

# DOPROC METHOD IMPROVEMENTS AND ITS APPLICATION IN STRUCTURAL FATIGUE ANALYSIS

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**Abstract:** Probabilistic methods allow better account of the randomness of input quantities in the design process and the reliability assessment. These computing approaches are currently enjoying an increasing popularity in civil engineering practice. New probabilistic procedures have been developing and implementing in commercial software. The main disadvantage is the inaccurate estimate of the resulting probability of failure and the high demands on computing technology and machine time. The presented DOProC method uses optimized numerical integration to calculate the probability of failure. The main improvement of this method is an increase in the accuracy of calculating the probability of failure by modification of the basic algorithm and reduction of computation time using parallelization. First experiences are gained in using the DOProC method on supercomputers. Probabilistic modeling and prediction of fatigue damage is one of the fields where the DOProC method is advantageously utilized and applied.

Keywords: Fatigue, steel structure, DOProC, probability of failure, parallelization.

## 1. Introduction

Probabilistic methods are used in engineering where a computational model includes random input variables, e.g. Park (2019) or Kala et al. (2017). When designing a structure, various reliability criteria should be met in accordance with the relevant standards Kotes and Vican (2013). Each random variable in the probabilistic computations contains uncertainties which may be widely classified into two main categories: *aleatoric* (stochastic) *uncertainties* of a random nature and *epistemic* (state of knowledge) *uncertainties* that arise owing to imperfect knowledge in analysis of the solved problem. Typical sources of aleatoric uncertainties are material properties, e.g. Kormanikova and Kotrasova (2018), Kralik (2010) or Major and Major (2017), and production and/or assembly inaccuracies in the geometry or the environment where the structure should be located. The final reliability of the structure is also affected by epistemic uncertainties which depend on the computational model used, statistical processing of input data, which also involves a human factor in the design process, and/or construction and use of the structure.

# 2. Refinement of the failure probability calculation

The DOProC method (Direct Optimized Probabilistic Calculation) is one of many probabilistic methods which has been developed since 2002. The DOProC method uses optimized numerical integration to calculate the probability of failure. The main advantages include: the high accuracy of the failure probability calculation and the high efficiency of the calculation for many probability tasks. Principle of the method

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was published e.g. by Janas et al. (2017) or Janas et al. (2015). If the values  $a_{1,i_1}, a_{2,i_2}, a_{3,i_3}, \cdot, a_{j,i_j}, \cdot, a_{n,i_n}$  are statistically independent, then the probability of occurrence of value  $b_k$  of histogram of resulting random variable B (e.g. reliability function Z in case of probabilistic reliability assessment) for a given combination k histogram classes of independent random variables  $A_j$  will be equal to the product of probabilities  $p(a_{j,i_j})$  of all intervals (classes)  $a_{j,i_j}$ :

$$p(b_k) = \left[ p(a_{1,i_1}) \cdot p(a_{2,i_2}) \cdot p(a_{3,i_3}) \cdot \ldots \cdot p(a_{j,i_j}) \cdot \ldots \cdot p(a_{n,i_n}) \right]_k .$$
(1)

Each of the values  $b_k$  will have the probability  $p(b_k)$  and will be assigned to the appropriate interval  $b_{i_B}$ . For all values of  $b_k$  inserted into the interval  $b_{i_B}$  will match the probability:

$$p(b_{i_B}) = \sum_k p(b_k) .$$
<sup>(2)</sup>

The simplest procedure for computing the probability of failure  $p_f$  is based on the analysis of the resulting histogram of the reliability function Z:

$$p_{f} = \sum_{\substack{i_{Z}=1\\i_{Z}=1}}^{i_{Z}=N_{z=0}-1} p(z_{i_{Z}}) + p(z_{N_{z=0}}) \cdot \left(1 - \frac{z_{N_{z=0}} + \frac{\Delta z}{2}}{\Delta z}\right) =$$

$$= \sum_{\substack{i_{Z}=1\\i_{Z}=1}}^{i_{Z}=N_{z=0}-1} p(z_{i_{Z}}) + p(z_{N_{z=0}}) \cdot \left(\frac{1}{2} - \frac{z_{N_{z=0}}}{\Delta z}\right) .$$
(3)

The experience and development of the method have shown that the resulting probability of a failure may not be determined in this way, but can to be computed by successive sums in the core of the basic algorithm:

$$p_f = \sum_{z_{(i_Z)} < 0} p(z_{i_Z}) + 0.5 \cdot \sum_{z_{(i_Z)} = 0} p(z_{i_Z}) .$$
(4)

Such a calculation method is much more accurate and depends only on the accuracy of the input data, its discretization and the computational error of the processor.

