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AN EXPLICIT TIME SCHEME WITH LOCAL TIME STEPPING FOR ONE-DIMENSIONAL WAVE PROPAGATION IN A BIMATERIAL BAR

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Abstract: *In this paper, we test a two-time step explicit scheme with local time stepping. The standard explicit time scheme in finite element analysis is not able to keep accuracy of stress distribution through meshes with different local Courant numbers for each finite element. The used two-time step scheme with the diagonal mass matrix is based on the modification of the central difference method with pullback interpolation. We present a numerical example of one-dimensional wave propagation in a bimaterial elastic bar. Based on numerical tests, the employed time scheme with pullback interpolation and local stepping technique is able to eliminate spurious oscillations in stress distribution in numerical modelling of shock wave propagation in heterogeneous materials.*

Keywords: Wave propagation in heterogeneous materials, Explicit time integration, Finite element method, Local stepping, Spurious oscillations.

1. Introduction

Currently, application potential for heterogeneous and, mainly, functionally graded materials (FGM) in industrial and engineering problems grows up. The reason is that 3D printing manufacturing processes and tools are available and financially attractive for a wider range of users. Further, functionally graded materials offer many advantages in real problems in comparison with conventional materials (Ebrahimi, 2016).

We focus on numerical solution of wave propagation in an elastic bar consisting of two different materials as the simplest problems of heterogeneous media. Wave propagation in functionally graded materials has been analyzed in (Chiu et al., 1999). More complex modelling of such heterogeneous materials has been done in (Berezovski et al., 2008).

In this paper, we use the finite element method (FEM) with explicit direct time integration based on the central difference method (Hughes, 2000). As it is known, the finite element method produces dispersion behaviour and spurious oscillations of stress components in numerical modelling of wave propagation in solids (Kolman at el., 2016b). Moreover, elastic waves run through bodies with different wave speeds. Further, elastic wave speed in heterogeneous media influences on local material parameters, therefore wave speed is different for each material position. This phenomenon means a big trouble for numerical methods, because wave speed affects stability limit for explicit schemes. Several numerical approaches for elimination of spurious oscillations in heterogeneous media have been developed, as the Park method (Park at el, 2012, Cho at el, 2013) based on pullback interpolation or the Idesman method based on post-processing filtering (Idesman 2014).

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2. Problem definition and explicit time scheme with local stepping

We consider the initial stage of wave propagation in an one-dimensional "thin" bimaterial bar in the framework of the classical small strain elasticity theory, see Fig.1. The linear constitutive equation in the form of Hooke's law is assumed, but material parameters are different for each part of a bar.

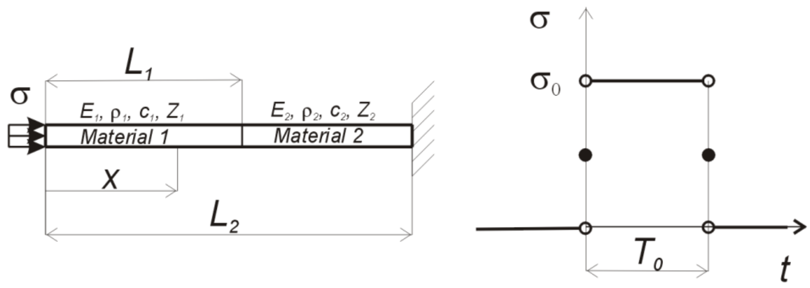

Fig. 1: Scheme of a free-fixed bimaterial elastic bar under shock loading.

One-dimensional wave motion in a bimaterial elastic bar (Fig. 1) is governed by the equations of motion for each part, see (Graff 1975), as

$$\rho_1 \frac{\partial^2 u_1}{\partial t^2} = E_1 \frac{\partial^2 u_1}{\partial x^2} \quad \text{on } [0, L_1]\text{x}[0,T] \text{ and } \rho_2 \frac{\partial^2 u_2}{\partial t^2} = E_2 \frac{\partial^2 u_2}{\partial x^2} \quad \text{on } [L_1, L_2]\text{x}[0,T] \qquad (1)$$

with boundary conditions on interface for displacement field $u_1(x = L_1, t) = u_2(x = L_1, t)$ and stress field $\sigma_1(x = L_1, t) = \sigma_2(x = L_1, t)$. Further, $t \geq 0$ denotes the time, ρ_1, ρ_2 are the mass densities, and E_1, E_2 denote the Young moduli. Wave speeds in each bar domain are given by relationships $c_{01} = \sqrt{E_1 / \rho_1}$, $c_{02} = \sqrt{E_2 / \rho_2}$. Based on analytical solution (Graff 1975), the transmitted and reflected amplitudes of the waves are prescribed as $T = \frac{2Z_2}{Z_1 + Z_2}$ and $R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$, where $Z_1 = \sqrt{E_1 \rho_1}$, $Z_2 = \sqrt{E_2 \rho_2}$ are the acoustic impedance for each material, respectively. This theoretical stress distribution along an elastic bimaterial free-fix bar under shock loading is used for the comparison with numerical solutions.

