

Svratka, Czech Republic, 15 – 18 May 2017

## MODELLING OF ENERGY DISSIPATION IN SHELL DAMPERS

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**Abstract:** The achievements of the authors in the analytical modeling of hysteretic energy dissipation in the shells with a filler at the expense of dry friction are presented. The last result describes the effect of maximum energy absorption by a shell damper. Importance of tribology settings of contact system, for which the dissipated energy of the external load reaches the maximum, is revealed.

Keywords: Vibration protection, Shell, Filler, Dry friction, Structural hysteresis.

## 1. Introduction

Vibration processes that occur in the operation of almost all, without exception, modern machines and mechanisms, typically lead to undesirable consequences. In the vast majority of cases, vibration decreases the strength, reliability and durability of industrial machines, mechanisms and structures, as well as affects health of personnel. Thus, the problem of vibration insulation proves to be quite urgent both in technical and social terms. One of the ways used to solve the formulated problem is connected with the application of vibration protecting devices, such as shock absorbers, dampers, dynamic vibration absorbers, etc. This is why the research and design works and theoretical investigations in the field of development of new means of vibration protection and methods for their numerical analyses are of crucial importance.

The authors have developed advanced design of vibration insulators (Shopa et al., 2002), among which the leading place belongs to shell elastic elements with deformable filler (Fig. 1). In the mechanical and mathematical modeling of the behavior of elastic elements under (generally speaking, non-monotonic) loading, we get a class of nonlinear non-conservative mixed contact problems of the frictional interaction of thin shells with deformable fillers (Popadyuk et al., 2003 and Shopa et al., 2015).



Fig. 1: Laboratory samples of shell dampers.

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The aim of the report is to acquaint the scientific audience with the results obtained due to the analytical modeling of hysteretic energy dissipation in the shells with a filler at the expense of dry friction. Thus, the latest achievement, i.e. the effect of maximum energy absorption by shell damper, is described.

## 2. Formulation of the problem

Consider an elastic deformable cylinder (filler) of radius R and length 2a placed into a cylindrical shell with thickness  $h_0$  (Fig. 2a). The filler is compressed on the end faces by perfectly rigid pistons subjected to the action of an external load Q non-monotonic as a function of time. Dry position friction occurs on contact surface of the shell with a filler. We need to study the phenomenon of structural damping in the given non-conservative system.

The shell, depending on the conditions of use of the structure, can be of closed (solid – Fig. 2b) or open (cut along a generating line – Fig. 2c) profiles. In the first case (Fig. 2b), the shell works on the tangential stress, and in the second case (Fig. 2c) – on the bending.



Fig. 2: Scheme of a shell damper.

### 3. Analysis of the results

Using applied shell and filler models the analytic algorithm of solving nonlinear mixed problem on frictional interaction of elastic filler with a shell at non-monotonic quasi-static loading has been developed. The solution of the problem at the stages of active loading, unloading and repeated loading has been proposed, characteristics of stress-strain state, stiffness and damping ability of the system have been obtained.

An approximate analytical description of the structural hysteresis loop obtained for shells with shearunresisting slight-compressible filler is provided in the works (Popadyuk et al., 2003 and Shopa et al., 2015). Thus, we describe the dependences without displacements of the piston and loading with arbitrary asymmetry cycle coefficient  $s = Q_{min} / Q_{max} \in [0, 1]$  by the following relations.

For active loading

$$\delta^{I} = \frac{Q}{c} \frac{1 - e^{-\lambda}}{\lambda}, \quad 0 \le Q \le Q_{\max};$$

for unloading

$$\delta^{II} = \begin{cases} \delta^{I}_{\max} - \frac{1}{c} \left[ b(Q_{\max} - Q) + \left( \sqrt{Q_{\max}} - \sqrt{Q} \right)^{2} / \lambda \right], & Q_{\max} \ge Q \ge s_{*} Q_{\max}; \\ & \frac{Q}{c} \left( b - \frac{e^{\lambda} - 1}{\lambda} \right), & s_{*} Q_{\max} \ge Q \ge 0; \end{cases}$$

for repeated loading

$$\delta^{III} = \begin{cases} \delta^{II}_{\min} + \frac{1}{c} \Big[ b \big( Q - Q_{\min} \big) + \big( \sqrt{Q} - \sqrt{Q_{\min}} \big)^2 / \lambda \Big], \ Q_{\min} \le Q \le \min \{ Q_{\min} / s_*, \ Q_{\max} \}; \\\\ \delta^{I}, \ \min \{ Q_{\min} / s_*, \ Q_{\max} \} \le Q \le Q_{\max}. \end{cases}$$

