

## MODELLING OF FLOW IN LINEAR BLADE CASCADE WITH THICK TRAILING EDGE AT A LOWER REYNOLDS NUMBER

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**Abstract:** *In this paper there are compared results of numerical simulation of compressible turbulent flow through low-pressure turbine blade cascade with relatively thick trailing edges for various physical models. Steady, unsteady, fully turbulent, transitional, two-dimensional and three-dimensional models are compared. Results show that: a) steady simulation gives an incorrect prediction of the boundary layer development, b) two-dimensional unsteady simulation leads to inaccurate prediction of the far wake development and c) only three-dimensional unsteady transitional simulation gives sufficiently accurate prediction. Results of numerical simulations are compared with the experimental data.*

**Keywords:** Linear blade cascade, Boundary layer transition, Vortex series.

### 1. Introduction

The contribution deals with modelling of unsteady compressible flow through linear blade cascade with a relatively thick trailing edge. The linear blade cascade presented in this work represents an unrolled section of low-pressure turbine wheel of jet engine. The operating chord-based Reynolds number of used blade cascade is in order of magnitude from  $10^4$  to  $10^5$ . These relatively low values indicate that the laminar/turbulent boundary layer transition plays an important role in forming of flow field.

Modelling of flow through the linear blade cascade belongs today to routine calculations in industrial practice. Commonly used approach is that the flow field is supposed as periodic and two-dimensional. It allows dramatically decrease the computational cost.

In cases, where the laminar/turbulent boundary layer transition plays an important role (such as presented case), appropriate turbulence models should be used (Langtry and Menter, 2009, Walters and Cokljat, 2008, Kubacki and Dick, 2016, Straka and Příhoda, 2010).

In case of the relatively thick trailing edge there is another important phenomenon which has an impact on forming of the flow field: the vortex series. In transonic and high subsonic regimes the vortex series generated in near wake causes pressure waves (weak acoustic waves), which influence development of the boundary layer on the suction side of the neighboring blade. It means that not only transition modelling is necessary, but also time accurate simulation for resolving the vortex series is needed.

One of the main outputs of flow simulation in linear blade cascade is prediction of the kinetic energy losses across the wake behind the blade cascade. In case of relatively thick trailing edge it means to simulate the development of the vortex series in the far wake. This is not possible correctly perform under the supposition of two-dimensional flow because of the three-dimensional principle of the phenomenon.

In this work there are compared following results: a) two-dimensional steady simulation of fully turbulent flow without the effects of the boundary layer transition and the vortex series, b) two-dimensional steady simulation of transitional flow without the effect of the vortex series, c) two-dimensional unsteady simulation of transitional flow including the effect of the vortex series but without three-dimensional development in the far wake, d) three-dimensional unsteady simulation of transitional flow.

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## 2. Turbulence model

The compressible turbulent flow is described by the system of the Favre-averaged Navier–Stokes equations. The system of governing equations is closed by two-equation  $k$ - $\omega$  nonlinear (quadratic) turbulence model (Rumsey and Gatski, 2001):

$$\frac{D(\rho k)}{Dt} = \gamma P_k - \gamma D_k + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_k \frac{\rho k}{\omega} \right) \frac{\partial k}{\partial x_j} \right], \quad (1)$$

$$\frac{D(\rho \omega)}{Dt} = P_\omega - D_\omega + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_\omega \frac{\rho k}{\omega} \right) \frac{\partial \omega}{\partial x_j} \right] + C_D, \quad (2)$$

where  $\rho$  is the density,  $k$  is the turbulent energy,  $\omega$  is the specific dissipation rate of the turbulent energy,  $P_k$  and  $P_\omega$  are the production terms,  $D_k$  and  $D_\omega$  are the destruction terms,  $C_D$  is the cross-diffusion term,  $\mu$  is the molecular viscosity and  $\sigma_k$  and  $\sigma_\omega$  are the model constants. The Reynolds stress tensor  $\tau_{ij}^t$  is given as a quadratic function of the strain rate tensor  $S_{ij}$ :

