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STATISTICAL ANALYSIS OF THE MODELS FOR ASSEMSSMENT OF THE PUNCHING RESISTANCE OF FLAT SLABS

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Abstract: *Flat slabs are one of the most widely used structural system for the construction of administrative buildings nowadays. Together with many advantages connected particularly with architecture and construction processes the system possesses also some drawbacks. From a structural point of view, the most dangerous is the concentration of shear forces at the vicinity of columns which may cause punching of a slab. Punching is a dangerous phenomenon due to its brittle mode of failure and its ability to spread over a whole structure, which can be followed by a progressive collapse. Several models for the assessment of punching capacity have been developed and calibrated using experimental results from laboratory tests. This paper deals with a statistical evaluation of the safety level of two models for punching resistance without transverse reinforcement. One of the models is fully empirical, the EC2 (2004), the second model reflect the physical nature of the phenomenon, CF CSCT (2017). Database which includes results of more than 400 experimental tests on flat slab specimens has been used for the statistical evaluation.*

Keywords: Shear, Punching resistance, Flat slab, Reliability, Safety.

1. Introduction

Reinforced concrete slabs supported by columns are common in residential and commercial buildings. The most usual and most dangerous damage of these systems is punching of the slab by support. Failure at one local support may lead to the overloading of neighboring areas and then may spread over the whole structure, resulting in progressive collapse, see Fig. 1.

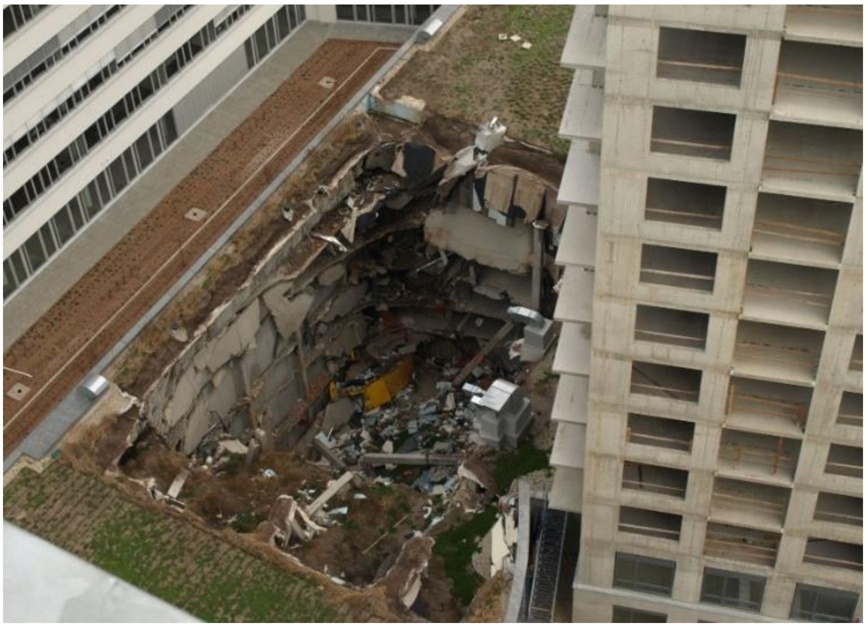

Fig. 1: Progressive collapse of a parking garage, Bratislava (2012).

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The punching provisions in codes of practice are based on different theories or empirical formulae, this in some cases leading to very different strength predictions. The design model for punching introduced in Eurocode 2 is based on the model originally presented in Model Code 1990. The model is empirical since the most influential parameters of the punching resistance were statistically evaluated using the results of experimental tests.

A new model for punching resistance was developed in the 1990s by Prof. A. Muttoni. The model is based on the Critical Shear Crack Theory (CSCT). The principles of the CSCT were published by Muttoni and Schwartz (1991) for the first time. For the sake of simplicity, a new model based on CSCT theory was developed and formulated by Muttoni and Ruiz (2017). The formula for punching resistance was expressed in a closed-form. The model was originally proposed for the second generation of Eurocode 2, however, then was further updated.

