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OPTIMAL DESIGN OF THE DOUBLE INVERTED PENDULUM

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Abstract: *The inverted pendulum is a classic example commonly found in control theory. Double inverted pendulum which is a combination of inverted pendulum and double pendulum makes the control problem more challenging. The purpose of this work was to design an optimal mechanical construction of a linear double inverted pendulum on a cart. Pendulums usually have the form of a freely attached rod to the cart moving in one dimension. The presented system assumes the functionality which ensures better efficiency. The main determinant of optimal parameters is the stabilization of the pendulum in vertical position and control of the position of the cart on the guide rail. The general concept outlined at the beginning presents the key elements necessary for the proper operation of the system. In some systems available in the literature and Internet, the engine moves with the cart, increasing the inertia forces acting on it. In the designed system, it was decided to fix the motor to the base. The engine in such a system should have the precise control and a high starting torque that allows stabilization of the pendulum. The presented conception base on the most optimal selection of all components.*

Keywords: Double inverted pendulum, Mathematical model, Nonlinear system

1. Introduction

The inverted pendulum is a nonlinear system classified as a balancing robot (Arakelian, 2015), which represents a high didactic value. It is understood as a physical pendulum whose centre of mass is located above its suspension point. This model is often used in control theory for analysis of modern control algorithms (Hung, Fernandez, 1993). Research in this field allowed for the development of self-balancing vehicles (Ciężkowski, Pawłuszewicz, 2015), mechanics of robot's gait etc (Astrom, Murray, 2008). The double inverted pendulum is a more non-linear and unstable dynamic system. This kind of system has three degrees of freedom, and the only parameter that can be controlled is the force applied to the pendulums cart (Eide et al, 2011). Its displacement sets the pendulum in motion. The purpose of this work was to design and build a double inverted pendulum on a cart operating in one dimension.

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2. Projects assumptions

The first step was to design the general concept of the double inverted pendulum. The key elements are presented in Figure 1. The main determinant for the optimal project is the stabilization of the pendulum in a vertical position. The guide must realize quick changes of the cart's position, high speeds and acceleration. In some of the inverted pendulum systems available in the literature and the Internet, the motor moves with the cart, increasing the forces of inertia. In the designed construction the motor was fixed to the base. Stabilization of the pendulum in a vertical position will also generate vibrations. That's why the angular position sensors (encoders) must be light and resistant.

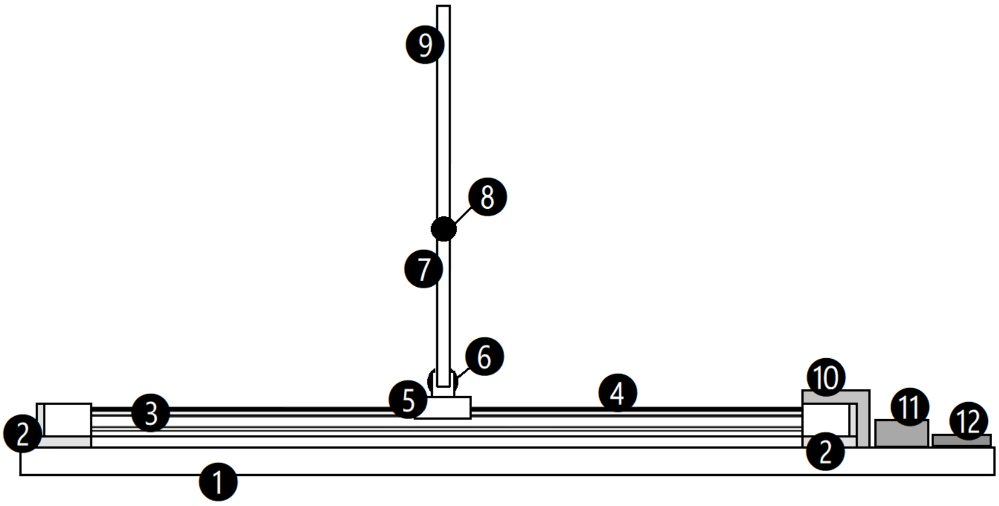

Fig. 1: *The conception of a double inverted pendulum system: 1. Base, 2. Mounting, 3. Guide rail, 4. Belt, 5. Cart, 6. Encoder no 1, 7. First joint, 8. Encoder no 2, 9. Second joint, 10. Stepper motor, 11. Motor drive, 12. Control module.*

2.1. The guide rail

The leadscrew system was excluded at fast positioning and continuous operation. The best solution is this case is sliding table moving on a guide rail using a toothed belt. The linear motion is ensured by the lubricant-free polymer inserts instead of balls, as shown in Figure 2. The use of linear slide bearings without rotating or ball elements give the possibility for work with high speeds up to 10 m/s and accelerations up to 100g. Linear bearings are therefore suitable for small loads, where it is necessary to increase the number of cycles. The use of hard anodized aluminium lowers the temperature of working bearings due to thermal conductivity. Thanks to this the system can work with a large one frequency, even at very short stroke lengths.