$$\tau_{ij}^t = \gamma \left\{ 2\mu_t \left( S_{ij} - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \rho k \delta_{ij} + 2\mu_t \left[ b_1 (S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj}) - 2b_2 \left( S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{lk} \delta_{ij} \right) \right] \right\}, \quad (3)$$

where  $\mu_t$  is the turbulent viscosity,  $\Omega_{ij}$  is the vorticity tensor and  $b_1$ ,  $b_2$  are the model constants (for more details see Rumsey and Gatski, 2001). In equations (1) and (3)  $\gamma$  is the intermittency coefficient. The role of the intermittency coefficient  $\gamma$  is to control the laminar/turbulent transition:  $\gamma = 0$  in the laminar part,  $\gamma = 1$  in the turbulent part and  $0 < \gamma < 1$  in transitional part of the boundary layer. In this work there is used the algebraic bypass and separation-induced transition model of Straka and Přihoda (2010), where the transition onset and the length of the transition region depend on the free-stream turbulence intensity, the longitudinal pressure gradient and on the wall-roughness (more details can be found in Straka and Přihoda, 2010).

## 3. Computational setup

In Fig. 1 there is shown a scheme of the linear blade cascade with definition of geometrical parameters and the flow regime (chord  $c$ , pitch  $t$ , trailing edge thickness  $d_{te}$ , stagger angle  $\gamma_{st}$ , inlet angle  $\alpha_1$ , outlet angle  $\alpha_2$ , outlet isentropic Mach number  $M_{2is}$ , outlet isentropic Reynolds number  $Re_{2is}$ , inlet turbulence intensity  $Tu_1$  and the turbulent to molecular viscosity ratio  $\mu_t / \mu$ ). More description can be found in (Straka and Přihoda, 2014 and Michálek and Straka, 2013).

