24th International Conference
ENGINEERING MECHANICS 2018
Svratka, Czech Republic, May 14 –17, 2018
Paper **#264**, pp. 561–564, doi: 10.21495/91-8-561

COMPUTATIONAL TIME REVERSAL METHOD BASED ON FINITE ELEMENT METHOD: INFLUENCE OF TEMPERATURE

M. Mračko[*] , R. Kolman[*], J. Kober[*], Z. Převorovský[*], J. Plešek[*]

Abstract: *Time reversal method is used to focus elastic waves to the location of the original source and reconstruct its source time function. The procedure consists of two steps: Frontal task and Reversal task. In the Frontal task, the medium is excited by an arbitrary source, elastic waves propagate through a body of interest and the dynamic response at few points on boundary is recorded. In the second step (say the Reversal task) the response signal is reversed in time and transmitted back into the medium resulting in focusing in the original source location. It is of practical importance to investigate a case when the medium changes its properties between the frontal and reversal wave propagation steps. An example is a problem of transferring experimentally recorded data to a computational model, where discrepancies in geometry, elastic properties and boundary conditions are expected. Our motivation is to develop a methodology for computation of time reversal problems in commercial finite element software. The results prove that this method is extremely sensitive to the change of temperature and one have to pay special attention to tuning of elastic parameters relevant to the experiment.*

Keywords: time reversal method, elastic wave propagation, finite element method, temperature sensitivity, refocusing

1. Introduction

Method of time reversal (TR), see e.g. (Fink et al., 2000) or (Givoli and Turkel, 2012), uses backward wave propagation for refocusing and reconstruction of the original source. One possible application of TR method is in non-destructive testing (NDT) for localization of cracks, imperfections or defects in materials. The time reversal process consists of two steps. In the first step – Frontal task, a real body is loaded at the given place with the defined time history and a response is recorded in a prescribed position of the body. In the second step – Reversal task, this responding signal is reversed in time and loaded into the computational model so as to locate so called scatterers (e.g. cracks). In computational Time Reversal method, both steps are performed numerically. Nonetheless, here we focus on refocusing and reconstruction of the original loading signal.

In this paper, the linear theory of elastodynamics is assumed with Hooke's law. For numerical solution, we use the linear finite element method, see (Bathe, 1996), with the lumped mass matrix and one-point Gauss integration rule. For direct integration in time the explicit central difference scheme is employed. The time scheme is conditionally stable and reversible in time, see (Givoli and Turkel, 2012). The dispersion of the finite element method and suggestion for setting the suitable mesh size have been presented in (Kolman at el., 2016b).

In previous work, (Mračko et al., 2017), we have studied the influence of different approaches of prescribing boundary conditions and loading for suitable refocusing of an original source. As a continuation, this work is a parametric study where we investigate the influence of temperature on reconstruction and refocusing of the original source. Following test represents a change of temperature between the Frontal task and the Reversal task.

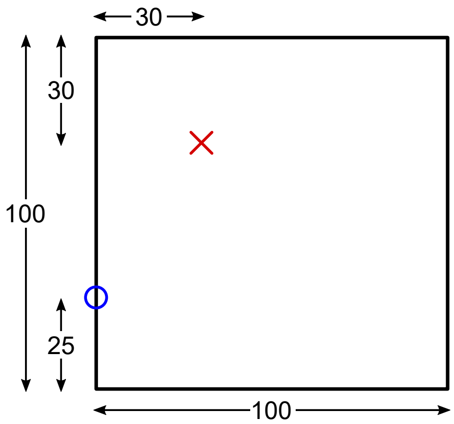
It is known a phenomenon in real applications that dimensions of bodies and their physical parameters are changed with temperature, see, e.g., (Ancaş and Gorbănescu, 2006). In this paper, we focus on situation

[*] Institute of Thermomechanics of the CAS, v. v. i.; Dolejškova 1402/5; 182 00, Praha 8; CZ, {mracko, kolman, kober, zp, plesek}@it.cas.cz

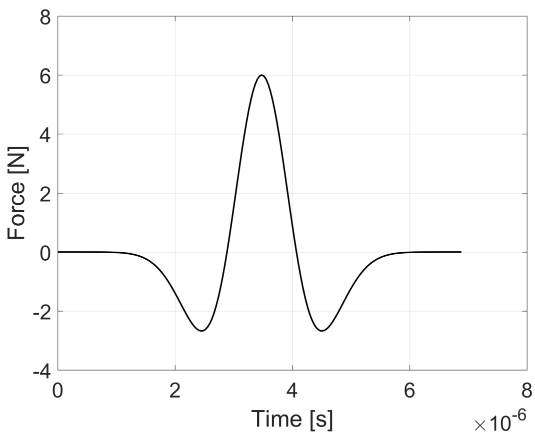
that temperature of frontal and reversal problem changes around several degrees of Celsius. Even though the change in shape can be neglected (Croxford et al., 2010), the change in elastic parameters plays an important role, see, e.g., (Převorovský and Kober, 2015), and this small temperature variation has to be taken into account in real applications of TR. The material parameters relevant to simulations of elastic wave propagation problems in solids are Young's modulus E, Poisson's ratio ν, and mass density ρ. For common structural steels, when the temperature increases, all previously stated material parameters decrease. This results in decrease of elastic wave speed. We simulate the change of temperature by modifying Young's modulus E to make our study dependent only on one parameter.

