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DETERMINATION OF TRANSMISSION ERROR AT HELICAL GEAR

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Abstract: *This paper deals with the static transmission error issue, which is one of the measured parameters of vibration in gear pairs and transmission systems, which is connected to noise and vibration. The transmission error can be determined in several ways, including a technical experiment. However, due to time and financial reasons, this is not always possible, and therefore, the possibility of using numerical simulations is offered. In this paper, the approach using stress-strain quasi-static contact analysis using the finite element method in Ansys Workbench software is used. The advantage is, among other things, a good comparability of results. The input 3D CAD geometry is a helical gear pair, which was created fully parametrically, so that various configurations could be generated and analyzed, including the gear teeth modifications. The peak-to-peak value of the transmission error is the subject of evaluation, since it turns out as a significant cause of vibration. Based on the result, the presented approach with numerical simulation can be used as a very useful tool for comparison.*

Keywords: Static transmission error, Helical gears, Gear mesh, Gear tooth modification, Tooth tip relief.

1. Introduction

Designing gears in order to fulfill high demands on strength, material fatigue and service life in all operating modes is a relatively complex process. When the requirements for low vibration and noise are added, the process becomes even more complicated. Nowadays, noise is often subjected to increasingly stricter demands across the industry, including the automotive one. With relatively quiet internal combustion engines or electric motors, the transmission systems can become one of the dominant sources of vibration and noise in the vehicle. With the onset of electromobility, this problem persists, as most electric cars have a single-stage reduction gearbox and the electric motor is quieter compared to an internal combustion engine. With demands on increasing driving distance multi-speed transmissions are used more often.

It turns out that the concept of the transmission error is closely related to vibration and noise. (Prokop et al., 2017). In an ideal world with perfectly rigid gears, shafts and other parts of the transmission system, without manufacturing and assembling inaccuracies, there would be no transmission error and the engagement of the gears would not cause vibrations and subsequent noise caused by vibrating thin-walled surfaces (gearcase, bodywork). In fact, this is not that case, so the transmission error needs to be dealt with.

Transmission error (TE) can be defined as the angular difference between the theoretical position of the driven gear (if the gears were precisely manufactured and perfectly rigid) and the actual position of the driven gear. Although the measurements result in angular displacements, these values tend to be multiplied (in radians) by the radius of the pitch or base circle. This gives the units of length (in micrometers). The advantage of this conversion is better comparability of the transmission error between the individual gears, as all gears of a given quality, regardless of their size (module, number of teeth, etc.), have approximately the same transmission error. The TE can be expressed by the following equations (Smith, 2003):

$$TE = \frac{\theta_1}{i} - \theta_2 , \tag{1}$$

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$$TE = r_1\theta_1 - r_2\theta_2 \,, \qquad (2)$$

where θ_1, resp. θ_2 is angular displacement of driving, resp. driven gear, i is transmission ratio, r_1 and r_2 are pitch circle radii. Expression $\frac{\theta_1}{i}$ represents theoretical angular position of the driven gear.

Transmission error can be divided into the static and dynamic TE. The static TE is caused by the alternation of the number of teeth in the engagement, deformation of the teeth and the whole system – shafts, bearings and gearcase. This error therefore depends mainly on the stiffness of the whole system and the variable stiffness of the engagement. It is measured at low speed and higher load (torque). The dynamic TE is measured at operating load and speed. Thus, the dynamic forces during rotation caused by the weight (moments of inertia) of the gears, as well as the internal damping of the gears, shafts and bearings, are also present. The dynamic TE also includes the static TE. (Prokop, 2016) The transmission error can be determined by numerical approach (Bartošová et al., 2018), or technical experiment (Kučera et al., 2018). This paper deals with the static TE, without considering the manufacturing and assembling inaccuracies, by using numerical simulation.

2. Methods

In this paper, the approach using numerical simulations – FEM analyses – was chosen to determine the static transmission error. The input geometry was a helical gear pair with the transmission ratio equal to one, normal module 3 mm, number of teeth 30, normal pressure angle and helix angle 20 °, gear width 24 mm and working center distance 96.026 mm. The number of teeth around the circumference was chosen to a smaller value to minimalize the number of mesh elements and nodes. The initial sensitivity study was performed to determine the number of teeth which has to be considered. The middle tooth pair was adjusted to be exactly at the beginning of the engagement.

First, the model was discretized, smaller elements were used in important areas in terms of equivalent stress (von Mises) and contact pressure (see Fig. 2) – tooth flanks and roots – and the mesh was created evenly. For each configuration, the mesh was set up in the same way so that the results could be better compared to each other. A sensitivity analysis of the mesh and the number of teeth used (out of the total number) was also performed. It turned out that in this case the element size of 0.25 mm (see Fig. 1) and the number of teeth around the circumference of 5 are sufficient in terms of results accuracy, computational time and the size of result files.