3. Numerical method for wave propagation in a heterogeneous bar

The used numerical method for wave propagation in heterogeneous materials is based on the algorithm presented in the Park's papers (Park et al., 2012) and (Cho et al., 2013). This scheme has been reformulated into the two-time step scheme in work (Kolman et al., 2016a). The used time stepping process is consisted of following two computational steps for predictor-corrector form:

Step 1: ***Pull-back integration with local stepping***

a) Integration by the central difference scheme with the local (elemental) critical time step size for each finite element (i.e. Δt_{c1} or Δt_{c2}).

b) Pull-back interpolation of local nodal displacement vector at the time $t_{n+1} = t_n + \Delta t$.

c) Assembling of local contributions of displacement vector from Step 1b.

Step 2: ***Push-forward integration with averaging***

a) Push-forward integration by the central difference scheme with the time step size Δt.

b) Averaging of the total displacement vectors at the time t_{n+1} form Steps 1c and 2a.

c) Evaluation of acceleration and velocity nodal vectors at the time t_{n+1}.

Implementation details and formulae for this two-time step scheme, one can see in (Park et al., 2016; Cho et al., 2013; Kolman et al., 2016a). A scheme of pullback interpolation for two different local critical time step sizes is depicted in Fig. 2.

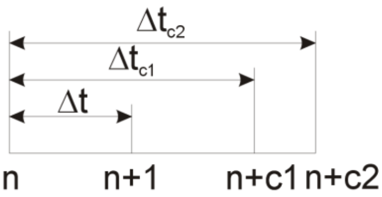

Fig. 2: Extrapolation of the displacement u(x, t) at steps (n + c1) and (n + c2) followed by interpolation at the next step (n + 1).

4. Formulation of bimaterial test and results

In this Section, we present results of numerical solution of wave propagation problem in a bimaterial elastic bar defined in the previous Section. Geometrical and material parameters of the task are set as: the domain lengths $L_1 = 1$ *m*, $L_2 = 2$ *m*, the cross-section $A = 1$ *m*^{2}, Young's modulus $E_1 = 16$ *Pa*, $E_2 = 1$ *Pa*, the mass density $\rho_1 = \rho_2 = 1$ *kg*/*m*^{3} and the amplitude of impact pressure $\sigma_0 = 1$ *Pa*, thus the applied force is $F_0 = A\sigma_0 = 1$ *N*. We use computational mesh with 240 uniform finite elements, the FE length is taken as $H = L_2 / NE = 0.0083$ m. Time duration of the loading is takes as $T_0 = 0.5 L_1 / c_1$, final time of computations is set as $t_{end} = 1.8 L_1 / c_1$. The critical time step sizes for finite elements of material 1 and material 2 are given as $\Delta t_{c1} = H / c_{01}$ and $\Delta t_{c2} = H / c_{02}$. In the case, $c_{01} > c_{02}$, the value Δt_{c1} dictates the global stability limit so $\Delta t_c = \Delta t_{c1}$. For computations by the Park method with and without local stepping, we use the time step size as $\Delta t = 0.5 \Delta t_{c1}$. For analysis of accuracy of the central difference method, results are computed for time step sizes $\Delta t = 0.5 \Delta t_{c1}$ and $\Delta t = 0.9 \Delta t_{c1}$. It means by the Courant numbers: $Co = 0.5$ and $Co = 0.9$. The transmitted and reflected amplitudes for the test are given as $T = 2/5$ and $R = 3/5$.