2. Problem description

We consider a two-dimensional domain under a plane stress condition with stress-free boundary conditions. Dimensions of the domain of interest are depicted in Fig. 1a. In the Frontal task, the position "X" shows the location of the loading signal and the position "O" shows the place of recording of the response. In the Reversal task, these points are swapped. The domain is divided into $250 \times 250 = 62500$ four-noded bilinear finite element elements.



(a) Scheme of the stated problem with dimensions [mm]



(b) Loading Ricker shape pulse

Fig. 1: Scheme of the problem and the loading pulse

The considered material parameters of a common structural steel are $E = 2 \cdot 10^{11}$ Pa, $\rho = 7\,850$ kgm^{-3} and $\nu = 0.3$. This gives the longitudinal wave speed (for 2D plane stress),

$$c_L = \sqrt{\frac{E}{(1-\nu^2)\rho}}, \tag{1}$$

as $c_L \doteq 5\,290$ ms^{-1}. For the following wave propagation simulations, the material parameters are nominal and we say *reference* values.

The Frontal simulation was computed with the reference values of material parameters. In TR simulations we consider an "increase of temperature". As it has been said, this is achieved by changing the value of Young's modulus E. As the determining quantity we chose the longitudinal wave speed c_L and Young's modulus E is then derived from the relation (1). In the Tab. 1, the values of c_L and E for particular tasks are presented. Roughly estimated, according to Ancaş and Gorbănescu (2006), these changes in velocity correspond to an increase of temperature between $1.5\,°C$ and $15\,°C$.

In the Frontal task, the point marked as "X" was excited by a force signal in the vertical direction with the time history given by a Ricker pulse, see Fig. 1a. The recorded value at the position "O" is the time history of the normal part of velocity on the outward boundary. This velocity time history is reversed in time and prescribed as loading signal, again as a force. Time of computation is $2 \cdot 10^{-3}$ s, which allows the longitudinal waves to reflect approximately 100 times between opposite edges. The time step size is $3.7 \cdot 10^{-8}$ s. Time of computation of reverse tasks was slightly extended in order to catch the reconstructed pulse.

Tab. 1: Change of longitudinal wave speed c_L and Young's modulus E in case study

Number of task	Decrease of c_L in %	c_L [ms^{-1}]	E [Pa]
Reversal task 0	0.00	5 291.265	$2.0000 \cdot 10^{11}$
Reversal task 1	0.01	5 290.735	$1.9996 \cdot 10^{11}$
Reversal task 2	0.02	5 290.206	$1.9992 \cdot 10^{11}$
Reversal task 3	0.05	5 288.619	$1.9980 \cdot 10^{11}$
Reversal task 4	0.08	5 287.032	$1.9968 \cdot 10^{11}$
Reversal task 5	0.10	5 285.974	$1.9960 \cdot 10^{11}$

Remark: *Used computational model is linear which makes it independent of a magnitude of the loading pulse and in principle, the reconstructed signal can be scaled arbitrarily. To suppress the effect of a singularity, the load is not prescribed only for one node. On the contrary, in the "X" position we use four nodes and in the "O" position we use two nodes. Output signal from the Frontal task is recorded in those two nodes, averaged over nodes, and prescribed as a load to the Reverse task in the averaged form for both nodes. Analogically, the output signal from the Reverse task is averaged among corresponding four nodes and used as a final output of reconstruction of the original time function of the source.*

3. Results

The influence of temperature on quality of a reconstruction of the original source time function has been studied in TR simulations for different perturbations of wave speed with respect to the reference value. In Fig. 2, one can see the reconstructed pulses for particular decreased wave speeds. Fig. 3 shows the wider time interval. It is apparent that for values over 0.05% of longitudinal velocity decrease, the reconstructed signal becomes hardly distinguishable.

