23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

NUMERICAL SIMULATIONS OF FREE SURFACE FLOWS USING A THREE-EQUATION MODEL

S. Plánička*, J. Vimmr**

Abstract: *Numerical results obtained from the application of a three-equation model for free surface flows are presented in the paper. The compressible two-phase flow model is suitable for complex free surface flow simulations that can, for example, include breaking waves. Compared to commonly used shallow water models, the model described in the paper offers certain advantages and wider applicability as it can deal with problems associated with complex interfaces. On the other hand, it should be noted that the model describes only flow field of one phase and the solution of pressure field is significantly simplified. Derived from the seven-equation Baer-Nunziato model and completed with the Tait equation of state, it consists of mass and momentum evolution equations in conservative form and of an advection equation for volume fraction in non-conservative form. A numerical code based on the finite volume method was developed, in which the inviscid numerical flux is approximated by an AUSM scheme. For spatial discretization of the non-conservative term contained in the model, an AUSM-based scheme is applied. The developed algorithm was successfully verified by a well-known dambreak test and further a bubble ascension problem was solved.*

Keywords: Free surface flow, Three-equation model, AUSM scheme, Finite volume method.

1. Introduction

Multi-phase flow phenomenons occur frequently in nature as well as in human-impacted activities such as industry. Considering their global significance, it is, therefore, useful to purse their computer modeling. Especially free surface flows need to be solved, which often include flows in open channels and ducts and propagation of waves on liquid surface. Currently there are many significantly different techniques, which were developed to solve free surface flow problems. The choice of an appropriate model depends on the degree of problem simplification. Two classes of models based on the finite volume method are usually distinguished: the interface tracking methods, which use boundary-fitted grids to precisely capture the free surface, and the interface capturing methods, which use a stationary grid, and as such are unable to describe the interface as sharp boundary (e.g. the MAC and VOF methods). Additionally, other efficient methods are developed such as the smoothed particle hydrodynamics or the multiphase lattice Boltzmann methods.

The main objective of this paper is to demonstrate the applicability of a simple three-equation model for the solution of complex free surface flow problems. For this purpose, a two-phase model proposed in the paper Dumbser (2011) is introduced and derived from a seven-equation model, which was designed by Baer-Nunziato (1986) for the modeling of compressible detonation two-phase waves. The numerical solution of the aforementioned flow problems is carried out by in-house computational software based on the finite volume method. The developed code was tested for the dambreak and bubble ascension problems. A comparison between the obtained numerical results and the sample results are presented.

2. Mathematical model

The three-equation model employed in this paper belongs to a class of Eulerian interface capturing methods, in which an amount of tracked fluid per volume is defined by the volume fraction α determined

* Ing. Stanislav Plánička: Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, Technická 8; 306 14, Plzeň; CZ, staplan@ntis.zcu.cz

** Assoc. Prof. Ing. Jan Vimmr, PhD.: European Centre of Excellence NTIS – New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia, Technická 8; 306 14, Plzeň; CZ, jvimmr@ntis.zcu.cz

from advection equation. The described model is applicable for the description of fluid flow with free surface, wherein the second phase is negligible (vacuum or fluid with a significantly lower density). By contrast, the approach is not suitable for solving dispersed flows, sedimentation and thermodynamics problems because of its characteristics and due to performed simplifications.

The original Baer-Nunziato model contains conservation equations of mass, momentum and energy for both phases and an advection equation of volume fraction. In this paper, the model is reduced following various simplifications proposed by Dumbser (2011). As mentioned above, the three-equation model completely neglects the secondary phase, making it possible to omit all the evolution equations belonging to the secondary phase as well as the corresponding variables from the set of equations. Additionally, it is necessary to determine appropriate values of unknown parameters at the interface, i.e., the interface pressure p_I and the interface velocity vector $\boldsymbol{v}_I$. The interface pressure is always kept on atmospheric pressure ($p_I = p_2 = p_{ATM} = 0\,Pa$), when choosing that all pressures value are relative to the atmospheric one. The interface velocity, which corresponds to the rate of advection of the volume fraction, is set to be equal to the velocity of the primary phase ($\boldsymbol{v}_I = \boldsymbol{v}_1$). In other words the interface propagates along with the primary fluid. This option is well-suited for free surface flow problems. The system was closed with the simple Tait equation of state for the primary phase