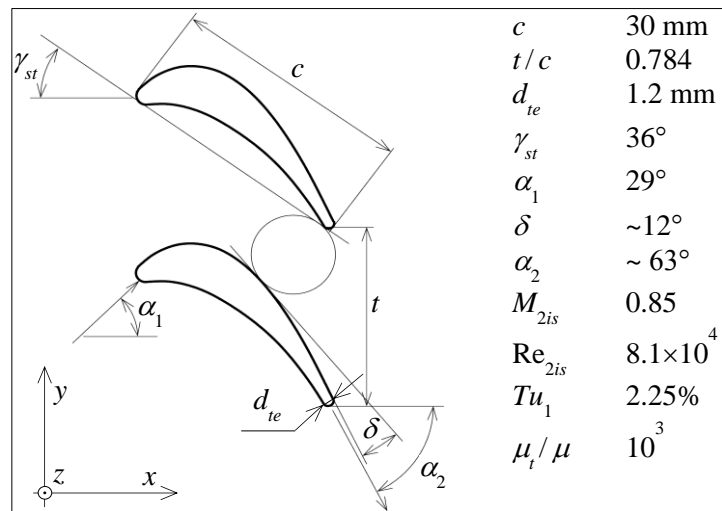


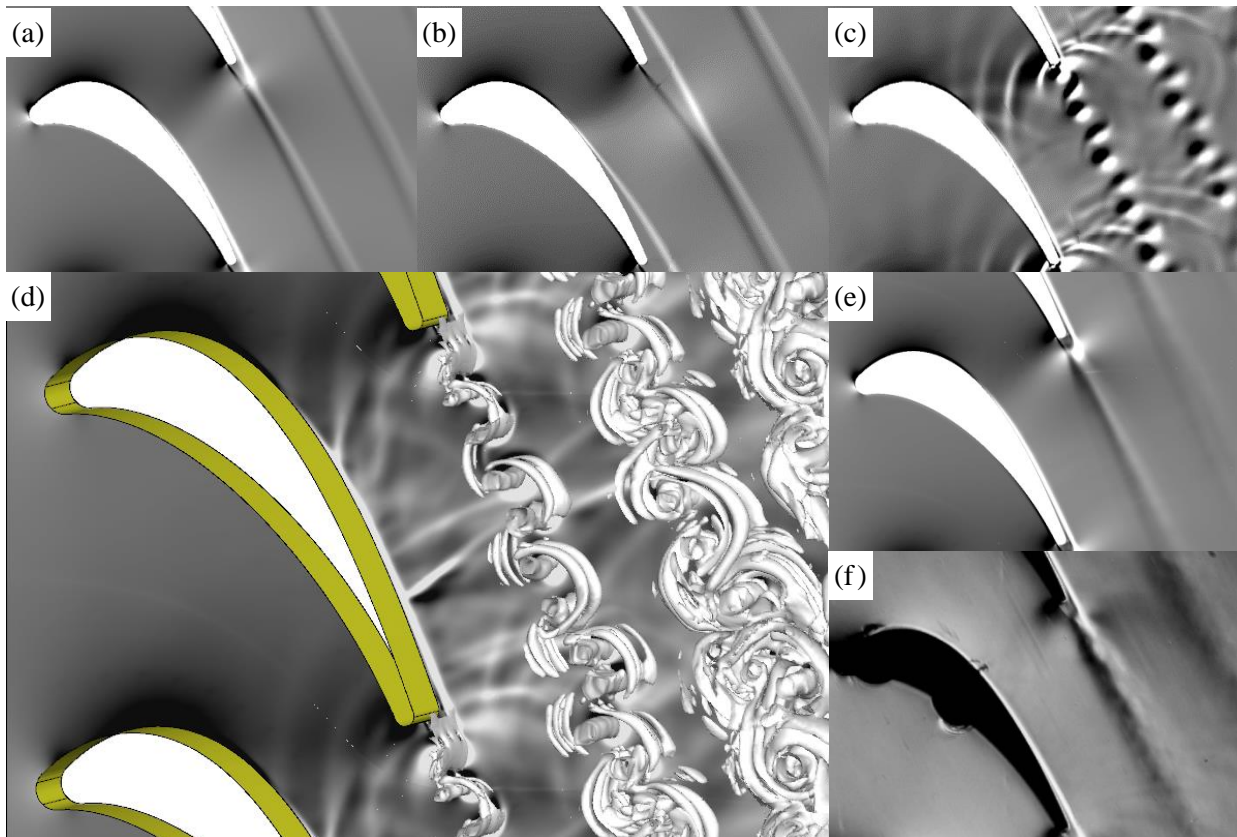
Fig. 1: Scheme of the linear blade cascade.

The computational mesh for two-dimensional simulation contains 51,000 quadrilateral cells, mesh for three-dimensional simulation contains 1,750,000 hexahedral cells. In both cases the mesh is refined near

the blade profile to ensure that there are at least five cells in the viscous sub-layer. The blade profile is discretized by 300 points. For three-dimensional simulation the geometry of the blade is pulled in “z” direction up to thickness of  $2\pi d_{te}$  with 35 cells across “z” direction.

#### 4. Results

In Fig. 2 there is a comparison of the schlieren images (directional derivative of density) of fully turbulent, transitional, steady, unsteady, two-dimensional and three-dimensional models and experiment (Michálek and Straka, 2013). Note that in part (d) of Fig. 2 there are also shown an instantaneous iso-surfaces of  $(|\Omega|^2 - |S|^2)$  for highlighting of the three-dimensionality of the far wake. The experimental schlieren photography in part (f) of Fig. 2 was taken with exposure time of approx. 20 to 30  $\mu$ s, while the frequency of the vortex series is around 45 kHz. Fig. 3 shows the distribution of the kinetic energy losses across the wake in the distance of 18 mm behind the trailing edges.