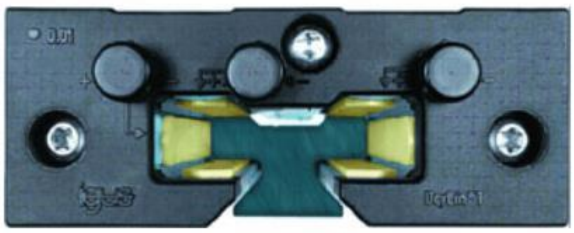

Fig. 2: *Polymer inserts used in the cart (https://www.igus.com/info/linear-guides-zlw-ca)*

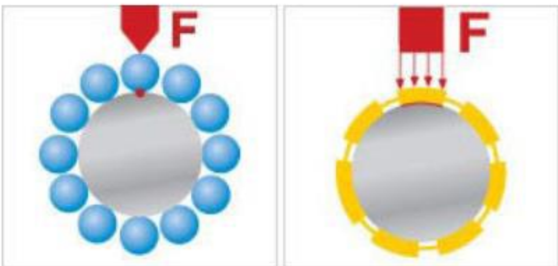

Fig. 3: *Load distribution on rolling elements (left) and lubricated plastic guides (right) (https://www.igus.com/info/linear-guides-zlw-ca)*

The plain bearings work on sliding elements as opposed to systems of rotating ball bearings as shown in Figure 3. Available surface it is larger, which reduces surface loads. The optimal force distribution allows using of non-reinforced rollers and non-metallic elements. Self-lubricating linear bearings have low coefficients of friction and are great for short-stroke applications.

2.2. The drive system

The used drive system is able to reach high acceleration values. The stepper motor NEMA23 (Figure 4) has also a built-in incremental encoder which increases the precision of control. Stepper motor allows to directly change the angle of rotation, which is proportional to the translation of the cart (Figure 5). This feature is a great advantage because the control system can affect acceleration, not force. In this case, neither mass or moment of inertia of the motor are eligible in the analysis.

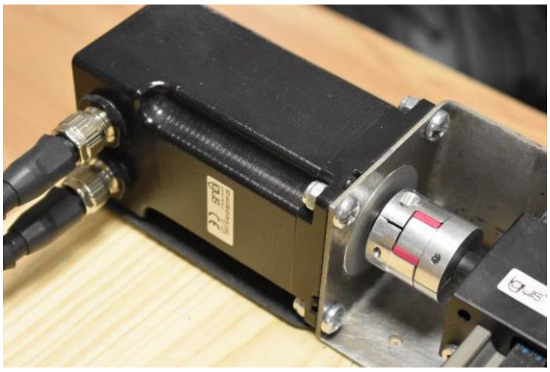

Fig. 4: *The stepper motor NEMA23 mounted to the system*

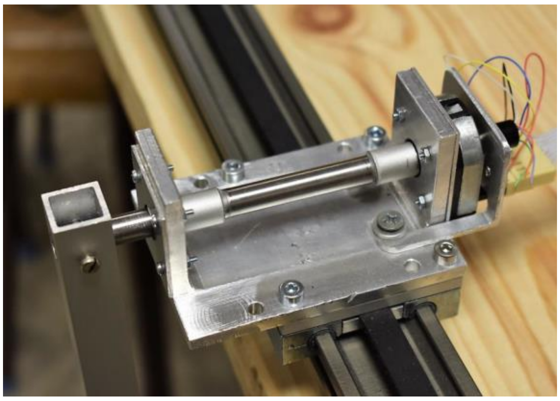

Fig. 5: *The carriage of the pendulum*

3. Mechanical construction

The pendulums carriage showed in Figure 5 is adapted to carry the wires inside. This solution allows no limited cyclic movements of the pendulum. The incremental encoder AMT 103-V is responsible for measuring the angle of the first joint. The cross-section of the cart with wires marked red is presented in Figure 6. One of the advantages is the through hole, which gives the possibility of mounting the encoder on the hollow shaft to guide the wiring from the second encoder.

The pendulum link is shown in Figure 7. The second joint was attached with a shaft with the second encoder magnet mounted at the end. The rolling bearing was placed directly in the first joint to increase the connection strength.

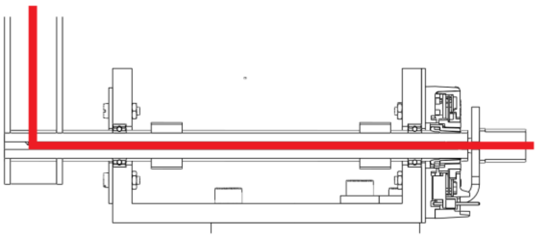

Fig. 6: *The cross-section of the first joint with encoder and slip ring*

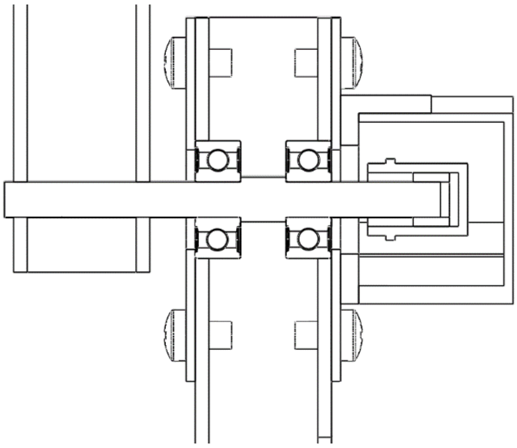

Fig. 7: *The cross-section of the second joint with encoder*

The designed and built system during operation is presented in Figure 8. After the determining of the mathematical model and equations of motion, the control law was designed. The linear-quadratic regulator (LQR) was successfully tested on the built system. The system has reached stable work in a vertical position and was able to withstand external disturbances like touch or impact.

Fig. 8: *Test stand of the double inverted pendulum during operation*

4. Conclusions

The applied construction solutions make the physical system does not differ than the derived mathematical model. The mechanical design of the test stands to study the problems of control system algorithms is often ignored, because scientists focus only on describing the regulation. This article presents how smart mechanical design can simplify the mathematical model of the system by getting rid of unnecessary mass and moments of inertia from moving parts of the system. It also shows how to change the type of controlled parameter by using a different type of motor. The direct application of acceleration in the stepper motor allows simplifying mathematical equations.

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