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MODELLING OF VIRUS VIBRATION WITH 3-D DYNAMIC ELASTICITY THEORY

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Abstract: *Elastic properties of virus shells (capsids) are important as they protect the virus genome and play important role in virus internalization (the process of virus entering the cell). These properties can also be measured experimentally by direct deformation of the capsid with a microscope's tip. A 3-D mathematical model of a virus under an external non-stationary load is proposed in this paper. The apparatus of the boundary value problems of mathematical physics was used during modeling. The stated initial boundary value problem of elasticity was solved with the help of the integral transformation method and the method of discontinuous solutions. As a result, the analytical solution of the problem was obtained in Laplace transformation domain. The numerical calculations of the virus elastic characteristics were illustrated for the case of a steady-state oscillation.*

Keywords: Virus, Elastic hollow sphere, Acoustic medium, Wave potentials, Exact solution.

1. Introduction

Mesoscopic properties of viruses as elastic bodies are important biologically as they allow to investigate the physics of the virus particles in various biologically relevant processes. Establishing a virus mathematical model is necessary to represent the effect of parameters variation on the behaviour of the virus as a dynamical system. Such mathematical models based on the reasonable biological assumptions were obtained earlier using three main interdisciplinary approaches:

1) based on a hydrodynamic theory (Markesteijn, 2014, Korotkin, 2016 and Scukins, 2015);
2) using the theory of numerical methods for solving hydrodynamic and elasticity non-linear problems (Polles, 2013 and Roos, 2010, Gibbons, 2007, Buenemann, 2007, Polles, 2013 and Zink, 2009);
3) based on the linear elasticity models (Zink, 2009, Buenemann, 2008 and Zandi, 2005).

These models allowed to obtain many important characteristics, but they could not fully describe the virus as a 3-D elastic object. In the proposed paper the authors first propose to use the full system of linear elasticity's motion equations for the virus wave field representation. It allows to take into consideration the virus 3-D structure and to obtain new qualitative characteristics of the virus stresses and displacements.

2. The statement of the problem

A virus PCV2 is modelled by an elastic hollow sphere occupying the area $R_1 < r < R_2, 0 < \theta < 2\pi, -\pi < \varphi < \pi$ in the spherical coordinate system. The equations of motion are written with regard to the displacements $u = u_r\left(r,\theta,\varphi,t\right), v = u_\theta\left(r,\theta,\varphi,t\right), w = u_\varphi\left(r,\theta,\varphi,t\right)$ (Nowacki, 1970). It is assumed that the virus is filled with an acoustic medium modelling the inside content of the virus composing of either the genome for the case of the real virus, or aqueous solution containing necessary ions in the case of an empty capsid (the so called Virus Like Particle). The virus is

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surrounded by the aqueous solution mimicking the cellular environment, which is also described by the ideal Newton liquid model. The wave potentials $\Phi_i(r,\theta,\varphi,t)$ of the external (i = 2) and the internal (i = 1) acoustic media satisfy the wave equations (Guz, 1982). It is assumed that adhesion takes place at the contact of the surfaces of the virus and the surrounding acoustic media