*Fig. 2: Numerical schlieren; 2D steady fully turbulent (a), 2D steady transitional (b), 2D unsteady transitional (c), 3D unsteady transitional (d), 3D unsteady transitional – time averaged (e), experiment (f).*

From comparison of the schlieren images and the kinetic energy losses distributions follows that the two-dimensional steady fully turbulent model predicts too narrow wake with high level of the kinetic energy losses, whilst the two-dimensional steady transitional model predicts massive separation of the boundary layer at the middle of the suction side, which leads to increase of the kinetic energy losses. The unsteady transitional model predicts generating of the vortex series in the near wake which causes the pressure waves (weak acoustic waves). These pressure waves prevent formation of the massive separation of the boundary layer on the suction side of the neighboring blade. The two-dimensional unsteady transitional model does not allow the decay of the vortex series, which leads to overestimation of the kinetic energy losses. Only the three-dimensional unsteady transitional model predicts correctly the development of the boundary layer on the suction side and the development of the far wake in a good agreement with the experimental data, as is shown in Fig. 3.

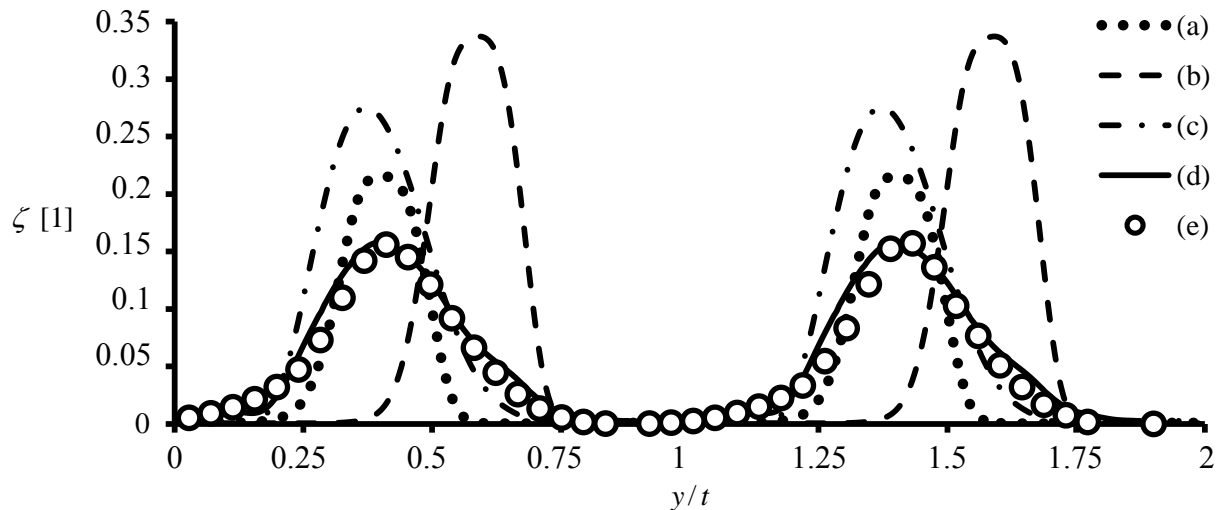


Fig. 3: Kinetic energy losses distribution across the wake; 2D steady fully turbulent (a), 2D steady transitional (b), 2D unsteady transitional (c), 3D unsteady transitional (d), experiment (e).

## 5. Conclusions

This work shows that although the simulation of flow in the linear blade cascade is really routine task nowadays, there are still cases with high demands on the physical model and on the computational costs. If we compare the computational time of used models with the computational time needed for the two-dimensional steady fully-turbulent simulation, we find that the computational time of the two-dimensional steady transitional simulation is comparable, the computational time of the two-dimensional unsteady transitional simulation is approx. twenty times higher and the computational time of the three-dimensional unsteady transitional simulation is more than seven hundred times higher. Therefore, the problem of sufficiently accurate and sufficiently efficient simulation of flow in the linear blade cascades with relatively thick trailing edges at lower level of the Reynolds number is still open.

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