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SOME REMARKS ON PREVENTIVE REPLACEMENT MODEL

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Abstract: *This paper presents a policy of preventive replacement that consists of the burn-in procedure and age-replacement. Criteria function in the derived maintenance model is a cost per unit time. Properties of criteria function are obtained .Illustrative example based on the derived maintenance model is given in this paper.*

Keywords: Lifetime distribution function, Reliability function, Failure rate function, Early failure, IFR-class, Burn-in procedure, Age-replacement, Criteria function.

1. Introduction

With development of the contemporary manufacturing technology, products have become more technologically advanced and more reliable. Burn-in is the procedure used to eliminate early failures in maintenance process. Preventive maintenance policy such as age-replacement is often used in maintenance. Age-replacement plays a key role in organization of the maintenance because it can significantly contribute to reducing cost of maintenance. As it is well known, in age-replacement model, if the technical object does not fail before a perspective time x, then it is replaced by a new one preventively; otherwise object is replaced at the failure time. Burn-in procedure consists of testing a new technical object for a given period before its active life in order to predict early failures. Burn-in procedure has been studied by several authors, including (Kuo and Kuo, 1983) who presented a main aspect of this procedure. In paper (Block and Savits, 1997), the burn-in optimization examples are derived. Many research studies combined policies of burn-in with age-replacement, for example (Jiang and Jardine, 2007; Canfield, 1975; Mi, 1994 and Drapella and Kosznik, 2002). The use of this combined policy is sometimes more effective than use of burn-in procedure without age-replacement. An example of combined policy is presented in (Mi, 1994). In this paper the period of inspection method and the replacement method are determined. Often the situation is that a population time to failure is heterogeneous and consists of two sub-populations, which represent early failures and wear-out failures. For the early failures the mean time to failure (MTTF) is "short", while for the wear-out failures the MTTF is "long". The sets times to failures is a mixture of two set early failures and wear-out failures. Mixture distributions model often arises for a reasons in statistical data of times to failures. Reduction of early failures is done in the burn-in process and failures in wear-out are removed by age-replacement. In this paper the sufficient conditions for the existence of the minimum per unit time for age-replacement are investigated. In this paper the sufficient condition of existence of minimum cost per unit time for age-replacements investigation. This is investigated assuming that the burn procedure was previously used to point time b. Distribution of lifetime T has non-decreasing failure rate function $\lambda(t)$. Criteria function considered in this paper was introduced in paper (Jiang and Jardine, 2007). A numerical example is analyzed to illustrate investigation of this paper.

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2. Model

Suppose that the population times consist of two sub-populations, one is weak with early failures and the other has with has a longer lifetime. To reduce the early failures, all technical objects experience a burn-in process with burn-in time b. In this article the following notation is used: T – time to failure, $F(t)$ – distribution function of T, $F(t)=P(T<t)$, $R(t)$ – reliability function of T, $R(t)=1-F(t)$, $f(t)$ – density function of T, $f(t)=F'(t)$, $\lambda(t)$ – failure rate function $\lambda(t)=f(t)/R(t)$, IFR – class of lifetime with non-decreasing failure rate function $\lambda(t)$, $ET(x)$ – mean time to failure or preventive replacement at moment x is $ET(x)=\int_0^x R(t)dt$, b – long of period burn-in, x – moment of preventive replacement, C_r – unit cost of repair in burn-in process, C_f – unit cost of repair after burn-in process, C_p – unit cost of preventive replacement, C_0 – unit cost of burn-in procedure.

In this paper we assume that:

Assumption 1: $C_f - C_p > 0$,
Assumption 2: $C_r - C_p > 0$.

