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BACK ANALYSIS FOR DETERMINATION OF TENSION-SOFTENING DIAGRAM

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Abstract: *Concrete and other cementitious materials are undoubtedly the most important and widely used construction material nowadays. However, there exist other building materials, such as wood or composites, which are on high demand as well. Different mathematical models describing the complex material behaviour of these mostly quasi-brittle materials are used. Their application often requires the knowledge of experimentally determined fracture properties of studied material. The total fracture energy can be often determined directly from experimental measurements as the total work divided by the ligament area. However, the determination of true tensile strength and the traction-separation diagram is usually problematic. The primary objective of this paper is to present a numerical scheme, that can be used to determine the traction-separation diagram in the framework of sequentially linear analysis (SLA).*

Keywords: Back analysis, Damage, Traction-separation diagram, Wood, Concrete.

1. Introduction

Concrete and other cementitious materials are undoubtedly the most important and widely used construction material nowadays. However, there exist other building materials, such as wood or composites, which are on high demand as well. Different mathematical models describing the complex material behaviour of these mostly quasi-brittle materials are used. Their application often requires the knowledge of experimentally determined fracture properties of studied material, e.g., the tensile strength, the fracture energy and the traction-separation (TS) diagram (strain-softening curve). The experimental setups used to determine the fracture properties are, e.g., uniaxial tension test, three- or four-point bending test, Brazilian splitting test, etc. While the fracture energy may be directly obtained from experiments by measuring the total work done by the test loads and dividing it by the ligament area, the determination of tensile strength and TS diagram appears to be technically difficult.

Several scholars have developed numerical procedures, different in complexity and experimental data requirements, to obtain the tension-softening curve. Li and Ward (1989) introduced a technique based on the J-integral to determine the tension-opening relation in cementitious composites. Cohesive force model analysis with poly-linear softening diagram was presented in (Kitsutaka et al., 1993), where the system of simultaneous equations, called crack equations, is solved by performing several iterations to obtain the parameters of poly-linear tension-softening curve. An indirect method utilising the bending test of a notched beam was proposed in (Nanakorn and Horii, 1996). In the approach, the slope of TS diagram is directly obtained for a given slope of the input load-deflection curve. Therefore, there is no iteration required for each incremental step. The interested reader can find another techniques, e.g., in (Skoček and Stang, 2008) or (Slowik et al, 2006).

The primary objective of this paper is to present a numerical scheme, that is used to determine the traction-separation diagram by means of the sequentially linear analysis (SLA). This scheme is particularly well suited to large-scale structural analyses (Rots and Invernizzi, 2004; Vorel and Boshoff, 2014; Vorel and Boshoff, 2015).

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2. Back analysis

Sequentially linear analysis was designed to overcome the difficulty of snap-back behaviour (Rots and Invernizzi, 2004), which typically occurs when full-scale structures are modelled. The procedure is based on the solution of physically nonlinear tasks by a sequence of linear (secant) steps. Relations between stresses and strains have to be defined as a collection of elasto-brittle teeth, called a ``saw-tooth" law. This approach is generally applicable for materials with nonlinear softening, jumps in constitutive law, or hardening (Billington, 2009) for which the failure process is stably controlled.

In the present study, a back analysis method to obtain the TS diagram is presented. More specifically, the cohesive interface elements are introduced in the position of the crack for the finite element analysis and the corresponding crack-opening diagram for these elements is determined during the SLA. As an input for the proposed procedure, any measured loading curve can be used, e.g., force load versus displacement, force load versus crack mouth opening, etc. However, the measure used to control the loading process must have strictly increasing order to determine the unique critical multiplier (λ).