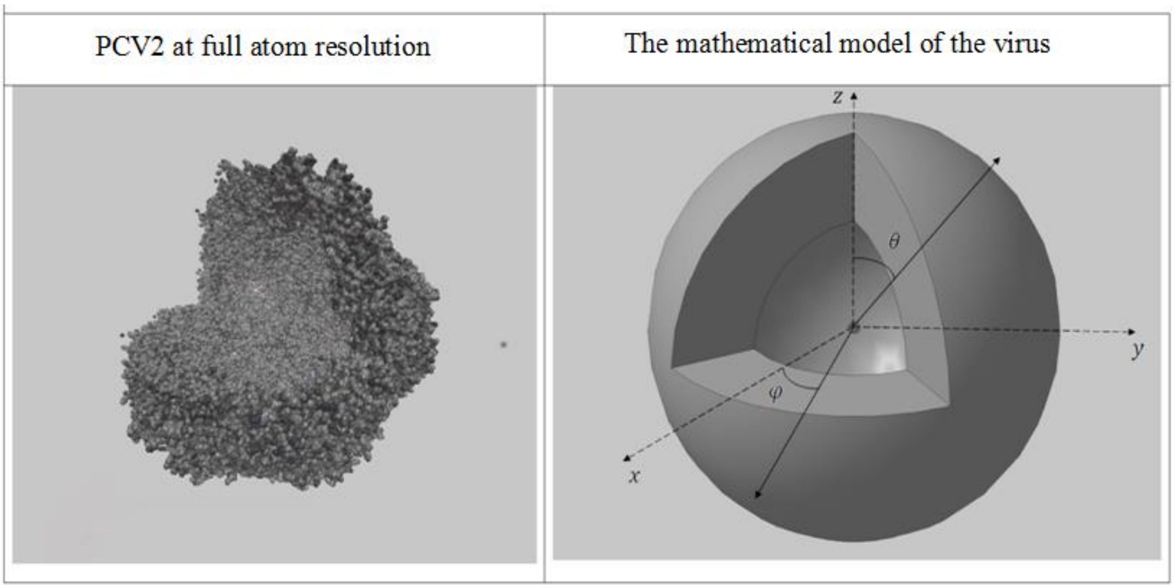

Fig. 1: The geometry of the problem.

$$\sigma_r\big|_{r=R_i} = -p_i(\theta,\varphi,t)\big|_{r=R_i} = \rho\frac{\partial\Phi_i}{\partial t}\bigg|_{r=R_i}, \tau_{r\varphi}\big|_{r=R_i} = \tau_{r\theta}\big|_{r=R_i} = T_i(\theta,\varphi,t), i = 1,2 \quad (1)$$

$$\begin{aligned} &u_r\big|_{\theta=0,2\pi} = \frac{\partial u_r}{\partial\theta}\bigg|_{\theta=0,2\pi} = q_1(r,\varphi,t), u_\theta\big|_{\theta=0,2\pi} = \frac{\partial u_\theta}{\partial\theta}\bigg|_{\theta=0,2\pi} = q_2(r,\varphi,t), \\ &u_\varphi\big|_{\theta=0,2\pi} = \frac{\partial u_\varphi}{\partial\theta}\bigg|_{\theta=0,2\pi} = q_3(r,\varphi,t), u_r\big|_{\varphi=\pm\pi} = \frac{\partial u_r}{\partial\varphi}\bigg|_{\varphi=\pm\pi} = g_1(r,\theta,t), \\ &u_\theta\big|_{\varphi=\pm\pi} = \frac{\partial u_\theta}{\partial\varphi}\bigg|_{\varphi=\pm\pi} = g_2(r,\theta,t), u_\varphi\big|_{\theta=\pm\pi} = \frac{\partial u_\varphi}{\partial\varphi}\bigg|_{\varphi=\pm\pi} = g_3(r,\theta,t) \end{aligned} \quad (2)$$

All functions on the right hand sides of the equalities (1), (2) are known functions. It is possible also to formulate on a virus's surfaces as the boundary conditions of first main elasticity problem, so and the mixed boundary conditions (in last case the solution is solved with the method of boundary integral equations). Zero initial conditions are fulfilled. One has to determine the wave field under the influence of a spherical pressure wave $\Phi_0(r,\theta,\varphi,t)$ falling on the virus external surface at the moment t = 0.

3. The methods of solution

Solution method is based on the application of the integral transformation method and the method of discontinuous solutions (Popov, 1982). The Laplace transformation with respect to the variable t and the finite Fourier transformation with respect to the variable φ are applied to the system of the equations of motion and the boundary conditions (1), (2). To construct the solution of the boundary value problem in the transformation domain one must use the discontinuous solutions of the motion equations for a spherical defect, which were constructed earlier in (Vaysfeld, 2002).

The transformations of the unknown functions are presented as a superposition of the functions $u_{sn} = u_{sn}^1 + u_{sn}^2, v_{sn} = v_{sn}^1 + v_{sn}^2, w_{sn} = w_{sn}^1 + w_{sn}^2$, where indexes s and n denote the parameters of the Laplace and the Fourier transformations respectively, the upper index 1 denotes the mechanical characteristics, which are discontinuous on the interior surface of the sphere, the upper index 2 denotes the mechanical characteristics, which are discontinuous on the external surface of the spherical shell

$$\left\langle u_{sn}\right\rangle\Big|_{r=R_1}=\left\langle u_{sn}^1\right\rangle\Big|_{r=R_1}+\left\langle u_{sn}^2\right\rangle\Big|_{r=R_1}=\left\langle u_{sn}^1\right\rangle\Big|_{r=R_1}=-u_{sn}^1\left(R_1+0,\theta\right),$$
$$\left\langle u_{sn}\right\rangle\Big|_{r=R_2}=\left\langle u_{sn}^1\right\rangle\Big|_{r=R_2}+\left\langle u_{sn}^2\right\rangle\Big|_{r=R_2}=\left\langle u_{sn}^2\right\rangle\Big|_{r=R_2}=u_{sn}^2\left(R_2-0,\theta\right),$$

where $u\in\left\{u_r,u_\theta,u_\varphi,\sigma_r,\tau_{r\theta},\tau_{r\varphi}\right\}$.

