

RUNNING FLUTTER WAVES IN BLADES CASCADE

L. Půst*, L. Pešek**

Abstract: The simplified mathematical model of dynamic properties of bladed cascade excited by wakes of flowing steam from the stationary cascade is derived. Interaction of this kind of forced excitation with aero-elastic self-excitation described by Van der Pol formula causes origin of flutter and its running waves. It is shown that the velocity, direction of flutter running waves and mode of vibration depend on the blades' number ratio and on kind of self-excitation forces on individual blade or on inter-blade distance.

Keywords: Running waves, Flutter, Self-excitation, Wakes, Ratio of blade numbers.

1. Introduction

Dynamic properties of blades cascades in turbines excited by the aero-elastic effects of flowing gas – flutter – have been intensively investigated and published during the several last decades up to the present time (Rao, 1991; Rządowski, 2007 and Pust, 2011). During experimental investigation of blades flutter properties, the existence of running waves was observed and mentioned (Yan, 1990 and Kielb, 2004). Comparatively small attention has been given to the explanation what is the cause of origin and properties of flutter running waves. Presented paper contains an attempt to reveal the possible impulses, which can excite and synchronize the flutter phenomenon. Flutter phenomenon is an intensive self-excited vibration of blade cascade structure. There are many types of description of flutter phenomena, however for simplicity an expression based on well-known Van der Pol model (Pust, 2016) is used in this study. Two modifications of this model are applied. The first one is oriented on self-excited vibration of each individual blade, the second one acts on relative motion between two neighboring blades. Different blades' number of rotating and of stator wheels causes the phase delays of excitation forces produced by the wakes of gas flow from the stator blades cascade. Consequently, the running waves of forced vibration exist, which initiate also the flutter running waves.

2. Methods

The dynamic properties of a model of turbine wheel with ten blades and excited by phase delayed harmonic forces caused by distorted stream of flowing gas from the stator blades cascade were studied in the paper (Pust, 2016) shown in Fig. 1. Another type of graphical presentation computational model of turbine blades row is shown in Fig. 2, where the tenth blade is connected with the first blade again by the same elastic element. The spring with stiffness k_1 [kg.s⁻²] represents elastic properties of disc or of shroud.

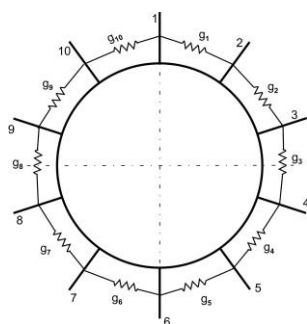


Fig. 1: Model of blade wheel.

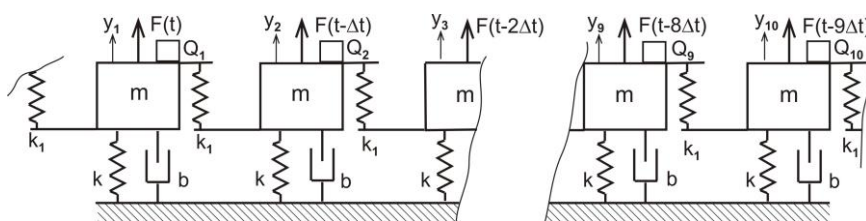


Fig. 2: Model of blade cascade.

* Ing. Ladislav Půst, DrSc, Institute of Thermomechanics AS CR, v.v.i., Dolejšková 5, 18200 Praha 8, pust@it.cas.cz

** Ing. Ludek Pešek, CSc, Institute of Thermomechanics AS CR, v.v.i., Dolejšková 5, 18200 Praha 8, pesek@it.cas.cz

Dynamic properties of blades are investigated in the narrow frequency range near the selected blade's eigen-frequency and modeled as a 1DOF system with parameters m, k, b . The wakes of steam flow from stator blades excite the blades on the rotating wheel by external periodic forces. Running periodic forces arise if the number l_r of blades on rotating wheel differs from the number l_s of blades on stator cascade $l_r \neq l_s$. These periodic forces can be simplified to one harmonic component $F_0 \cos(\omega t - (i-1)\Delta\varphi)$, where the phase delay $\Delta\varphi$ between the neighboring excitation forces depends on ratio l_s/l_r according the following relations: $\Delta\varphi = 2\pi * (1 - \frac{l_s}{l_r})$. As example for $l_s/l_r = 9/10$ is $\Delta\varphi = \pi/5$.