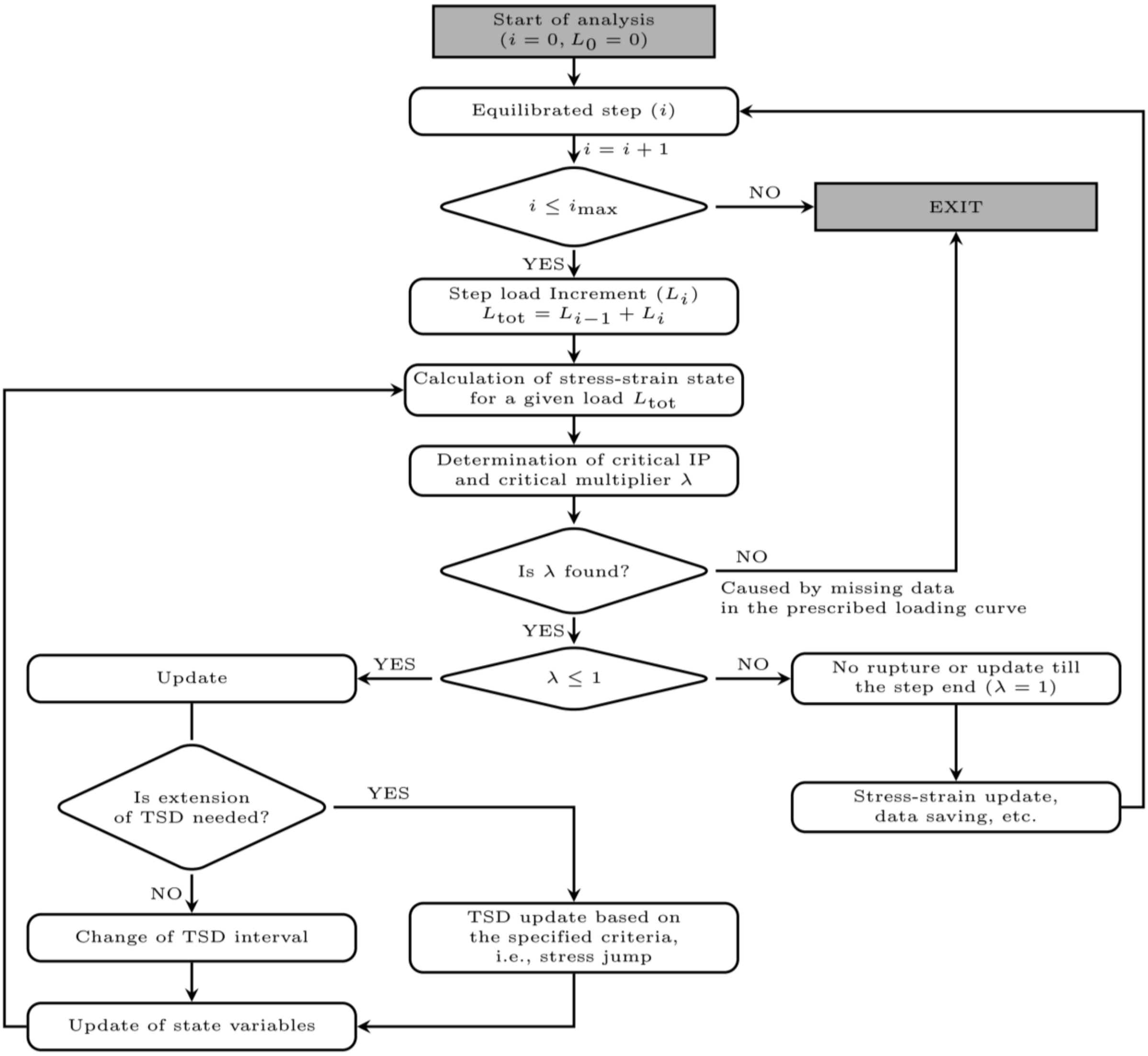


Fig. 1: Flowchart of proposed back analysis.

In the initial state, no tensile strength and no parameters of TS diagram have to be defined. The only parameters needed to be setup by the user are: 1) allowable overshooting of the prescribed diagram (F_{err}); 2) reduction of the damage parameter ($\Delta\omega$); 3) initial normal and shear stiffness of the interface element (k_n, k_s). In this paper the F_{err} value is set as the percentage of the maximum load determined from the experimental data and can be seen as the error in the experimental measurement, and

$$\Delta\omega = \frac{k_n^{(i+1)} - k_n^{(i)}}{k_n} = \frac{\Delta\sigma_{\mathrm{cr}}}{w_{\mathrm{cr}}^{(i)} k_n}, \tag{1}$$

where index $i = 1, 2, \dots$ refers to point number of the searched TS diagram, $k_n^{(\cdot)}$ denotes current and new normal stiffness of the element, and $w_{\mathrm{cr}}^{(\mathrm{i})}$ is the opening for the current critical multiplier. $\Delta\sigma$ stands for the maximum allowable stress jump which characterises the damage change and is prescribed as the percentage of the determined tensile strength, i.e., the first stress value of searched TS diagram.

The flow chart of the proposed procedure is shown in Fig. 1. This procedure allows combination of elements used to determine the search TS diagram together with the standard elements for the SLA where the "saw-tooth" approximation is prescribed in advance. The critical multiplier λ is determined as the minimum load needed to exceed a given failure criterion or to exceed the prescribed loading curve. The determined TS diagram is subsequently utilised in the remaining elements involved in the back analysis. In most cases, e.g., three-point bending test, Brazilian splitting test, etc. only one element governs the shape of softening (hardening).