Here,  $\lambda = 2f a/R$ , *f* is the coefficient of dry friction in the shell-filler pair;  $s_* = e^{-2\lambda}$ ; *c* is the linear stiffness of the conservative system, which depends on type, size and physical and mechanical characteristics of the shell and filler.

$$c = \frac{\pi R^2 E}{a} \frac{1}{3(1-2\nu) + (2-\nu_0)\varepsilon}, \ b = \frac{\nu_o \varepsilon}{3(1-2\nu) + (2-\nu_o)\varepsilon} \text{ and } c = \frac{\pi R^2 E}{a} \frac{1+36\frac{R^2}{h_o^2}\varepsilon}{36\frac{R^2}{h_o^2}\varepsilon}, \ b = 0$$

for a solid shell, for a shell cut along a generating line respectively;  $\varepsilon = ER/(E_o h_o)$ ; E,  $\nu$ ,  $E_o$ ,  $\nu_o$  are Young's modulus and Poisson's ratio of the materials of the filler and shell respectively.

System "shell–deformable filler" with parameters  $h_o/R = 0.1$ ,  $E/E_o = 0.001$ , v = 0.499,  $v_o = 0.3$ , a/R = 2 was chosen as an example. A diagram of cyclic deformation of damper is provided in Fig. 3.



*Fig. 3: The loop of constructive hysteresis: active loading (line OA), unloading (line AO), repeated loading (lines BA and DEA).* 

The value of dissipated energy was defined as the area of the hysteresis loop.

$$\psi = \begin{cases} \frac{Q_{\max}^2}{3c\lambda} (1-s)(1-\sqrt{s})^2, \ s_* \le s \le 1; \\ \frac{Q_{\max}^2}{6c\lambda} \left[ \left(2+e^{-\lambda}\right)(1-e^{-\lambda})^2 - s^2\left(e^{\lambda}-1\right)^2\left(2+e^{\lambda}\right) \right], \ 0 \le s \le s_*. \end{cases}$$

Fig. 4 features a typical dependence of the normalized value of the dissipated energy  $\tilde{\psi} = \psi/A$  on the coefficients of dry friction f and asymmetry of the load cycle s. Energy of the elastic deformation of a conservative system was taken as a normalizing factor  $A = Q_{\text{max}}^2/(2c)$ .

For each value of stress ratio there is a value  $\lambda_*(s)$ , at which function  $\psi(\lambda, s)$  reaches the extreme  $\max_{\lambda} \psi(\lambda, s) = \psi(\lambda_*(s), s) = \psi^*(s)$ . Fig. 5 features results of such analysis. The global maximum is attained for pulsating cycle at  $f \approx 0.253$ . Then  $\lambda_*(0) = 4f = 1.012$ , and  $\max_{\lambda, s} \tilde{\psi}(\lambda, s) = \tilde{\psi}(1.012, 0) \approx 0.304$ .



Fig. 4: The value of dissipated energy.



Fig. 5: Stationary value  $\lambda$  and the highest values of dissipation, which are achieved for them.

Existence of the extreme value is explained by the fact that in systems with dry positional friction distribution of the friction forces depends on the deformations of the contact pair (the filler and shell), and deformations, in turn, depend on the friction forces. Such a close relationship identifies specific, often intuitively unpredictable behaviors of such structures. In this situation, the increase in the coefficient of dry friction f leads to a reduction in the area of mutual slippage of the shell and filler, and, consequently, reduction of structural energy dissipation, which occurs only in this area.

### 4. Conclusions

The article presents the main analytical results of the modeling of structural hysteresis at frictional interaction of solid and cut shells with a filler. It has been first established that for a fixed ratio of cycle asymmetry with increasing coefficient of friction between the shell and filler, the amount of energy dissipated per cycle grows, reaches an extremum, and then gradually decreases. The maximum capabilities of the shell damper as for the energy absorption have been determined, and the optimal coefficients of friction in the contact pair have been calculated. Therefore, the idea of optimization of shell vibration protecting devices according to the criterion of maximum energy absorption of external influences by providing the desired tribologic properties of the contacting pairs is declared.

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