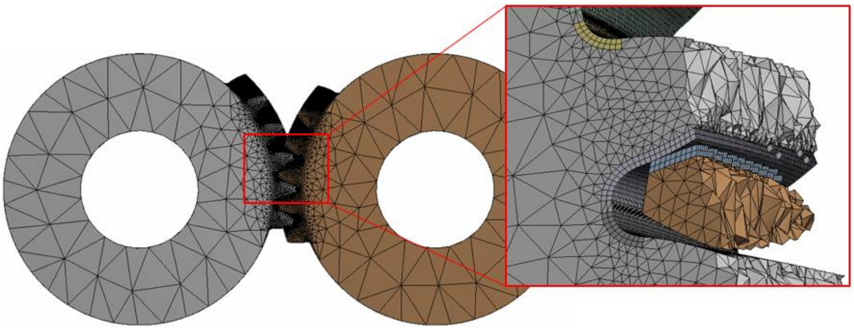

Fig. 1: Finite-element mesh of the helical gear pair.

The driven gear was loaded with a braking torque of 100 N·m, which preloaded the teeth of both gears, and was allowed to rotate freely around its axis. The driving gear rotated gradually with a given step until the middle tooth pair went through its entire engagement. The full torque was present throughout this rotation. Frictional contacts were also set onto the flanks of the teeth with a coefficient of friction of 0.1.

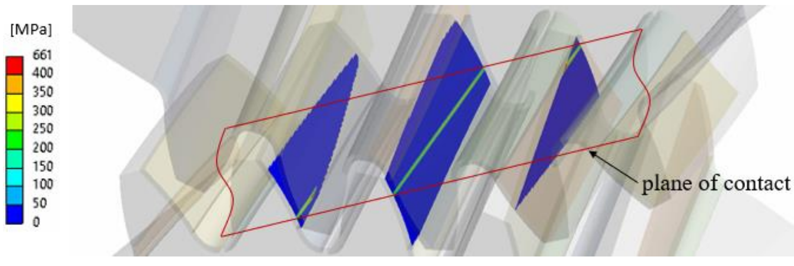

Fig. 2: Contact lines in the plane of contact.

After the simulation was successfully completed, the angular displacements of the driving and driven gear were evaluated. These values were substituted into the equation (2) to obtain the transmission error in microns, and subsequently a graph of the TE as a function of the driving gear rotation.

Regarding the tip relief modification, its amount was determined from the average transmission error (within the engagement of one pair of teeth) and the length (extent) by (Palmer et al., 2012). The modification is linear in terms of roll distance. This means that the modified part of the tooth profile is also involute but with a different pressure angle. The design (in terms of the lowest TE) and the maximum (in any terms) load were equal, so it is a long modification. The root relief modification was determined analogical.

3. Results

Fig. 3 compares the graphs of the static TE of spur and helical gears for the same load. The parameters of spur gears were the same as the parameters of helical gears in the transverse plane. In the case of spur gears, one and two pairs of teeth alternate in the gear mesh. The minimum TE occurs with a two-pair mesh, the maximum with a single-pair mesh. Moreover, the number of tooth pairs alternates suddenly. With the helical gearing, two and three pairs of teeth alternate in the gear mesh. The minimum TE occurs with a two-pair mesh, the maximum with a change in the number of tooth pairs in the mesh. In both cases, the TE is periodic with the period of one circular pitch. The peak-to-peak value of the TE (PTPTE) is almost 6 times higher at the spur gears than at the helical ones, thus the advantage of helical gears has been confirmed.

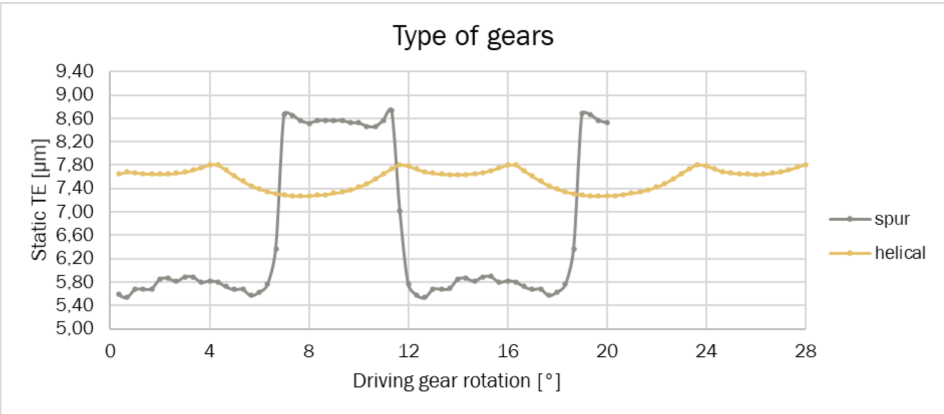

Fig. 3: Graphs of the static TE for the spur and helical gears.

Next, the effect of tooth modifications was analyzed. Modifications of tip relief, root relief and a combination of both on one gear while leaving the other gear unmodified turned out to be almost equivalent (see Fig. 4). The PTPTE value decreased from 0.54 µm to 0.11 µm. The average TE increased due to the higher bending stress in the roots, which was caused by the different distribution of the contact pressure on the tooth flanks. Too long relief tip (starting on the pitch cylinder) did not bring such a significant improvement. Tip corner rounding (without other modifications) caused a slight increase in the PTPTE. The reason is probably a shortening of the active involute profile.