$$p = K\left((\rho / \rho_0)^\gamma - 1\right), \quad (1)$$

which directly relates the pressure p and the density ρ. For more information, see, for example, the work MacDonald (1966). K, γ are constants affecting the compressibility of the fluid and ρ_0 is the density at the reference pressure. The determination of speed of sound uses a common formula, which is valid for general equation of state during isentropic process. A major advantage of this formulation is that the density is calculated directly from the corresponding evolution equations and afterwards the pressure is determined using the equation of state in the simple algebraic form. This approach makes the energy equation unnecessary and enables us to omit it from the final set of equations. The resulting mathematical model in two dimensions can be written as

$$\frac{\partial}{\partial t}(\alpha\,\rho) + \frac{\partial}{\partial x}(\alpha\,\rho\,u) + \frac{\partial}{\partial y}(\alpha\,\rho\,v) = 0, \quad (2)$$

$$\frac{\partial}{\partial t}(\alpha\,\rho\,u) + \frac{\partial}{\partial x}\left(\alpha\,(\rho\,u^2 + p)\right) + \frac{\partial}{\partial y}(\alpha\,\rho\,v\,u) = 0, \quad (3)$$

$$\frac{\partial}{\partial t}(\alpha\,\rho\,v) + \frac{\partial}{\partial x}(\alpha\;\rho\,u\,v) + \frac{\partial}{\partial y}\left(\alpha\,(\rho\,v^2 + p)\right) = \alpha\rho g, \quad (4)$$

$$\frac{\partial}{\partial t}\alpha + u\frac{\partial}{\partial x}\alpha + v\frac{\partial}{\partial y}\alpha = 0, \quad (5)$$

where u, v denote the velocity components and the independent variables of space and time are labelled as x, y and t, respectively. The source term on the right hand side of Eq. (4) represents the gravitational force per unit volume with the acceleration of gravity g. The nonlinear system of Eqs. (2) – (5) is hyperbolic when the fluid density is positive ($\rho > 0$) and the volume fraction is $0 < \alpha < 1$. This means that the primary phase or even more the neglected secondary phase cannot vanish within whole domain. Under these conditions, eigenvalues $\{u - a,\ u,\ u,\ u + a\}$ and $\{v - a,\ v,\ v,\ v + a\}$ are real.

3. Numerical method

With the goal to examine the suitability of the presented model for free surface flow simulations, a numerical code written in Matlab was developed. The finite volume method was used for spatial discretization of the system of equations. The inviscid numerical flux was approximated by the first order scheme in order to damp instabilities of the two-phase flow model. Specifically, an AUSM scheme was used due to its simplicity, efficiency, stability and accuracy, for more information see for example papers Evje (2003) or Paillére (2003). The numerical flux vector through the k-th edge belonging to the cell L with the adjacent cell R with the unit vector of the outward normal $\left[n_x^e,\ n_y^e\right]$ is then defined as

$$\boldsymbol{F}_k^{AUSM} = \frac{M_{LR}}{2}(\boldsymbol{w}_L a_L + \boldsymbol{w}_R a_R) - \frac{|M_{LR}|}{2}(\boldsymbol{w}_R a_R - \boldsymbol{w}_L a_L) + P_{LR}\left[0, n_x^e, n_y^e, 0\right]^T, \quad (6)$$

where $M_{LR}(M_L^n, M_R^n)$ and $P_{LR}(M_L^n, M_R^n, p_L, p_R, \alpha_L, \alpha_R)$ represent the splitting functions. Unlike the approximate Riemann solvers or the characteristic flux schemes, the AUSM scheme does not require any characteristic analysis. This is a significant advantage for two-phase flow simulations, which enables us to use the AUSM scheme in a whole range of different two-phase models.

For numerical solution, the advection equation for volume fraction (5) in the non-conservative form presents a certain complication. For this reason, it was rewritten into a new form

$$\frac{\partial}{\partial t}\alpha + \frac{\partial}{\partial x}(\alpha\, u) + \frac{\partial}{\partial y}(\alpha\, v) - \alpha\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \qquad (7)$$

where the second and the third additional terms form basically a conservative convective flux of volume fraction and the fourth term is a non-conservative source term. Note that Eq. (7) was utilized in the numerical code instead of Eq. (5). The AUSM scheme was used for the inviscid flux discretization of the entire set of Eqs. (2) – (4), (7). The non-conservative term in Eq. (7) was discretized in the sense of the AUSM scheme due to consistency. The entire coupled system of Eqs. (2) – (4), (7) was time-iterated by an explicit two stage Runge-Kutta scheme.

