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MULTI-PARAMETER MTS AND SED CRITERION APPLIED ON A CRACK IN AN ALKALI-ACTIVATED CONCRETE SPECIMEN UNDER MIXED-MODE LOADING

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Abstract: *The paper is devoted to an analysis of a crack propagation in a semi-circular concrete specimen under three-point bending. I+II mixed-mode loading is ensured via an inclined crack. Crack deflection angle is investigated by means of the generalized/multi-parameter form of the Maximum Tangential Stress (MTS) criterion and Strain Energy Density (SED) criterion as well as experimentally. The multi-parameter fracture mechanics concept applied in this work is based on the approximation of the crack-tip stress field by means of the Williams series expansion (WE). It has been proved that this approach can help to describe crack behavior in some kinds of materials better than the classical (one-parameter: stress intensity factor) fracture mechanics. This conclusion is discussed also in this paper.*

Keywords: Multi-parameter fracture mechanics, Alkali-activated concrete, Mixed-mode loading, MTS, SED.

1. Introduction

Recently, several works (Chen, 2002 and Ramesh et al., 1997) have been devoted to investigations on importance of higher-order terms of the Williams expansion (Williams, 1957). They prove that not only the stress intensity factor (first term of the WE) is crucial when the stress field around the crack tip is to be approximated as accurate as possible in particular materials and specimen geometries. Multi-parameter fracture criteria for assessment of crack deflection angle are derived for instance in Smith et al. (2001) and Malíková (2015). Also in this work the generalized Maximum Tangential Stress (MTS) criterion and Strain Energy Density (SED) criterion are applied in order to estimate the angle of the further crack propagation under mixed-mode loading. The analysis is performed experimentally as well as numerically via finite-element method. Results are mutually compared, discussed and conclusions are stated.

2. Theory and applied methodology

For investigations of the crack deflection angle it is necessary to know the crack-tip stress field. Then selected fracture criteria can be applied.

2.1. Williams expansion

Williams derived expressions for the individual stress tensor components near a crack in a homogeneous elastic isotropic body subjected to an arbitrary remote loading:

$$\sigma_{ij} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n,\theta) A_n + \sum_{m=1}^{\infty} \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m,\theta) B_m \qquad (1)$$

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In Eq. 1 i, j stand for the stress tensor components indexes, (r, θ) represent polar coordinates with the system's origin at the crack tip, f_{ij}, g_{ij} denote the known functions corresponding to loading mode I and II, respectively and A_n, B_m designate unknown coefficients of the terms of the WE. These coefficients are to be determined numerically. In this paper, the over-deterministic method (Ayatollahi and Nejati, 2011) is utilized because of its advantages. This method is based directly on the definition of the displacement fields in the vicinity of the crack tip, analogically to Eq. (1). Displacements of selected nodes obtained from a common finite-element solution are used as inputs for a further procedure.

2.2. Multi-parameter fracture criteria

When the coefficients of the WE are known, an arbitrary crack-tip stress field component can be expressed at an arbitrary distance from the crack tip. Then, a selected fracture criterion can be applied.

MTS criterion

The MTS criterion (Erdogan and Sih, 1963) consists in the idea that the crack deflects in the direction where the tangential stress $\sigma_{\theta\theta}$ reaches its maximum. Thus, the tangential stress component is approximated via WE and its maximum is searched numerically at a particular distance.

SED criterion

The idea of the SED criterion assumes that the crack grows in the direction where the strain energy density factor Σ reaches its minimum, see e.g. Sih (1973) and Sih (1974):

$$\Sigma = \frac{1}{2\mu}\left[\frac{\kappa+1}{8}(\sigma_{rr} + \sigma_{\theta\theta})^2 - \sigma_{rr}\sigma_{\theta\theta} + \sigma_{r\theta}^2\right] \qquad (2)$$

In Eq. (2) the symbols have the following meaning: μ is shear modulus of elasticity, κ is Kolosov's constant and σ_{ij} are stress tensor components (radial, tangential and shear stress). Again, a suitable distance from the crack tip must be chosen for application of the criterion (Susmel and Taylor, 2008).