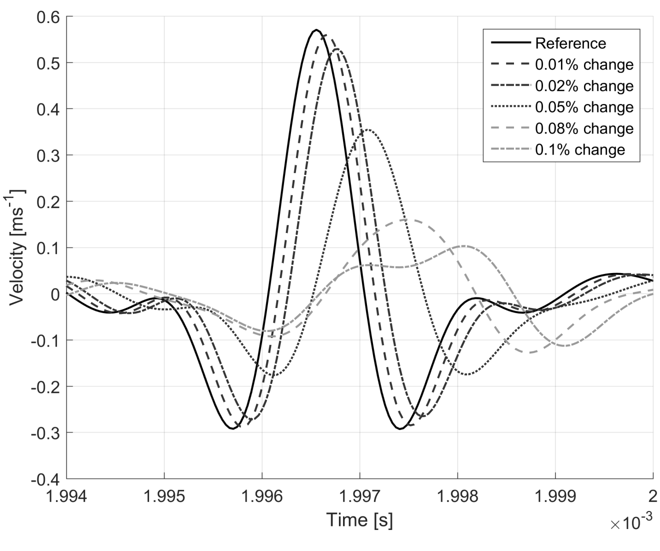


Fig. 2: Reconstruction of the original pulse for decreased longitudinal velocities

4. Conclusions

As we have shown, the successfulness of a reconstruction of the loading signal depends on precise tuning of material parameters. Even a slight change of a temperature, in order of ones of $^\circ C$, can lead to a lost of

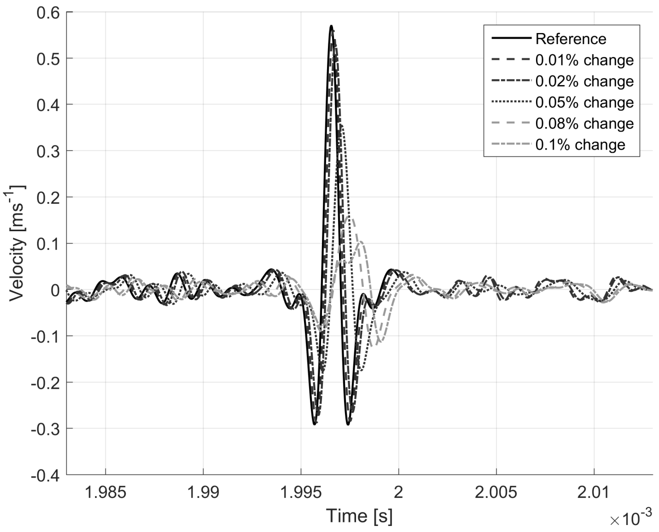


Fig. 3: Reconstruction of the original pulse - wider look

accuracy of reconstruction of the source. In a future work we will focus on study of the sensitivity analysis of the Time reversal method in depth with special emphasis on material parameters. Also we will focus on finding an approach of elimination of the sensivity of the TR method on material parameters, as is the influence of the change of the temperature. Futher, a special technique for modelling of wave propagation will be a point of interest, as the non-spurious time scheme (Kolman at el., 2016b).

Acknowledgments

The work of M. Mračko and J. Plešek was supported by the European Regional Development Fund under Grant No. CZ.02.1.01/0.0/0.0/15 003/0000493 (Centre of Excellence for Nonlinear Dynamic Behaviour of Advanced Materials in Engineering). The work of J. Kober, R. Kolman and Z. Převorovský was supported by the grant project CSF with No. 17-22615S within institutional support RVO:61388998.

References

Ancaş, A.-D. and Gorbănescu, D. (2006), Theoretical models in the study of temperature effect on steel mechanical properties. *The Bulletin of the Polytechnic Institute of Jassy, Construction. Architecture Section*, Vol 52, No. 1-2, pp 49-54.

Bathe, K.-J. (1996), *Finite Element Procedures*, Prentice Hall.

Croxford, A. J., Moll, J., Wilcox, P.D. and Michaels J. E. (2010) Efficient temperature compensation strategies for guided wave structural health monitoring. *Ultrasonics*, Vol. 50, No. 4-5, pp 517-528.

Fink, M., Cassereau, D., Derode, A., Prada, C., Roux, P., Tanter J.-L. T., Wu, F. (2000), Time-reversed acoustics. *Reports on Progress in Physics*, Vol. 63, pp 1933-1995.

Givoli, D. and Turkel, E. (2012), Time reversal with partial information for wave refocusing and scatterer identification. *Computer Methods in Applied Mechanics and Engineering*, Vol. 213, pp 223-242.

Kolman, R., Plešek, J., Červ, J., Okrouhlík, M. and Pařík, P. (2016a) Temporal-spatial dispersion and stability analysis of finite element method in explicit elastodynamics. *International Journal for Numerical Methods in Engineering*, Vol. 106, No. 2, pp 113-128.

Kolman, R., Cho, S.S. and Park, K.C. (2016b) Efficient implementation of an explicit partitioned shear and longitudinal wave propagation algorithm. *International Journal for Numerical Methods in Engineering*, Vol. 107, No. 7, pp 543-579.

Mračko, M., Kober, J., Kolman, R., Převorovský, Z., Plešek, J., Masák, J. and Kruisová, A. (2017), On finite element modelling in time reversal problem. In: *Proc. IXth NDT in progress*, Vol 1, pp 46-50.

Převorovský, Z. and Kober, J. (2015), Some Factors Affecting Time Reversal Signal Reconstruction. *Physics Procedia*, Vol 70, pp 604-608.