3. Self-excited oscillation

Steam flowing through the blade-cascade can cause decrease of damping and rise of flutter. Exact mathematical model of this aero-elastic phenomenon is very complicated, therefore we will use in this study the Van der Pol model, (Pust and Pesek, 2016) described by equation

$$G = -\mu(1 - (x/r)^2)\dot{x}, \quad (1)$$

where G is the aerodynamic force, x, \dot{x} are general displacement and velocity, r is displacement of blade at which the aerodynamic force changes its sign, μ gives intensity of this non-linear damping.

4. Position of flutter activity

The flowing steam from the stationary blade cascade influence both individual blades and also the interaction forces between neighboring blades. These situations can be graphically modelled by different positions of points of action. The position of direct self-excitation G on individual blade is depicted in Fig. 3. The inter-blades self-excitation G is shown in Fig. 4.

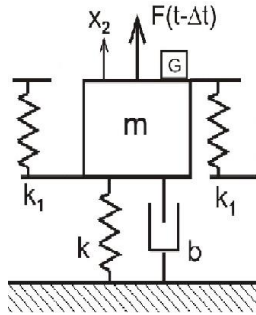


Fig. 3: Blade's self-excitation.

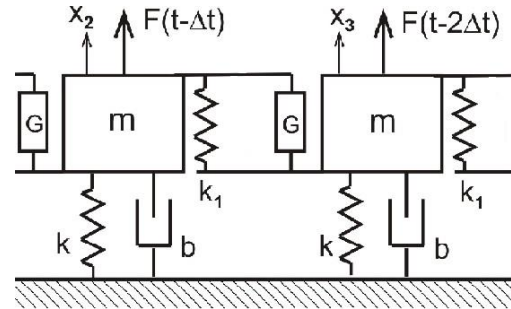


Fig. 4: Inter-blades self-excitation.

$$\text{In the first case it is: } G_i = -\mu_1(1 - (x_i / r_1)^2)\dot{x}_i, \quad (2a)$$

where index i denotes the number of blade. The self-exciting force in the second case is

$$G_{i,i+1} = -\mu_2(1 - ((x_i - x_{i+1}) / r_2)^2)(\dot{x}_i - \dot{x}_{i+1}) \quad (2b)$$

5. Vibrations of forced and aero-elastic excitation

Ten differential equations describe motion of blade cascade

$$m\ddot{x}_i + b\dot{x}_i + kx_i + g_i - g_{i-1} + G_i + G_{i,i-1} + G_{i,i+1} = F_0 \cos(\omega t + (i-1)\Delta\varphi), \quad (3)$$

where g_i describes connections between g blades $g_i = k_1(y_i - y_{i+1}) \quad i=1, \dots, 10, \quad y_{11} = y_1$.

Response curves are computed in the following examples for one blade's mass, stiffness and damping coefficient: $m = 0.182 \text{ kg}$, $k = 105000 \text{ kg.s}^{-2}$, $b = 2 \text{ kg.s}^{-1}$. The blades' interconnections stiffness is $k_1 = 1000 \text{ kg.s}^{-2}$ and amplitude of external wakes force is $F_0 = 1 \text{ N}$ with frequency $\omega = 762 \text{ rad/s}$.