3. Experimental set-up and numerical model

All the analysis has been performed on a novel material that is denoted as environment-friendly one. Particularly, a type of alkali-activated concrete has been chosen for the investigations. Specimen geometry can be seen in Fig. 1 with the following dimensions:

- specimen radius R = 50 mm,
- half-span between the supports S = 40 mm,
- crack inclination angle β = 30 °, 40 ° and 50 °,
- crack length a = 25 mm.

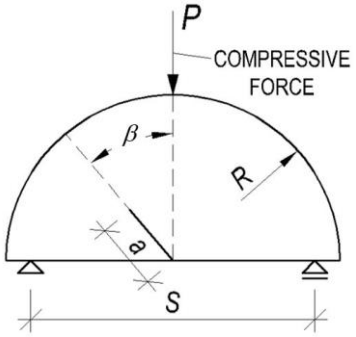

Fig. 1: Semi-circular disc with an inclined crack under three-point bending.

Based on the experimental set-up a numerical model in a finite-element code ANSYS was created. Besides the geometry, the elastic material properties must have been measured: Young's modulus of 35 GPa and Poisson's ratio of 0.23. The numerical model was created as two-dimensional with plane stress conditions

because the experimental results were obtained optically from the surface of the concrete specimen. The element PLANE183 was used to mesh the specimen and a special mesh refinement around the crack tip was utilized. Loading force $P = 1$ kN and zero displacements at the places of the supports were programmed. From the numerical analysis, displacements and polar coordinates of a selected set of nodes at a radius around the crack tip were used for estimation of the coefficients of the WE. The multi-parameter fracture criteria were applied at various radial distances (r_c = 0.1, 0.5, 1.0 and 1.5 mm), various numbers of the terms of the WE (N, M = 1, 2, …, 10) were considered and the various results were discussed.

4. Results and discussion

In Figs. 2 and 3 the obtained dependences of the crack deflection angle γ on the mixed-mode ratio K_I/K_{II} can be seen. Comparison between the experimental results (a set of 3 tests have been made for each configuration) and data achieved numerically by means of the described multi-parameter fracture criteria (MTS – full lines, SED – dashed lines) are introduced. Fig. 2 represents the dependences for the critical distance of 0.5 mm and Fig. 3 the critical distance of 1.5 mm; both figures contain results for the cases when 1, 2, 3 and 5 initial terms of the WE are considered for approximation of the stress tensor components.