Similar representations for the wave potentials are constructed. They lead to

$$\left\langle \Phi_{1sn}\right\rangle\Big|_{r=R_1}=\Phi_{1sn}\left(R_1-0,\theta\right),$$
$$\left\langle \Phi_{2sn}\right\rangle\Big|_{r=R_2}=-\Phi_{2sn}\left(R_2+0,\theta\right)+\Phi_{0sn}\left(R_2,\theta\right).$$

The proposed solution method allowed to construct the representations for the wave potentials, displacements, and stresses of the elastic medium in terms of the Laplace transformations.

$$\Phi_{js}\left(r,\theta,\varphi\right)=\frac{R_j^2}{2\pi}\left[\int_{-\pi}^{\pi}\left\langle \Phi'_{js}\left(R_j,\tau,\phi\right)\right\rangle\int_0^{\pi}\sum_{n=-\infty}^{\infty}e^{in(\phi-\varphi)}K_n\left(\theta,\tau;r,R_j\right)\sin\tau\,d\tau\,d\phi-\right.$$
$$\left.-\int_{-\pi}^{\pi}\left\langle \Phi_{js}\left(R_j,\tau,\phi\right)\right\rangle\int_0^{\pi}\sum_{n=-\infty}^{\infty}e^{in(\phi-\varphi)}\frac{\partial}{\partial R_j}K_n\left(\theta,\tau;r,R_j\right)\sin\tau\,d\tau\,d\phi\right]$$

where: $K_n\left(\theta,\tau;r,R_j\right)=\sum_{k=|n|}^{\infty}\sigma_{k,|n|}\Gamma_{ik}\left(r,R_j\right)P_k^{|n|}\left(\cos\theta\right)P_k^{|n|}\left(\cos\tau\right)$,

$$\sigma_{k,|n|}=\frac{\left(k-|n|\right)!\left(k+1/2\right)}{\left(k+|n|\right)!},$$

$$\Gamma_{ik}\left(r,R_j\right)=\frac{1}{\sqrt{rR_j}}\begin{cases}I_\nu\left(R_jq_i\right)K_\nu\left(rq_i\right), & r>R_j,\nu=k+1/2\\ I_\nu\left(rq_i\right)K_\nu\left(R_jq_i\right), & r<R_j,k=0,1,2,...\end{cases},$$

$q_i^2=\dfrac{s^2}{c_i^2}$, $I_\nu\left(z\right),K_\nu\left(z\right)$ are modified Bessel functions, $P_k^n\left(z\right)$ are associated Legendre functions.

$$u_s\left(r,\theta,\varphi\right)=\frac{1}{2\pi\mu b^2}\sum_{n=-\infty}^{\infty}e^{in\varphi}\left\{\sum_{k=|n|}^{\infty}\int_0^{\pi}P\left(n,k,\vartheta\right)\int_{-\pi}^{\pi}e^{-in\phi}\left\{\sum_{j=1}^{2}\left(-1\right)^{j+1}\Psi\left(R_j,\vartheta,\phi\right)\frac{\partial^2}{\partial R_j\partial r}\Gamma_{a,k}\left(r,R_j\right)P_\sigma\left(n,k,\theta\right)+\right.\right.$$
$$+\frac{1}{r}\frac{n^2}{\sin^2\theta}\sum_{j=1}^{2}\left(-1\right)^{j+1}\Psi\left(R_j,\vartheta,\phi\right)\Gamma_{b,k}\left(r,R_j\right)P_\sigma\left(n,k,\theta\right)-\frac{1}{r}\sum_{j=1}^{2}\left(-1\right)^{j+1}\Psi\left(R_j,\vartheta,\phi\right)\Gamma_{b,k}\left(r,R_j\right)\frac{\partial^2}{\partial\theta^2}P_\sigma\left(n,k,\theta\right)-$$
$$\left.\left.-\frac{1}{r}ctg\theta\sum_{j=1}^{2}\left(-1\right)^{j+1}\Psi\left(R_j,\vartheta,\phi\right)\Gamma_{b,k}\left(r,R_j\right)\frac{\partial}{\partial\theta}P_\sigma\left(n,k,\theta\right)\right\}d\phi\,d\vartheta\right.$$

where: $P\left(n,k,\vartheta\right)=\sin\vartheta P_k^{|n|}\left(\cos\vartheta\right),P_\sigma\left(n,k,\theta\right)=\sigma_{k,|n|}P_k^{|n|}\left(\cos\theta\right)$,

$\Psi\left(R_j,\vartheta,\phi\right)=R_j^2\rho\dfrac{\partial\Phi_j}{\partial t}\left(R_j,\vartheta,\phi\right)$. The formulae for the displacements and the stresses are analogous.