4. Test cases

The developed numerical code was verified by the two-dimensional test case – the dambreak with bottom step into a wet bed area. The computational domain $\Omega = ([-5;\, 5] \times [0;\, 3]) \setminus ([0;\, 5] \times [0;\, 0.2])$ m is divided with an unstructured triangular mesh. The domain initially containing the fluid ($\alpha = 0.95$) is $\Omega_L^{IC} = ([-5;\, 0] \times [0;\, 1.46]) \cup ([0;\, 5] \times [0.2;\, 0.51])$ m. The gravity acceleration in the y-direction is $g = -9.81\ \mathrm{m/s^2}$ and the parameters of the Tait equation (1) are artificially set as $\gamma = 1$, $\rho_0 = 1000\ \mathrm{kg/m^3}$, $K = 0.637\ \mathrm{MPa}$ to improve the solution convergence. The initial velocities are $u = v = 0\ \mathrm{m/s}$ and the pressure is set to be equal to the hydrostatic one with zero pressure level at height $h = 1.46$ m. A wall boundary condition is applied to all boundaries of the computational domain. The parameters of the performed numerical simulation are the same as the parameters in the original work Dumbser (2011). In doing so, the purpose is to compare the results obtained using our developed solver with the published results, which were achieved with discontinuous Galerkin schemes. Both simulations used unstructured grids of comparable quality. The comparison of volume fraction results is shown in Fig. 1: developed numerical code – isolines, Dumbser (2011) – blue/red field (trimming used as $\alpha < 0.5 \ \rightarrow$ blue, $\alpha > 0.5 \ \rightarrow$ red), shallow water equations – thick black line.

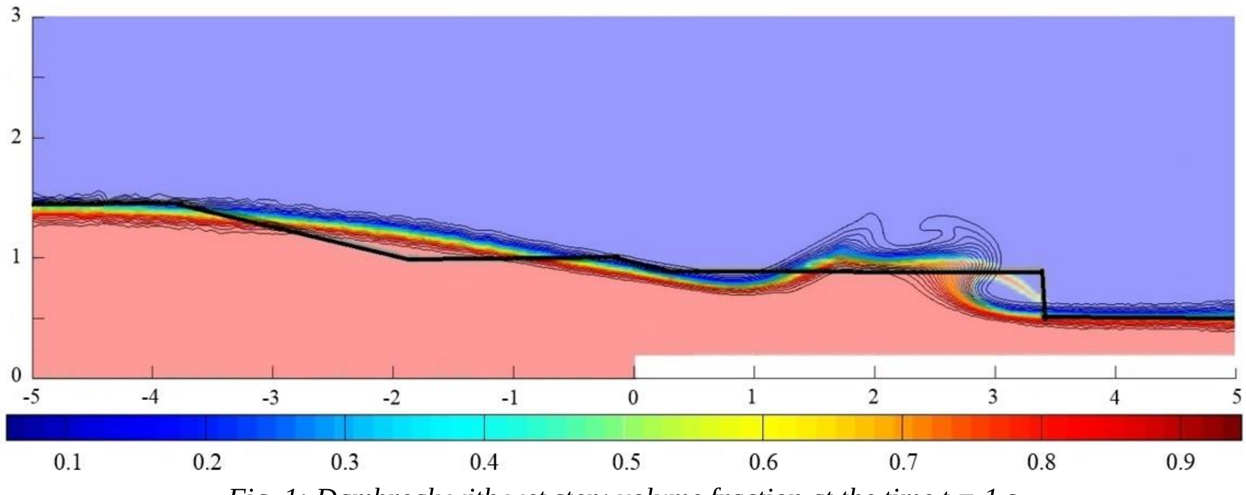

Fig. 1: Dambreak with wet step: volume fraction at the time t = 1 s.