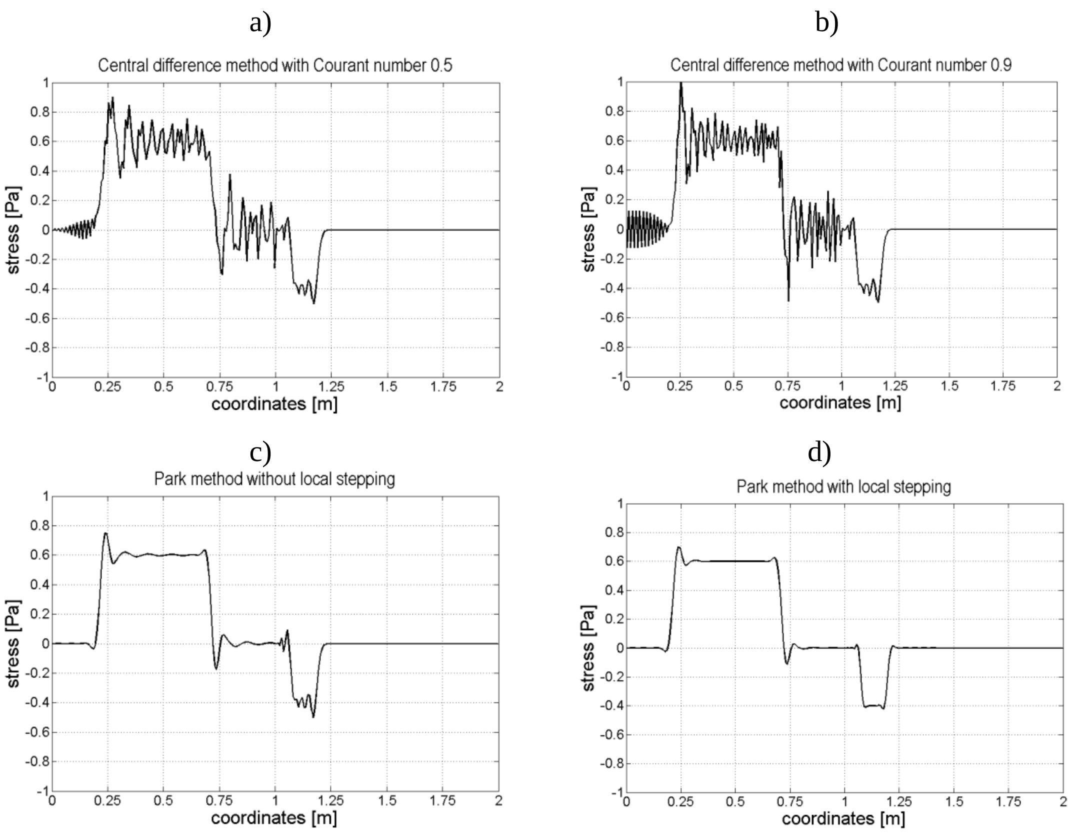

Fig. 3: Stress distributions at a bimaterial elastic bar under shock loading obtained by a) the central difference method with Courant number 0.5; b) the central difference method with Courant number 0.9; c) the Park method without local stepping; d) the Park method with local stepping.

In Fig. 3, one can see results of stress distributions in a bimaterial elastic bar defined above. We employ the central difference method with Courant numbers 0.5 and 0.9, the Park method without and with local stepping technique. Based on comparison, we can say that the Park method with local stepping technique cardinally improves the stress distribution and, in principle, spurious oscillations are eliminated, but only small cusps on the corners of stress discontinuities can be observed.

5. Conclusions

In this paper, we have tested the two-time step explicit scheme based on pullback interpolation with local stepping for accurate tracking of elastic wave in heterogeneous media. The bimaterial bar test has showed accuracy of the presented scheme. Further, the scheme is able to eliminate spurious oscillations in numerical modelling against the central difference method. In the future, we will focus on using the presented scheme for wave modelling in layered and functionally graded materials.

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References

Berezovski, A., Engelbrecht, J. and Maugin, G.A. (2008) Numerical Simulation of Waves and Fronts in Inhomogeneous Solids, World Scientific Pub Co Inc.

Ebrahimi, F. (2016) Advances in Functionally Graded Materials and Structures. InTech.

Graff, K.F. (1975) Wave motion in elastic solids, first ed., Oxford University Press.

Hughes, T.J.R. (2000) The Finite Element Method: Linear and Dynamic Finite Element Analysis. Dover Publications: New York.

Chiu, T.-C. and Erdogan, F. (1999) One-dimensional wave propagation in a functionally graded elastic medium. Journal of Sound and Vibration, 222, pp. 453-487.

Cho, S.S., Park, K.C. and Huh, H. (2013) A method for multi-dimensional wave propagation analysis via component-wise partition of longitudinal and shear waves. International Journal for Numerical Methods in Engineering 95, 3, pp. 212-237.

Idesman A. (2014) Accurate finite-element modeling of wave propagation in composite and functionally graded materials. Composite Structures 117, pp. 298-308.

Kolman, R., Cho, S.S. and Park, K.C. (2016a) Efficient implementation of an explicit partitioned shear and longitudinal wave propagation algorithm. Inter. Journal for Numerical Methods in Eng. 107, 7, pp. 543-579.

Kolman, R., Plešek, J., Červ, J., Okrouhlík, M. and Pařík, P. (2016b) Temporal-spatial dispersion and stability analysis of finite element method in explicit elastodynamics. International Journal for Numerical Methods in Engineering, 106, 2, pp. 113-128.

Park, K.C., Lim, S.J. and Huh, H. (2012) A method for computation of discontinuous wave propagation in heterogeneous solids: basic algorithm description and application to one-dimensional problems. International Journal of Numerical Methods and Engineering, 91, 6, pp. 622-643.