The expected burn-in and replacement cost per unit time is considered by (Jang and Jardine, 2007) given by

$$C(x,b)=\frac{C_B(b)+C_R C(x,b)}{w(x,b)} \tag{1}$$

where $C_B(b)$ is the expected burn-in cost and

$$C_B(b)=\frac{C_r F(b)}{R(b)}+C_0 T_b$$

$$T_b=\frac{1}{R(b)}\int_0^b R(t)dt$$

$$C_R(x,b)=C_p+(C_f-C_p)+\frac{F(x+b)-F(b)}{R(b)}$$

After a simple transformation of $C(x,b)$ given by (1) is now

$$C(x,b)=\frac{(C_f-C_p)F(x+b)+(C_r-C_f)F(b)+C_0ET(b)+C_p}{ET(x+b)-ET(b)} \tag{2}$$

The first partial derivative respect to b of criteria function $C(x,b)$ is given by

$$\frac{\partial C}{\partial b}(x,b)=\frac{(C_f-C_p)\left[H(x+b,x+b)-H(x+b,b)\right]+(C_r-C_p)\left[H(b,x+b)-H(b,b)\right]}{[ET(x+b)-ET(b)]^2}+ \\ \frac{C_0\left[R(b)ET(x+b)-ET(b)R(x+b)\right]-C_p[R(x+b)-R(b)]}{[ET(x+b)-ET(b)]^2} \tag{3}$$

where $H(x,y)=f(x)ET(y)-F(x)R(y)$.

The first partial derivative respect to y of the function $H(x,y)$ is given by

$$\frac{\partial H}{\partial y}(x,y)=f(x)ET(y)+F(x)R(y)$$

Function $H(x,y)$ is increasing under y. Now, we conclude that

$$H(x+b,x+b)-H(x+b,b)\ge 0, \tag{4}$$

$$H(b,x+b)-H(b,b)\ge 0. \tag{5}$$

For last but one term of nominator of (2), we obtain

$$C_0\left[R(b)ET(x+b)-ET(b)R(x+b)\right]=C_0ET(x+b)ET(b)\left[\frac{R(b)}{ET(b)}-\frac{R(x+b)}{ET(x+b)}\right]$$

The function $v(x)=\dfrac{R(x)}{ET(x)}$ is decreasing, and

$$C_0\left[R(b)ET(x+b)-ET(b)R(x+b)\right]\geq 0 \tag{6}$$

For last term of nominator of (2), we obtain

$$C_p\left[R(x+b)-R(b)\right]\leq 0 \tag{7}$$

By assumptions A1, A2, and (4 – 7), we obtain

$$\frac{\partial C}{\partial b}(x,b)\geq 0$$

Corollary 1. The criteria function $C(x,b)$ is non-decreasing with respect to b.

We will analyze of the first derivative of criteria function $C(x,b)$ with respect to x.

$$\frac{\partial C}{\partial x}(x,b)=\frac{1}{R^2(x+b)}\Big\{\Big[\left(C_f-C_p\right)\left[H(x+b,x+b)-f(x+b)ET(b)\right]-\left(C_r-C_p\right)F(b)R(x+b)\Big]$$
$$-C_pR(x+b)-C_0ET(b)R(x+b)\Big\}$$

Let $H_1(x,b)=H(x+b,x+b)-f(x+b)ET(b)$.
We can see $H_1(0,b)=R(x+b)F(x+b)<0$ and $H_1(\infty,b)=0$.

$$\frac{\partial C}{\partial x}(x,b)=\frac{1}{R(x,b)}\left[\left(C_f-C_p\right)h(x,b)-A(b)\right] \tag{8}$$

where $h(x,b)=\lambda(x+b)\left[ET(x+b)-ET(b)\right]$ and $A(b)=\left(C_r-C_p\right)F(b)+C_p+C_0ET(b)$.

We can see that $h(0,b)=-F(b)\leq 0$ and by assumption A1 is $A(b)>0$.

The first derivative with respect to x of $h(x,b)$ is given by

$$\frac{\partial h}{\partial x}(x,b)=\lambda'(x+b)\left[ET(x+b)-ET(b)\right]$$

If $T\in IFR$ then

$$\frac{\partial h}{\partial x}(x,b)\geq 0$$

Theorem. If $C_f-C_p>0$, $C_r-C_p>0$, $T\in IFR$ and

$$\lambda(\infty)>\frac{\left[\dfrac{A(b)}{C_f-C_p}+1\right]}{\left(ET-ET(b)\right)} \tag{9}$$

then exactly one minimum of criteria function $C(x,b)$ exists.