2. EC2 model (2004)

Punching shear resistance without shear reinforcement can be determined using formula (1), according to current EC2 model (2004). This model is based on a formula proposed by Zsutty (1968). The model was further refined and the final formula (1) was published in Model Code 1990.

$$V_{Rd,c} = C_{Rd,c} * k * (100 * \rho_l * f_{ck})^{1/3} * d * u_1 \text{ [MPa]} \tag{1}$$

Where design value of empirical factor $C_{Rd,c} = C_{Rk,c}/\gamma_C$ [MPa], with $C_{Rk,c} = 0.180$ MPa, ρ_l is reinforcement ratio [-], f_{ck} is characteristic cylinder compressive strength of concrete, d is an effective depth of a slab [m] and u_1 length of the control perimeter at a distance $2*d$ from the face of a column.

3. Critical Shear Crack Theory Model (CSCT)

CSCT is a mechanical model for the assessment of punching resistance. The model was verified by the results of 99 experiments. The principles of the theory came out from the assumption of critical crack development at the vicinity of the column. Punching resistance is ensured by aggregates interlocking in the critical crack and by the tensile strength of the concrete. Shear resistance then depends on friction in the critical crack. The friction is descending with a growing crack width. Crack width is proportional to the slab rotation ψ.

$$v_{Rd,c} = k_\psi * (f_{ck})^{1/2}/\gamma_C \text{ [MPa]} \tag{2}$$

$$k_\psi = (1/1.5+0.9 * k_{dg} * \psi * d_v) \text{ [-]} \tag{3}$$

Where f_{ck} is characteristic cylinder compressive strength of concrete [MPa], d_v is an effective depth of a slab for shear [m], k_{dg} is factor depending on the maximum aggregate size $d_{g,max}$ [-] and ψ is slab rotation [-].

4. Closed form of the CSCT model (CF CSCT (2017))

This model is based on the CSCT theory. The evaluation of the closed-form of the CSCT model (2017) was published by Muttoni and Riuz (2017). To simplify the design of flat slabs for designers, the basic formula (2) has been changed and expressed in closed form (4). This closed form looks now very similar to the EC2 (2004) formula.

$$V_{Rd,c} = k_b/\gamma_C * (100 * \rho_l * f_{ck} * d_{dg}/a_v)^{1/3} * d_v * b_0 \leq 0.6 * (f_{ck})^{1/2}/\gamma_C * d_v * b_0 \text{ [MPa]} \tag{4}$$

$$k_b = (8 * \mu * d_v/b_0)^{1/2} \text{ [-]} \tag{5}$$

Where d_{dg} is coefficient that takes into account the type of concrete and its aggregate properties [m], a_v is shear span, the geometric average of the shear spans in both orthogonal directions and not less than $2.5\ d_v$ [m], μ is parameter accounting for the shear force and bending moment in the region of the shear [-] and b_0 is length of the control perimeter at a distance $d_v/2$ from the face of a column.

5. Database of experimental tests

An experimental test database was created at RTWH Aachen and collected by Carsten Sibrug. The results of a total of 660 tests are recorded in the database for specimens without transverse reinforcement tested under axis-symmetric conditions. Only the tests on slabs with an effective depth greater than 100 mm were included in the analysis the dataset “A”. The dataset “B” included results of all slabs. For the statistical evaluation, a detailed review of the database has been carried out to exclude non - standard tests or tests where some important parameters were missing.

After a detailed review of the database, the total results of 295 tests (dataset “A”) and 408 tests (dataset “B”) were included in the statistical analysis for the EC2 (2004) model and 190 tests (dataset “A”) and 238 tests (dataset “B”) for the CF CSCT (2017) model. The difference between the number of tests is caused by the absence of some important information; e.g., in the case of CF CSCT (2017) model $d_{g,max}$ or shear span a_v were mostly unknown. In the case of the EC2 (2004) model the reinforcement ratio ρ_l was sometimes missing.