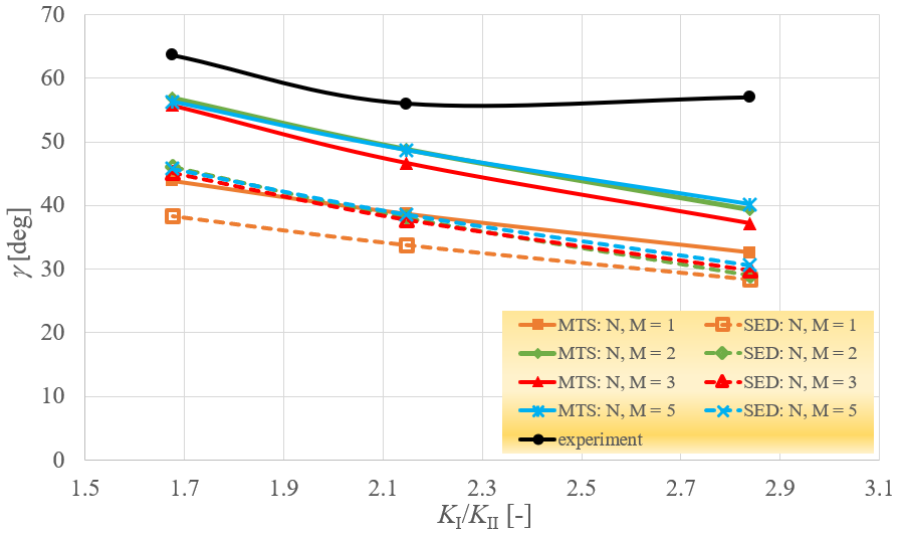

Fig. 2: Dependence of the crack deflection angle on the mixed-mode ratio for the critical distance r_c = 0.5 mm: comparison between experiment and multi-parameter MTS and SED criterion considering 1, 2, 3 and 5 initial terms of the WE.

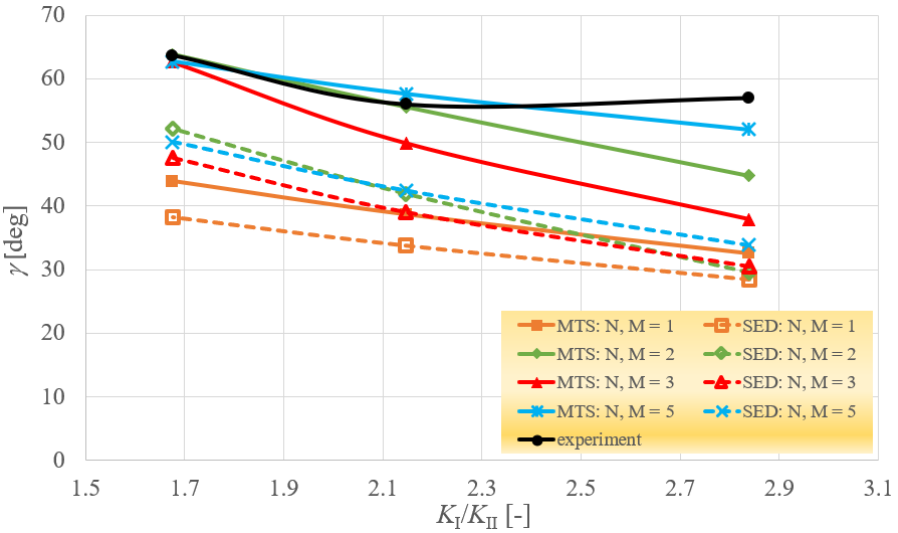

Fig. 3: Dependence of the crack deflection angle on the mixed-mode ratio for the critical distance r_c = 1.5 mm: comparison between experiment and multi-parameter MTS and SED criterion considering 1, 2, 3 and 5 initial terms of the WE.

From the results presented in Figs. 2 and 3 several tendences are obvious and are discussed in the following text:

- Values of crack deflection angles obtained at larger distances from the crack tip (Fig. 3) are closer to experimental results.
- Multi-parameter SED criterion gives generally lower crack deflection angles than multi-parameter MTS criterion regardless the considered number of the initial terms of the WE. Thus, MTS criterion could be rather recommended for this configuration and material.
- Classical (one-parameter) form of both criteria is very far from the experimental results and considering arbitrary number of the WE terms makes the results always closer to the experimental ones.

5. Conclusions

Crack deflection angles have been investigated on an environment-friendly alkali-activated concrete specimen. Particularly, a semi-circular disc with an inclined crack was subjected to three-point bending experimentally (mixed-mode loading conditions were set), as well as a numerical analysis was performed. Angle of the further crack propagation was estimated by means of the multi-parameter MTS and SED criterion. The parametric study was performed via considering various critical distances for application of the fracture criteria and various numbers of the initial terms of the WE considered for stress tensor components approximation. The results show that MTS criterion is more suitable for assessment of the crack propagation than SED criterion. Additional recommendations for more precise estimation of the crack path are larger critical distances where the fracture criterion is applied and/or assuming more initial terms of the WE, because the first singular parameter is not sufficient for this analysis when quasi-brittle materials are treated.

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