

# TIME-DOMAIN SIMULATIONS OF GROOVING PROCESS WITH STIFFNESS UNCERTANTITIES

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**Abstract:** In the world of nonlinear dynamics, it is necessary to consider the fact that a small change of input parameter can cause large changes in results. However, several methods deal with uncertainty. This paper compares two methods for uncertainty analysis, polynomial chaos expansion, and Monte Carlo method. The analysis was performed on the structure of the machine tool slide. The two main components - linear roller guideway with preload and linear ball screw, they both present with the nonlinear characteristic. The stiffness of these components is dependent on the deformation as well as the direction of load. The results of both methods match reasonably in a stable condition. For the unstable machining should be surrogated model improve for further analysis. The main advantage of PCE is a shorter computation time in contrast to time-consuming MC. This approach enables not only the stimulation of the machining process in local conditions but in the future, it might be also extended for the analysis of machine tool stability in the full condition range.

# Keywords: Polynomial chaos expansion, Monte Carlo, Turning model, Uncertainty analysis, Nonlinear stiffness.

### 1. Introduction

The frequent problem of mechanical structure analysis is the uncertainty of input parameters. This is the main problem in the nonlinear structures where a small change of input values could cause a large change in the system response. Therefore, one must always keep in mind the uncertainty of a nonlinear system.

Thanks to the topicality of uncertain parameters problem, several works describing the analysis of structures with uncertain parameters can be found. Nevertheless, due to long computational time of large-scale structure, most works focus on systems with a small number of uncertain parameters. The Sandus analyzed the quarter car model with wheel-surface contact, with uncertain define by a random variable, the generalized polynomial chaos (gPC) was used for analysis and the results were validated by Monte Carlo method (MC) (Sandu and Sandu et al., 2006). Furthermore, they extended their original model and focused on parametric uncertainty (Sandu and Sandu, 2015). Zhang described the problem of the stochastic response of offshore wind turbines using Monte Carlo (MC) (Zhang and Høeg et al., 2019). The usage of polynomial chaos expansion PCE was described by Poursina on the example of the multi-body system (Poursina, 2015). The same method was applied for the analysis of aeroelastic systems with parametric uncertain parameters by (Desai and Sarkar, 2010), as a reference method was again used MC.

The machining process is highly influenced by dynamic behavior. The issue is not caused by the selfexcited vibration, well known as chatter, but it is also affected by the precision and surface quality. The evaluation of uncertain parameters is necessary for robust stability prediction. In 2016, Hajdu presented the analysis of robust stability prediction in the turning process (Hajdu and Insperger et al., 2016). The two key works of chatter prediction in the milling process were presented by Tlusty and Altintas with Budak and

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later extended by Duncan with uncertainty propagation (Tlusty and Zaton et al., 1983; Altintas and Budak, 1995; Duncan and Kurdi et al., 2006).

This paper presents a dynamic model of the machine-tool slide with structural nonlinearities. In this structure two main components, both linear roll guideway and ball screw are characterized by nonlinear stiffness. The model of stiffness covers the uncertainties that may influence the behavior of the structures. Two methods of PCE were used for the analysis of the results and they were subsequently validated with MC.

#### 2. Polynomial chaos expansion

The main motivation for the use of PCE was the shortening of computational time. The main application is the situation where there are limited numbers of uncertain parameters with high uncertainty in a model of nonlinear behavior. In such cases, direct methods, such as MC, would be highly impractical.

The main principle is to build a surrogate model  $M^{PC}$  based on finite polynomials series which substitutes the original computation model M. Where X is the vector of inputs and Y is the vector of outputs.

$$Y = M(X) \tag{1}$$

$$\hat{Y} = \mathsf{M}^{PC}(\mathsf{X}) = \sum_{k=0}^{P} \hat{y}_k \psi_k(\mathsf{X})$$
<sup>(2)</sup>

The  $\psi_k$  is basis function and  $\hat{y}_k$  are coefficient which needs to be computed. The PCE is made of multivariate orthonormal polynomials. The sum of the Hermits polynomials builds the basis of Hilbert space. The key step is the evaluation of coefficients, usually nonintrusive least square method is used. For the model cross-validation, the method leave-one-out was used. The whole procedure is described in detail by Sudret (2017).

#### 3. Model of machine tool slide with nonlinear stiffness

The model of the machine-tool slide is represented by its momentum of inertia, mass and three supports, which connect the slide with the base the system can be seen in Fig. 1. The cutting force is split into two orthogonal components Fc, Ff, by the angle of 30 °. All damping parameters are modeled as linear with the assumption of 2 % critical damping of the linearized system.



Fig. 1: The scheme of modeled structure.

The springs k1 and k2 represents the behavior of linear guideway with nonlinear characteristics. The spring k3, which is representing the ball-screw, also has the nonlinear representation. The representation used in our model was presented by Dolata (Dolata and Jastrzębski, 2015). Both characteristics, the linear guideway, and the ball screw are not symmetrical by origin. The ball screw was approximated by a third-order polynomial, which must be split into two parts - positive and negative, which differs by the linear coefficient. The linear ball guideway characteristic is represented by a second-order polynomial. The nonlinear coefficients were considered as uncertain. In the case of the linear guideway, it was the quadratic coefficient in the case of ball screw quadratic and cubic. In the end, there are four uncertain parameters with the normal distribution. Fig. 2 shows the distribution of coefficients. The stiffens characteristic with uncertainty can be seen in Fig. 3., where the ball screw is on the left side and the linear guideway on the right side.



Fig. 2: The normal distribution of each coefficient.



Fig. 3: Characteristic under uncertainty: the left represents the ball screw; the right represents the linear guideway.

The key part of this work is the time-domain simulation which represents the regenerative chatter model presented by Schmitz (2009). The model is described by a Lagrange differential equation for three degrees of freedom and it was simulated in the Matlab-Simulink, this model was used for both PCE and MC. The main difference lies in the input cycle. The whole simulation model is described in detail by Svobodová (2020).

#### 4. Results

The results of two different grooving conditions are displayed in Fig. 4. The results of simulation in purely stable conditions are shown in part (a) and partially in (b). It is noticeable that for the edge of stability MC gives slightly different results comparing to PCE, where the distribution range is wider. On the other hand, in the stable conditions, where the character of deflection has mainly static nature, results are almost identical. The problem of worse prediction on the edge between stable and unstable conditions probably lies in computing basis coefficient, which creates the room for the method improvement. The question is, if the time for the PCE model tuning does not consume more time than the whole MC, in the systems with changeable behavior.



Fig. 4: Machining in the stabile region (a) and on the boundary of the stabile machining condition.

#### 5. Conclusion

This paper presents an example of the growing process analysis with structural uncertainties. Two methods have been presented PCE and MC, which was also used for PCS base coefficient calculation. The results show that the PCE method gives reliable results in the 80 % shorter time comparing to MC. However, the weak point can be the preparation time of the metamodel, which can beat the time savings during the simulation. Mainly the base function computation and its validation must be considered, during the planning of the simulation. The simulation was done for two machining conditions. Firstly, it was performed in stable region and subsequently, at the edge of stability. The distribution of maximal deflection during the grooving process is not surprising and corresponds with the stiffness distribution. For the future analysis of the whole combination of possible machining conditions, the optimal number of distribution combination must be analyzed first, to do analysis as efficient as possible.

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