Proof. Let $u(x,b)=\left(C_f-C_p\right)h(x,b)-A(b)$. By $C_f-C_p>0$, we conclude that $u(0,b)<0$ and $u(x,b)$ increasing under x. If $u(\infty,b)>0$ then the first derivative (8) exactly one change of sign for – to +.
The condition $u(\infty,b)>0$ is equivalent to inequality (9).

Corollary 2. If $\lambda(\infty)=\infty$ then, the assumption (4) is true and criteria function $C(x,b)$ approaches one minimum.

3. Numerical example

This section presents a numerical example to illustrate our results obtained in Section 2. The time to failure of a unit is assumed to follow a Weibull distribution and have the reliability function:

$R(t)=exp\left(-\left(\frac{t}{a}\right)^{C}\right)$ for $\boldsymbol{a}, \boldsymbol{c} > \mathbf{0}$, $\boldsymbol{t} \geq \mathbf{0}$, the failure rate function $\lambda(t)=\left(\frac{c}{a}\right)\left(\frac{x}{a}\right)^{C-1}$, $t \geq 0$.

Also, we assume that $b=1.25$, $C_r=8$, $C_f=10$, $C_p=1$, $C_0=0.2$

Limit value of criteria function is

$$C(\infty,b)=\frac{\left(\left(C_r-C_f+C_p\right)+C_pET(b)+C_f-C_p\right)}{\left(ET-ET(b)\right)}$$

In this example for every Weibull distributions parameter $c\in\{2.5,3,3.5,4\}$ we compute a value of parameter a such that $C(\infty,b)=2.4$. Fig. 1 shows the graphs that describe the cost per unit time. We observe that for every $c\in\{2.5,3,3.5,4\}$ function $C(x,b)$ approaches the minimum cost per unit time.

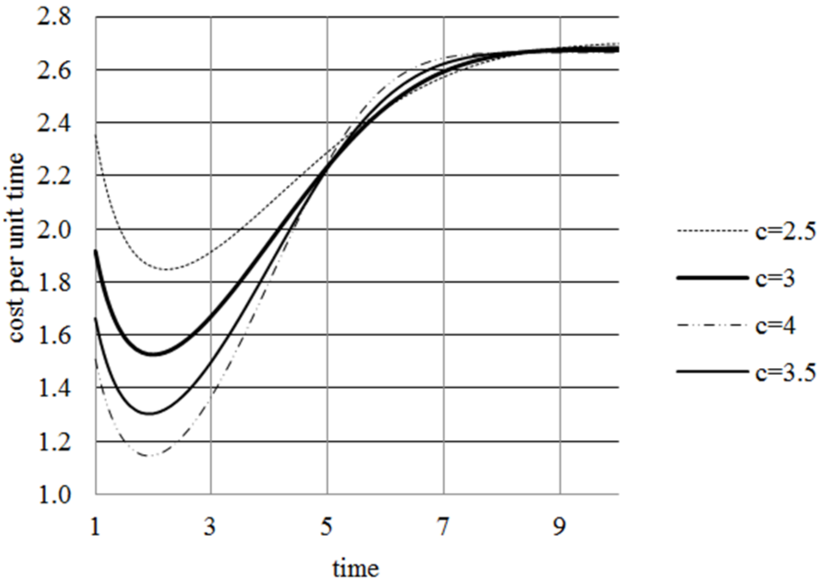

Fig. 1: Cost C(x, b) as function of time x of preventive replacement.

4. Conclusions

Cost function $C(x,b)$ proposed by (Jiang and Jardine, 2007) approaches a minimum respect to time of age-replacement. In this paper the sufficient conditions for the existence of the minimum per unit time for age-replacement are formulated.

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