## 3. Parallelization of the DOProC computation

Calculations by DOProC method can be very time consuming. Certain parts of the calculations may take place simultaneously. The DOProC algorithm described previously is advantageous for use on computers with two or more processor CPUs or cores. In the basic computational algorithm, it is possible to divide the total number of computational operations into as many parts as are available number of computing units. After partial calculations, the histogram of the resulting variable (e.g. reliability function Z in probabilistic reliability assessment) can be compiled from the partial results.

Parts of resulting random variable histogram B are not matched by the relationship (1), but by choosing the same width of the intervals  $\Delta b_{1,p}$  for all parts, ie also for the random variable histogram B, and are summed in each probability interval of all parts of histograms  $B_p$  of random variable B for  $p = (1, \ldots, p, \ldots, s)$ . This procedure ensures that the result of the parallelized calculation is identical to the unparalleled calculation.

Computational operations running on more complex tasks, such as probabilistic prediction of fatigue damage in steel structures from Krejsa et al. (2017) (number of load cycles per year – normal distribution,  $\mu = 10^6$ ,  $\sigma = 10^5$ ; initial size of fatigue crack – log-normal distribution,  $\mu = 0.2$  mm,  $\sigma = 0.05$  mm; detectable size of fatigue crack – normal distribution,  $\mu = 2$  mm,  $\sigma = 0.2$  mm; deterministic material characteristics m = 3,  $C = 2.2 \cdot 10^{-13}$  MPa<sup>m</sup>m<sup>(m/2)+1</sup>), can be easily adjusted to run in parallel, such as Lan et al. (2018) or Cho and Hall (2012). If two or more processors are used, then the calculation time can be shortened considerably. Different programming system has to be used for application of parallelization in DOProC method on supercomputers (eq in IT4Innovations national supercomputing center). Matlab seems to be as a very advantageous software for this purpose. The algorithm of DOProC method is adapted for application in Matlab software on platforms with more than one cores using the SPMD (Single program,



Fig. 1: Parallel algorithm scaling: decrease of computing time with increasing number of processor units, input histograms described by 128 classes



*Fig. 2: Parallel algorithm scaling: decrease of computing time with increasing number of processor units, input histograms described by 256 classes* 

multiple data) parallel programming. This is the most common method of parallel programming that enables to split up and run the same task with different input and output data on more CPUs or CPU cores. This technique can also split up data arrays, which is an important feature when the calculation runs on cluster of more computers that do not necessarily share all of the memory resources. The parallel algorithm is divided into three parts. The first part that prepares input data runs on one client worker (processing core of multicore CPU or one CPU in cluster). Then the second part runs simultaneously on all workers with different inputs and outputs. This part contains the main multiple for loop of DOProC method and it is the most intense part of processing. Finally, in the third part client collects the different outputs and compiles the partial results into the overall result. The probabilistic reliability assessment of the load-bearing element loaded with cyclic load with respect to the occurrence of fatigue damage was made using this programming technique. There is a significant reduction in computing time (see Figs. 1 and 2) while preserving the accuracy of the solution (see Tab. 1).

Number of intervals	64		128		256		1024	
Core count	Time	$p_f$	Time	$p_f$	Time	$p_f$	Time	$p_f$
1	0.39	0.013495	2.71	0.01641	21.91	0.018427	-	-
3	0.13	0.013495	0.90	0.01641	7.10	0.018427	-	-
6	0.08	0.013495	0.50	0.01641	3.77	0.018427	-	-
9	0.07	0.013495	0.36	0.01641	2.62	0.018427	-	-
12	0.06	0.013495	0.32	0.01641	2.03	0.018427	88.03	0.019917

Tab. 1: Comparison of calculation times t [min] depending on the number of cores and the number of classes in input histograms including the resulting probability of failure  $p_f$ .

## 4. Conclusions

In this paper, it has been presented the improvements of the probabilistic method - DOProC, which is still under development, such as more accurate estimation of probability of failure and reducing calculation time using parallelization. It turns out that the DOProC method is very suitable for solving various engineering tasks such as modelling of fatigue problems and prediction of fatigue damage.

#### Acknowledgments

This contribution has been developed as a part of the research project GACR 17-01589S "Advanced computational and probabilistic modelling of steel structures taking account fatigue damage" supported by the Czech Grant Agency and also has been completed thanks to the financial support provided to VSB-Technical University of Ostrava by the Czech Ministry of Education, Youth and Sports from the budget for conceptual development of science, research and innovations for the 2019 year. Access to computing and storage facilities owned by parties and projects contributing to the National Grid Infrastructure MetaCentrum provided under the programme "Projects of Large Research, Development, and Innovations Infrastructures" (CESNET LM2015042), is greatly appreciated also.

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