4. Numerical results

The obtained formulae were used to illustrate the case of steady-state oscillations, the homogeneous boundary conditions (2) and the conditions $T_i\left(\theta,\varphi,t\right)=0, i=1,2$. The elastic constants of the virus, namely the Young module and the Poisson ratio, were determined with the help of LAMMPS Molecular Dynamics package. The sea water was selected as the external acoustic medium, and water was selected as the internal acoustic medium. The mechanical parameters of these liquids were taken for the modelling of the liquids inside and outside the virus. The stresses on the surfaces of the sphere were calculated. The analysis was conducted depending on the incident wave's angle.

5. Conclusions

1. 3-D mathematical model of a PCV2 virus was constructed on using dynamic elasticity boundary value problem. The elastic constants of the virus (the Poisson ratio, the Young module) were determined with the help of LAMMPS package.
2. Formulae determining the virus wave field under the acoustic pressure wave were obtained.
3. This model will serve as the first step in developing a more realistic models of viruses with varying density of the capsid, its geometry and, possibly, elastic properties.

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References

Buenemann, M. and Lenz, P. (2007) Mechanical limits of viral capsids. PNAS, 104, 24, pp. 9925-9930.

Buenemann, M. and Lenz, P. (2008) Elastic properties and mechanical stability of chiral and filled viral capsids. PhysRevE.78.051924 PACS.

Gibbons, M.M. and Klug, W.S. (2007) Nonlinear finite-element analysis of nanoindentation of viral capsids. PhysRevE.75.031901.

Guz, A.N. and Kubenko, V.D. (1982) Theory of non-stationary aero-hydro-elasticity of shells. Naukova dumka, Kyiv.

Korotkin, I., Nerukh, D., Tarasova, E., Farafonov, V. and Karabasov, S. (2016) Two-phase flow analogy as an effective boundary condition for modelling liquids at atomistic resolution. Journal of Computational Science.

Markesteijn, A., Karabasov, S., Scukins, A., Nerukh, D., Glotov, V. and Goloviznin, V. (2014) Concurrent multiscale modelling of atomistic and hydrodynamic processes in liquids. Phil. Trans. R. Soc. A 372, pp. 2021.

May, E.R. and Brooks, C.L. (2012) On the Morphology of Viral Capsids: Elastic Properties and Buckling Transitions. J. Phys. Chem. B, 116, pp. 8604-8609.

Nowacki, W. (1970) Theory of elasticity. Warszava.

Polles, G., Indelicato, G., Potestio, R., Cermelli, P., Twarock, R. and Micheletti, C. (2013) Mechanical and Assembly Units of Viral Capsids Identified via Quasi-Rigid Domain Decomposition. PLoS Comput Biol, 9, 11.

Popov, G.Y. (1982) The elastic stress' concentration around dies, cuts, thin inclusions and reinforcements. Nauka, Moskow (in Russian).

Roos, W.H., Gibbons, M.M., Arkhipov, A., Uetrecht, C., Watts, N.R., Wingfield, P.T., Steven, A.C., Heck, A.J.R., Schulten, K., Klug, W.S. and Wuite, G.J.L. (2010) Squeezing Protein Shells: How Continuum Elastic Models, Molecular Dynamics Simulations, and Experiments Coalesce at the Nanoscale. Biophysical Journal, 99, 4, pp. 1175-1181.

Scukins, A., Nerukh, D., Pavlov, E., Karabasov, S. and Markesteijn, A. (2015) Multiscale molecular dynamics/hydrodynamics implementation of two dimensional “Mercedes Benz” water model. European Physical Journal, 224, 12, pp. 2217-2238.

Vaisfel'd, N.D. and Popov G.Y. (2002) Nonstationary dynamic problems of elastic stress concentration near a spherical imperfection. Mechanics of Solids. 37, 3, pp. 77-88.

Zandi, R. and Reguera, D. (2005) Mechanical properties of viral capsids, Phys. Rev. E 72, 021917.

Zink, M. and Grubmüller, H. (2009) Mechanical Properties of the Icosahedral Shell of Southern Bean Mosaic Virus: A Molecular Dynamics Study. Biophysical Journal, 96, 4, pp. 1350-1363.