3. Numerical simulation

This section deals with the presentation of the obtained results for wood compact tension specimen shown in Fig. 2. Two different experimental data, presented in (Blass, 2011) are utilize in this study. The wood is modelled as the orthotropic material and is characterised by the user-defined properties (Kasal and Blass, 2013; Kasal and Heiduschke, 2004)

- Experiment No. 1: $E_{xx} = 18.9$ GPa, $E_{yy} = E_{zz} = 0.8$ GPa, $G_{xy} = G_{xz} = 0.62$ GPa, $G_{yz} = 0.19$ GPa, $\nu_{yz} = \nu_{xz} = \nu_{xy} = 0.3$, $k_n = 18.9$ MN/m, $k_s = 18.9 \cdot 10^4$ MN/m.
- Experiment No. 2: $E_{xx} = 18.9$ GPa, $E_{yy} = E_{zz} = 0.74$ GPa, $G_{xy} = G_{xz} = 0.57$ GPa, $G_{yz} = 0.19$ GPa, $\nu_{yz} = \nu_{xz} = \nu_{xy} = 0.3$, $k_n = 18.9$ MN/m, $k_s = 18.9 \cdot 10^4$ MN/m.

The presented back analysis is implemented for plane stress problems in the open source object oriented finite element program OOFEM (Patzák and Bittnar, 2001). The isoparametric four-node quadrilateral plane-stress finite elements and the interface cohesive elements with linear approximation of displacement field are used for the finite element calculations. The element size in the region of interest is equal to 1 mm. The comparison between the experimental data and the calculated load-CMOD diagrams is shown in Fig. 3a. The calculated traction-separation diagrams is depicted in Fig. 3b.

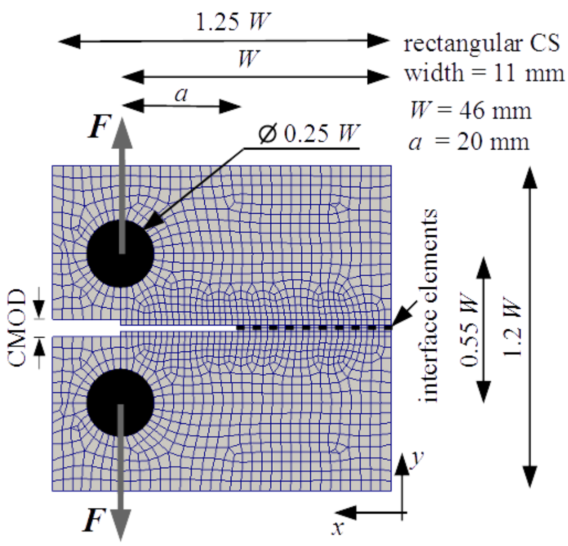
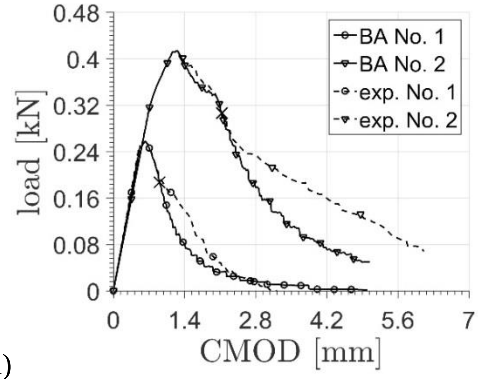


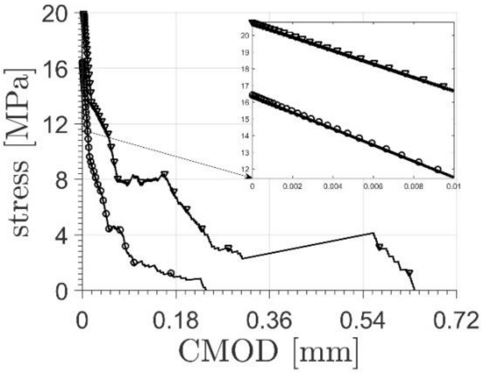
Fig. 2: Compact tension test setup.

4. Conclusions

The new procedure for the determination of traction-separation diagrams was introduced in the paper. The method is utilised to obtain the fracture properties of wood samples. More specifically, the traction-separation diagrams are presented in Fig. 3b. As can be seen from the presented results, the agreement between the experimental data and numerical analysis is sufficient. However, it has to be stressed out that the back analysis depends on the quality of experimental data, especially the part used for the determination of the traction-separation diagram. In Fig. 3a, this part of diagram lies between the origin and the cross mark.



a)



b)

Fig. 3: Compact tension test: a) load-CMOD diagram; b) TS diagram.

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