability analysis (Zapoměl, 2007) of the equation of motion (1) is determined by Floquet theory. The frequently used method for Floquet multipliers obtainment is the 2n-pass method (Peletan et al., 2013). This method operates in the time domain and evaluates the perturbation evolution by time integration. Unfortunately, this method requires enormous computational effort. Hill's method (Peletan et al., 2013) can produce the same results but operates in frequency domain. The need for filtering eigenvalues presents a problem, which is still in field of research (Krack, 2019).

3. Verification Example for Testing of the Developed Computational Procedure

The first demonstration of the developed procedure capabilities was performed on simple two DOF oscillator with cubic springs (Krack, 2019), which is illustrated in Fig. 1

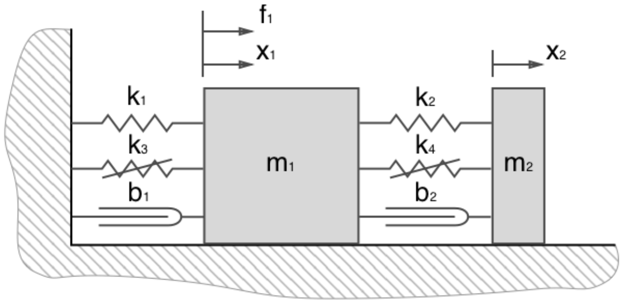

Fig. 1: Scheme of the two DOF oscillator with cubic springs.

The system is being excited at the first DOF by a harmonic force. The behaviour of this system is described by the following equation of motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \\ + \begin{bmatrix} k_3 & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} x_1^3 \\ x_2^3 \end{Bmatrix} + \begin{Bmatrix} k_4 \\ -k_4 \end{Bmatrix}(x_1 - x_2)^3 = \begin{Bmatrix} f_1 \\ 0 \end{Bmatrix}. \qquad (4)$$

$m_1 = 1\ \mathrm{kg}$, $m_2 = 0.05\ \mathrm{kg}$, $b_1 = 0.002\ \mathrm{kg.s^{-1}}$, $b_2 = 0.013\ \mathrm{kg.s^{-1}}$, $k_1 = 1\ \mathrm{kg.s^{-2}}$, $k_2 = 0.0453\ \mathrm{kg.s^{-2}}$ are the masses, damping, and stiffness coefficients, respectively. x_1 and x_2 are the displacements. The cubic spring stiffnesses are $k_3 = 1\ \mathrm{kg.m^{-2}.s^{-2}}$, $k_4 = 0.042\ \mathrm{kg.m^{-2}.s^{-2}}$, and the excitation force is given by $f_1 = p_1 \cos\omega t$, where $p_1 = 0.11\ \mathrm{kg.m.s^{-2}}$ is the amplitude of the excitation force.

The results in Fig. 2 show amplitude-frequency response of the first DOF. There are two sections of the response curve, where the maximum of Floquet multipliers absolute values is greater than one, which indicates the occurrence of the unstable motion. Neimark-Sacker (NS) bifurcation and the limit point were identified in the amplitude-frequency response. The Fig. 3 presents a comparison of the 2n-pass method and Hill's method with Floquet multipliers filtering by the values of imaginary parts, where both methods provide accurate results in area of resonance. The implemented version of Hill's method unfortunately differs in areas outside the resonance, which is due to the filtering issue. These results are in good agreement with the literature (Krack, 2019) and serve as verification of the developed numerical procedure.

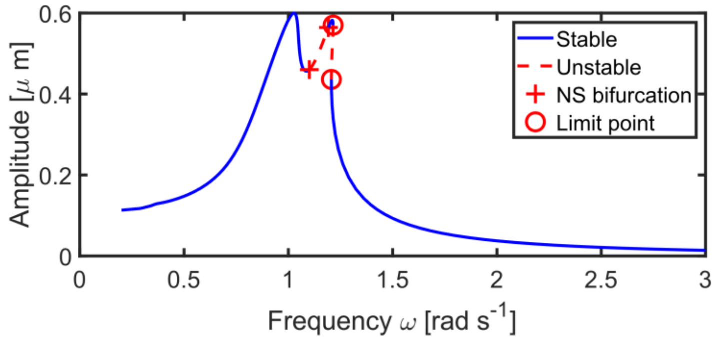

Fig. 2: Amplitude-frequency response of the two DOF model with cubic springs.