6. Statistical evaluation of the models for punching resistance

A statistical evaluation of the models for assessing punching resistance without a shear reinforcement has been carried out, based on formula (1) and formula (4) with the partial safety factor $\gamma_C = 1.0$. The actual cylinder strength of the concrete f_c (the mean value) introduced by the test authors was used for the concrete strength. Control perimeters were assumed at distance 2d from the face of a column for the EC2 (2004) model and d/2 for the CF CSCT (2017) model.

Main statistical variable in the evaluation was the model uncertainty observation $\theta_i = (V_{R,test}/V_{Rd,c})_i$, where “$i$” is number of a test, $V_{R,test}$ is a resistance obtained from an experimental test and $V_{Rd,c}$ is punching resistance obtained from the theoretical model. Only variables θ_i which satisfy condition $0.62 < \theta_i < 1.63$ for EC2 (2004) model and $0.58 < \theta_i < 1.45$ for CF CSCT (2017) have been used in statistical evaluation. The limits of the θ_i were determined by statistical analyses where so-called box and whisker graphs were used to depict the groups of data through their quartiles. The method allows for detecting outstanding data (outliers), which are far from the other data and can affect the analysis. A special outstanding data test was used, based on the quartile deviation:

$$I = [Q1 - k(Q_3 - Q_1, Q_3 + k(Q_3 - Q_1))], \quad (6)$$

where: Q_1, Q_3 are lower and upper quartiles,
k - coefficient: if $k = 1.5$ and $x \notin I$, then x are outliers,
if $k = 3$ and $x \notin I$, then x are far out data.

Mean value θ_m was calculated using formula (7) where n is a number of assumed tests. The characteristic value was determined as 5 % fractile for Gaussian distribution according to formula (8), where V_θ is coefficient of variation $V_\theta = \sigma_\theta/\theta_m$ and σ_θ is standard deviation, formula (9). Obtained results are summarized in Tab. 1.

$$\theta_m = (\Sigma\theta_i)/n \quad (7)$$

$$\theta_{k,0.05} = \theta_m(1\text{-}1{,}645.V_\theta) \quad (8)$$

$$\sigma_{\theta 2} = [\Sigma(\theta_i\text{-}\theta_m)^2]/(n\text{-}1) \quad (9)$$

Tab. 1: Statistical evaluation of model safety.

Model		Number of tests included [n]	Average value [θ_m]	Variation coefficient [V_θ]	Characteristic value [$\theta_{k,0.05}$]
EC2 (2004)	“A“	295	1.08115	0.15156	0.81159
	“B“	408	1.14599	0.16549	0.83401
CF CSCT (2017)	“A“	190	0.93841	0.13028	0.73729
	“B“	238	0.96447	0.13852	0.74469

7. Conclusions

A statistical evaluation of the models for prediction of the punching resistance has been carried out using experimental test database. The database included only tests without transverse reinforcement. Not all results were included in the evaluation. Only the tests with all parameters needed for correct prediction have been used and the outliers were excluded from the analyses.

The target value of 5 % fractile $\theta_{k,0.05}$ is 1.0 according to EN 1990. However, resistance models can be assumed reliable if $\theta_{k,0.05} > 0.85$, because the experimental specimens were isolated flat slab fragments. In actual construction, the punching shear resistance is higher due to membrane forces and redistribution of internal forces due to cracking above columns that is resulting in movement of the radial bending moments contraflexure line closer to the column.

The EC2 (2004) model can be considered only partially reliable with the result of $\theta_{k,0.05} = 0.812$ (dataset "A") and $\theta_{k,0.05} = 0.834$ (dataset "B"). The quality of the model is quite high when CoV is slightly above 0.15 in the case of the dataset "A".

The CF CSCT (2017) model does not have required safety with result of $\theta_{k,0.05} = 0.737$ (dataset "A") and $\theta_{k,0.05} = 0.745$ (dataset "B"). However, the quality of the model is higher than the EC2 model, because CoV reached a value of 0.130 and 0.138, respectively. The model needs to change failure criterion used to attain a higher mean value of the ratio θ_m.

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