Further, the suitability of the three-equation model for the description of the bubble ascension problem was analyzed. This task is very different from the problem of breaking waves, for which the model was primarily designed. In this case, the bubble rises under the influence of gravity acting on the surrounding fluid, making the phase interface very complex. Initially, the circular bubble $\Omega_{Bub}^{IC} = circ([1;\, 0.3], 0.2)$ m is at rest in a closed box $\Omega = ([0;\, 2] \times [0;\, 2])$ m filled with fluid. The gravity acceleration in the y-direction is $g = -9.81\ \mathrm{m/s^2}$ and the parameters of the Tait equation are the same as in the previous case. The initial pressure field has a hydrostatic profile with zero pressure level at height $h = 1$ m. The results of numerical simulations carried out by the developed software are shown in the Fig. 2 (left) at time $t = 1$ s. For illustration, Fig. 2 (right) shows the results from the paper Murrone (2005). Note that Murrone and Guillard used a more sophisticated five-equation model, different parameters of the Tait equation of state and a finer mesh. In our case, the simulation was performed on a computational grid with square cells. The results are entirely symmetrical until approximately the time $t = 0.5$ s, when the symmetry is broken probably due to accumulation of numerical errors.

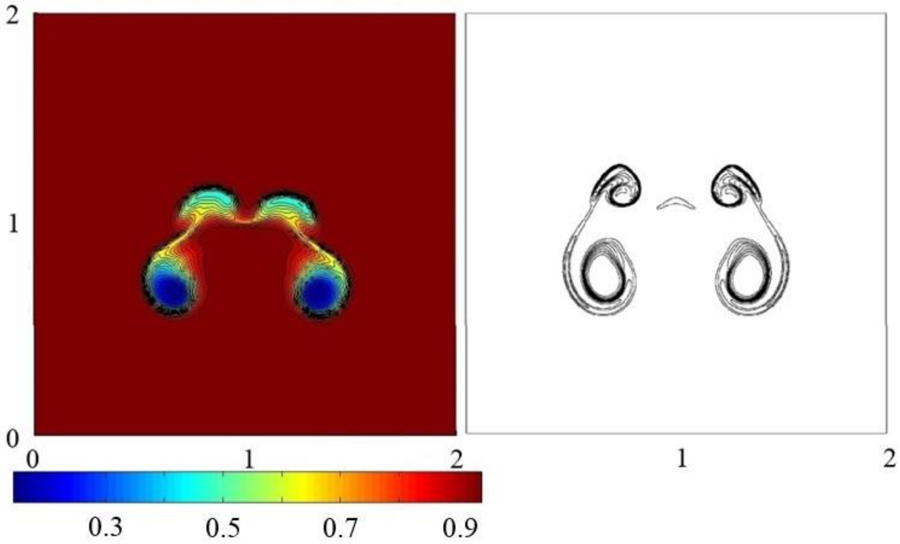

Fig. 2: Bubble ascension problem: volume fraction at the time t = 1s.

5. Conclusions

The results presented in this paper demonstrated the applicability of the three-equation model for the solution of free surface flow problems using FVM and AUSM scheme. Compared to other high-order solvers (e.g., DGFEM), our algorithm exhibits higher artificial viscosity, but is able to capture the initial breaking wave better than the shallow water equations model, see Fig. 1. Good results were also achieved for the bubble ascension problem, which demonstrated the ability of the model to handle complex free surface interfaces.

Acknowledgement

This study was partly supported by the internal student grant project SGS-2016-38 of the University of West Bohemia and partly by the project LO1506 of the Ministry of Education, Youth and Sports of the Czech Republic.

References

Baer, M.R. and Nunziato, J.W. (1986) A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials. International Journal of Multiphase Flow, 12, 6, pp. 861-889.

Dumbser, M. (2011) A simple two-phase method for the simulation of complex free surface flows. Computer methods in applied mechanics and engineering, 200, pp. 1204-1219.

Evje, S. and Fjelde, K.K. (2003) On a rough AUSM scheme for a one-dimensional two-phase model. Computers and fluids, 32, pp. 1497-1530.

MacDonald, R. (1966) Some simple isothermal equations of state. Reviews of modern physics, 38, 4, pp. 669-679.

Murrone, A. and Guillard, H. (2005) A five equation reduced model for compressible two phase flow problems. Journal of Computational Physics, 202, pp. 664-698.

Paillére, H., Corre, C. and García Cascales, J.R. (2003) On the extension of the AUSM+ scheme to compressible two- fluid models. Computers and fluids, 32, pp. 891-916.