6. Individual blade self-excitation

The graphical presentation of influence of forced wake vibration interaction with direct self-excitation G_i of individual blades is shown in Fig. 3. There are two damping forces acting on each blade: positive structural damping with coefficient $b = 2 \text{ kg.s}^{-1}$ and negative aero-elastic damping with coefficient μ_1 . Self-excited oscillations can arise if $\mu_1 > b = 2$, but it needs an initial impulse for its expansion. If external wake force does not exist ($F_0 = 0$), no self-excited oscillation arises, in spite of high negative aerodynamic coefficient $b - \mu_1 = 2 - 2.5 = -0.5$ as shown in Fig. 5. However, the wakes of steam flowing from the stator blade cascade at amplitude $F_0 = 1 \text{ N}$ initiate increase of self-excited vibrations as it is shown in Fig. 6 for the same numbers of stator and rotor blades $l_s / l_r = 1$ and so for $\Delta\varphi = 0$. Registered time courses of all blades are shown in these figures. The lowest and upper bold line belong to the first blade, upper dashed line belongs to the 10th blade.

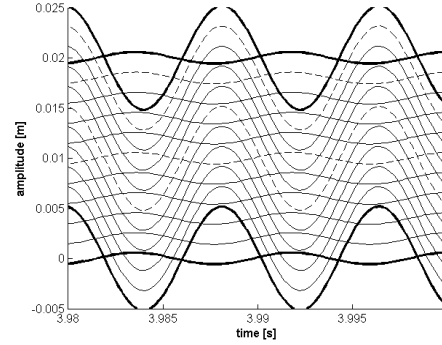
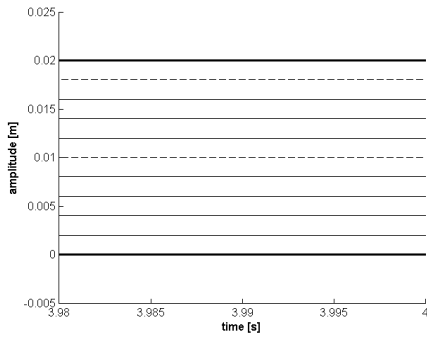


Fig. 5: Self-excitation without initial impulses.

Fig. 6: Self-excitation with initiation by wakes.

If the ratio of stator and rotor blades is $l_s / l_r = 9/10$ then $\Delta\varphi = \pi/5$ and backward running wave occurs as it is seen in Fig. 7. The change of blade's numbers to $l_s / l_r = 8/10$ causes increase of phase shift to $\Delta\varphi = 2\pi/5$ and the backward running wave velocity is half, the period of one wave's revolution is double, as shown in Fig. 8. The corresponding wave mode contains two cosines forms, as distinct from the case in Fig 8, where the form is described by one cosine form.

If the ratio of stator and rotor blades numbers is greater than 1 e.g. $l_s / l_r = 11/10$ then $\Delta\varphi = -\pi/5$ and running waves have forward direction as it is seen in Fig. 9.

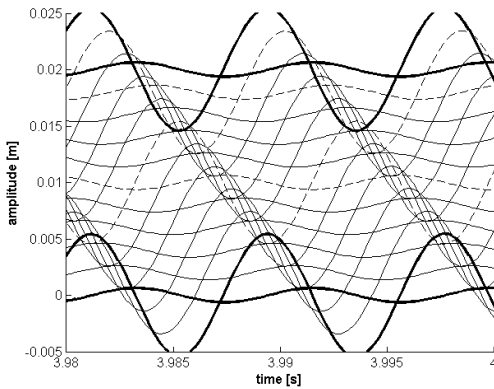


Fig. 7: Backward waves at $\Delta\varphi = \pi/5$.

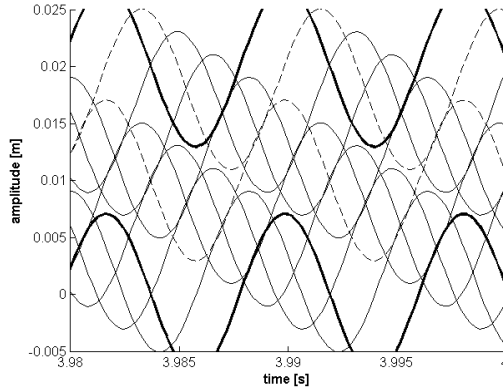


Fig. 8: Backward waves at $\Delta\varphi = 2\pi/5$.