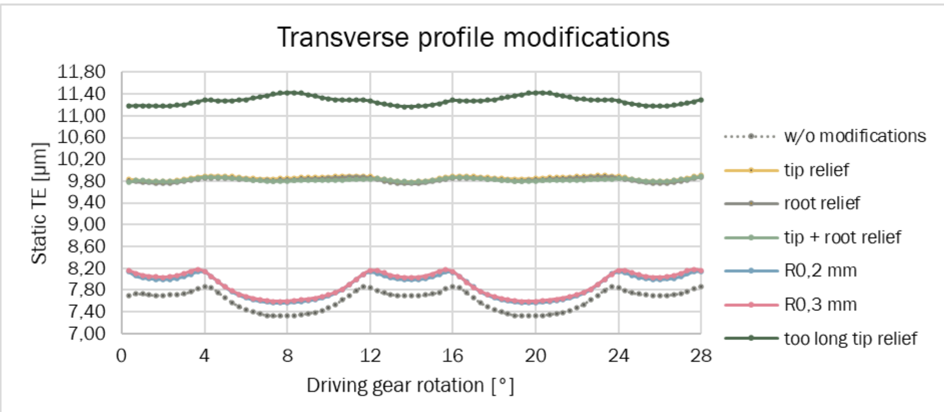

Fig. 4: Graphs of the static TE for the transverse profile modifications.

The influence of the overlap ratio was also analyzed (see Fig. 5). Its value was changed by changing the helix angle. Theoretical assumptions were confirmed – the local minimum of the PTPTE value (in this case 0.24 μm) occurs at the integer value of the overlap ratio. Using the tip relief modification, the PTPTE value decreased to a similar value (0.09 μm) as at the default configuration with this modification.

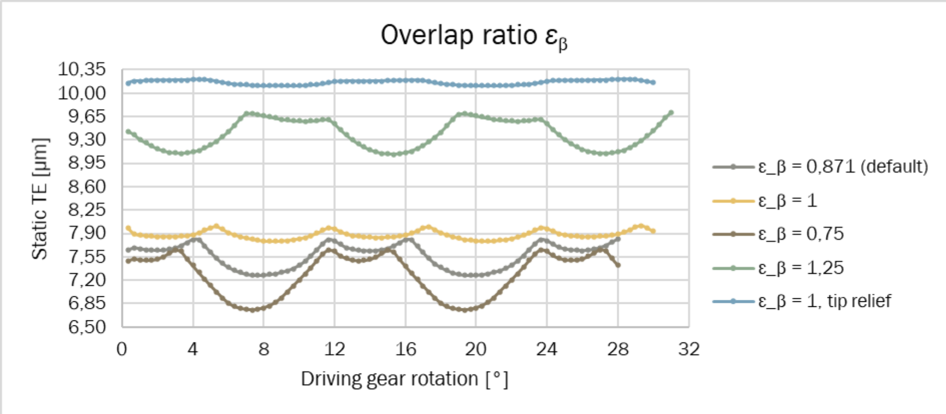

Fig. 5: Influence of the overlap ratio on the static TE.

4. Conclusions

To determine the static transmission error, the approach using numerical simulations – FEM – was chosen. The advantage of this approach was the ability to analyze a relatively large number of arbitrary gear configurations without the need to manufacture real components, which is suitable from a financial and time point of view. A parametric 3D CAD model of gears and gear pairs was created, including gear teeth modifications. Subsequently, numerical simulations were performed on the 3D model in FEM software. The modifications contribute to the reduction of TE but only in the given boundary condition and operation conditions, thus it is necessary to perform a wide range of numerical simulations to create knowledge database. In the future, the mathematical/computational model can be extended to shafts and the effect of flank line modifications can be investigated, and the final design will be manufactured and tested.

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References

Bartošová, D., Otipka, V. and Řehák, K. (2018) Determination of transmission error in spur gear by numerical approach. Vibroengineering Procedia, 19, pp. 284-288.

Kučera, P., Píštěk, V., Prokop, A. and Řehák, K. (2018) Transmission error analysis for heavy-duty gearbox. Vibroengineering Procedia, 18, pp. 113-116.

Palmer, D. and Fish, M. (2012) Evaluation of Methods for Calculating Effects of Tip Relief on Transmission Error, Noise and Stress in Loaded Spur Gears. GearTechnology, pp. 56-67.

Prokop, A. (2016) Powertrain vibration modelling using virtual prototypes. Doctoral thesis, BUT, Brno (in Czech).

Prokop, A., Kopečková, B. and Řehák, K. (2017) Gear Drive System Simulation of Input Parameters Effect on Rattle. Acoustics and Vibration of Mechanical StructureS. Springer Proceedings in Physics, pp. 381-388.

Prokop, A. and Řehák, K. (2017) Virtual prototype application to heavy-duty vehicle gearbox concept. Engineering Mechanics, pp. 810-813.

Smith, J. D. (2003) Gear noise and vibration. 2nd edition. New York: Marcel Dekker.