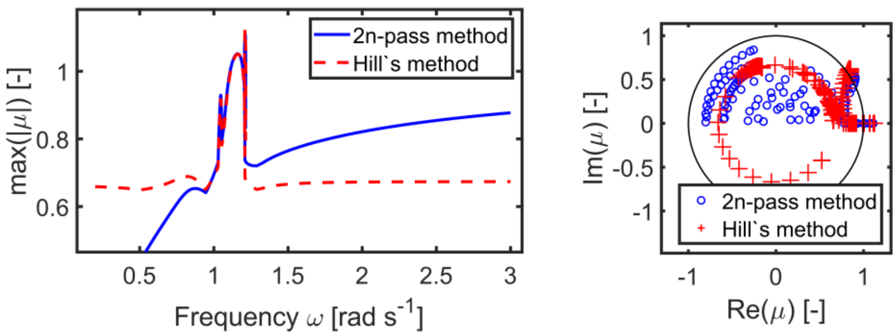

Fig. 3: Comparison of the 2n-pass method and Hill's method on stability analysis results of the two DOF model with cubic springs. On the left: Absolute values of Floquet multipliers. On the right: Real and imaginary part of Floquet multiplier values.

4. Finite Element (FE) Model of Rotor System Mounted on Squeeze Film Dampers (SFDs)

The developed and verified procedure was applied on FE model of the rotor mounted on SFDs. The analysed rotor (Fig. 4) consists of a flexible shaft and two rigid discs. The rotor is supported by rolling element bearings that are coupled with the stationary base by SFDs. The task was to investigate the steady-state response and judge its stability.

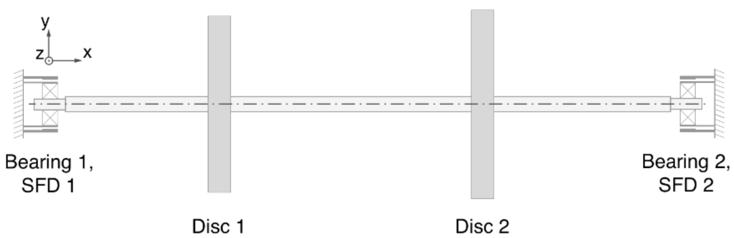

Fig. 4: Scheme of the rotor system with SFDs.

The dimensions of the studied rotor are: length 1400 mm, shaft diameter 70 mm, shaft diameter in place of bearings 60 mm, length of shaft parts with reduced diameter 100 mm, diameter of the discs 300 mm, 350 mm, positions of discs on shaft 400 mm, 900 mm, its unbalances 1.889 mm, 2.610 mm (Disc 1 and 2, respectively), and their thicknesses 15 mm. The density of the rotor material is $7850\ \mathrm{kg.m^{-3}}$, its Young's modulus is 210 GPa, Poison's ratio is 0.3, coefficients of material viscous and external damping are $2.0 \cdot 10^{-6}\ \mathrm{s^{-1}}$, $4\ \mathrm{s^{-1}}$. The parameters of SFDs are following: damper length 22 mm, damper diameter

95 mm, damper clearance 250 μm, and dynamic viscosity 0.0312 Pa. s. The stiffness and damping coefficients of the bearings are equal to $4 \cdot 10^6$ N. m^{-1} and 1000 kg. s^{-1}.

The equation of motion is the same form as (1). The number of DOF associated with the solved FE model is equal to the 60 and the shaft is discretised by a two-node beam FE (Zapoměl, 2007). The pressure distribution of SFD is determined with the short damper assumption (Ferfecki et al., 2018).

The resulting amplitude-frequency curves in Fig. 5 display comparison of model variations, where damper clearance and the damper length were varied. The results show that an unstable motion occurs near the resonance peak.

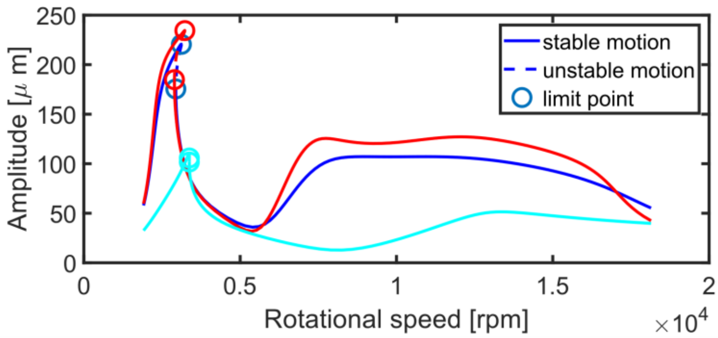

Fig. 5: Amplitude-frequency response of the rotor system with SFDs. The color of the curve represents model with variations of the damper clearance and the damper length: the blue curve (250 μm, 22 mm), the red curve (250 μm, 18 mm), and the cyan curve (125 μm, 22 mm). The amplitude represents eccentricity maximum.

5. Conclusions

The presented study offers a demonstration of the created numerical procedure on the FE model of the rotor system with SFDs, where it provides accurate results in the form of amplitude-frequency curves of the steady-state response and its stability. The parameter study of this rotor system shows a significant change in the response, depending on the damper clearance and length. The created computational procedure is implemented with the goal of a reduction of the computational time and future deployment on high performance computing architectures.

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