7. Inter-blades excitation

The behavior of running waves in the case that the flowing steam from the stationary blade cascade influences the interaction between neighboring blades (see Fig. 4) distinguishes from the properties of blades' cascade with the individual blade self-excitations.

The dynamic properties of rotor blade cascade with $l_s / l_r = 9/10$ ($\Delta\varphi = \pi/5$) are shown in Fig. 10. The mode of vibration has five cosine forms on the periphery and there exist two running waves forward and

backward – designated by arrows in figure. The velocities of these running waves are five times lower than running waves depicted in Fig. 7 or 9.

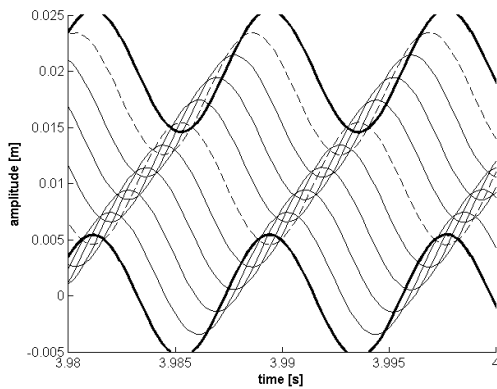


Fig. 9: Forward waves at $\Delta\varphi = -\pi/5$.

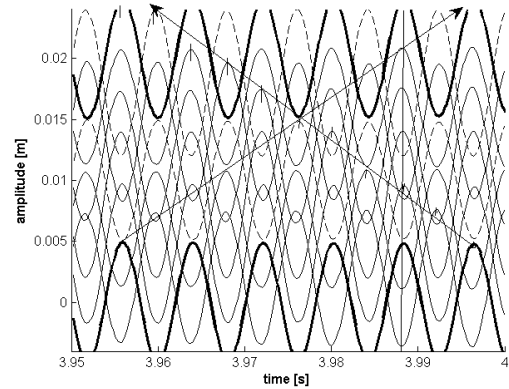


Fig. 10: Waves at inter-blades self-excitation.

8. Conclusions

The presented paper deals with investigation and ascertaining of conditions for origin of flutter in turbine including its running waves.

It is shown that the ratio of blades' numbers of stationary and of rotating disk influences due to the wakes of gas flow the phase shift between neighboring blades and consequentially also direction and velocity of forced or flutter running waves. The flutter self-excitation is realized by Van der Pol model.

There are different blade-cascade responses in the cases when the self-exciting effect of flowing steam acts direct on individual blades or by interaction forces between neighboring blades. The running velocities as well as modes of vibration in both cases are different.

Presented study is the first stage of deeper analysis. In the future the bladed cascade will be improved by addition of torsion DOF and the self-excitation elements will be modified according to the results of prepared experiments.

Acknowledgement

This work has been supported by the grant project of the Czech Science Foundation No. 16-04546S "Aero-elastic couplings and dynamic behavior of rotational periodic bodies".

References

- Kielb, R.E., et al. (2004) Flutter of low pressure turbine blades with cyclic symmetric modes: A preliminary design method, *Journal of Turbomachinery- transactions of the ASME* 126, pp. 306-309.
- Pust, L. and Pesek L. (2016) Interaction of self-excited and delayed forced excitation on blade bunch, *Proc VETOMAC XII*, Warsaw, 2016, pp. 139-148.
- Rao, J.S. (1991) *Turbomachine Blade Vibration*, Wiley Eastern Limited, New Delhi.
- Rzadkowski, R. and Gnesin, V. (2007) A3D Inviscid self-excited vibration of the last stage turbine blade row, *Journal of Fluids and Structure* 23, pp. 858-873.
- Yan, L.-T. and Li, Q.-H. (1990) Investigation of Travelling Wave Vibration for Bladed Disk in Turbomachinery, in: *Proc. 3rd Int. Conf. on Rotordynamics-IFTOMM*, (Lalanne, M., ed.), Lyon, pp. 133-135.
- Pust, L. and Pesek, L. (2011) Vibration Of Circular Bladed Disk With Imperfections, *International Journal of Bifurcation And Chaos*, Vo. 21, No. 10, pp. 2893-2904.