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NONLINEAR VIBRATION – STOCHASTIC APPROACH

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Abstract: *This paper presents a stochastic approach to the free vibration of a solid body with nonlinear damping. Simulations were conducted in MATLAB and Anthill, using the Monte Carlo method. The results of practical damping tests were used and approximated to create a mathematical model of vibration. For the stochastic calculations, the input parameters of the mathematical model were assigned deviations using a histogram of probability distribution reflecting real operational situations.*

Keywords: Vibration, Nonlinear damping, Dynamics, Stochastic mechanics, Monte Carlo.

1. Introduction

This paper describes a solution for the technical free vibration of a solid body taking into account the influence of nonlinearity in viscous dampers. The calculations are based on motion equations of technical vibration including nonlinearity of damping via the mathematical expression of the characteristics of the damping elements. As it is very problematic to find a closed-form solution, the vibration is solved numerically. An integral part of the paper is a stochastic calculation of vibration incorporating variance in input values. The stochastic calculations are conducted using the Monte Carlo method in Anthill software. The resulting curves for the investigated parameters can find practical application in the optimization of products such as rotary machines, where the parameters of mass, centre of gravity position etc. change during operation. An ideal example is a washing machine, whose mass and centre of gravity vary depending on the quantity of water or clothing inside it, meaning that stochastic variance realistically represents the normal operation of the machine. Variables also include the characteristics of suspension springs or damping, as shown in the probability calculations. For more details see Brousil (1989), Juliš (1987), Timoshenko (1960) and Frydrýšek (2011), Frydrýšek (2012).

2. Nonlinear free vibrations

In order to determine the curves for the vibration of a body, it is necessary to specify the damping function F_b [N], see Fig. 1b, which best corresponds with real data. The appropriate power function was specified in the following form:

$$F_b(v) = sign(v) \cdot p_1 \cdot (|v|)^{p_2}, \qquad (1)$$

where the coefficients p_1 = 140 kg/s, p_2 = 1/5 and v [m/s] is the velocity of linear motion of the damper.

It is very difficult, if not impossible, to find a closed-form solution for self-induced or externally-induced vibration with the given nonlinear damping function. Possible variants therefore include the linearization of the damping function or numerical calculation. From our observations, initial conditions and measuring follows the simplification based on the 1D model (i.e. the other 5 generalized coordinates can be neglected).

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(a) (b)

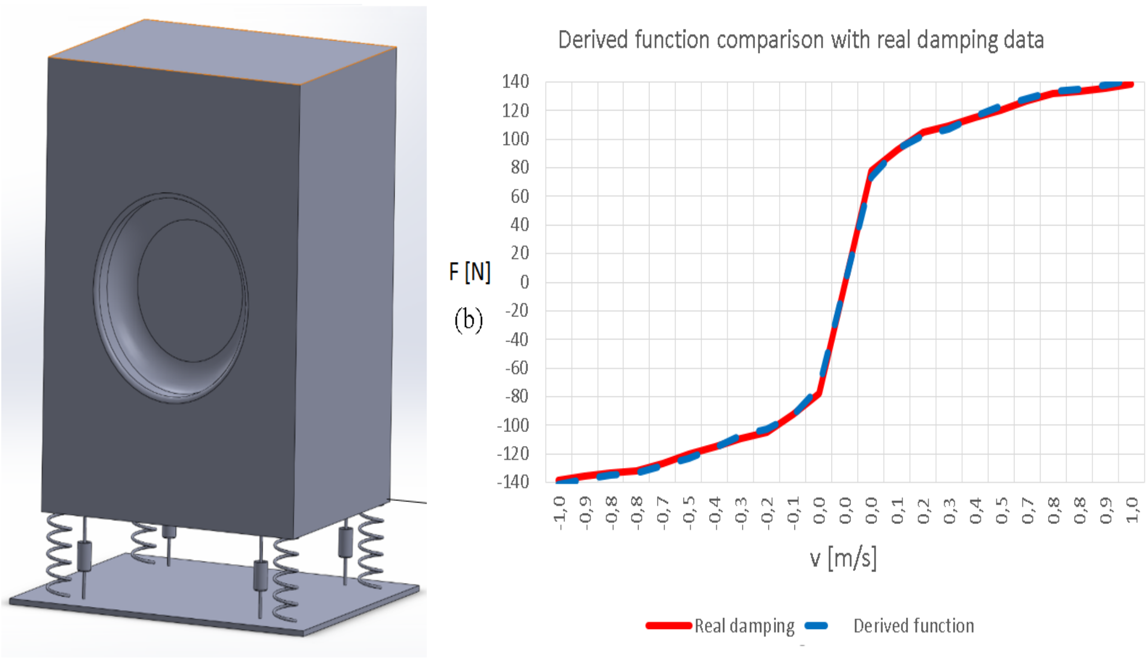


Fig. 1: a) Illustration of the solid body; b) comparison of derived damping function with real data.

The advantage of numerical calculation is the option of using nonlinear damping characteristics, which better represent reality than the linearization of the damping function. The calculation assumes an even acceleration of motion at very short intervals dt = 0.0001 s. The solution is thus performed by iteration, with the initial displacement x_0 [m] and the initial velocity v_0 [m/s]. The relation for acceleration is derived from the motion equation as follows:

$$a = \frac{-F_k - F_b}{m_t} = \frac{-k_c \cdot x - sign(v) \cdot p_1 \cdot (|v|)^{p_2}}{m_t} \quad \left[\frac{m}{s^2}\right], \tag{2}$$

where F_k [N] is the directional force of the suspension springs, F_b [N] is the directional force of the dampers, m_t [kg] is the mass of the body, k_c [N/m] is the total suspension spring stiffness, and x [m] is the trajectory of motion.

After introducing the initial values x_0 [m] and v_0 [m/s], we obtain the initial acceleration a_0 [m/s^2]. It is then necessary to select an appropriate number of integration steps for the numerical solution dependent on the time step dt.

The numerical calculation can be carried out using the loop "for" in MATLAB software; see Fig. 2:

```
mt=173;kc=27200;x0=0.03;v0=0;
p1=140;p2=0.2;
dt=0.0001;
n=20000;

for i=1:n
    a0(i)=-(kc*x0(i)+sign(v0(i))*p1*abs(v0(i))^p2)/mt;
    v(i+1)=v(i)+a(i)*dt;
    x(i+1)=x(i)+v(i)*dt+0.5*a(i)*dt^2;
    a(i+1)=-(kc*x(i+1)+sign(v(i+1))*p1*abs(v(i+1))^p2)/mt;
end;
```

Fig. 2: Numerical calculation of vibration (MATLAB program).

The resulting curves for nonlinear vibration (trajectory, velocity and acceleration) are shown in Fig. 3.

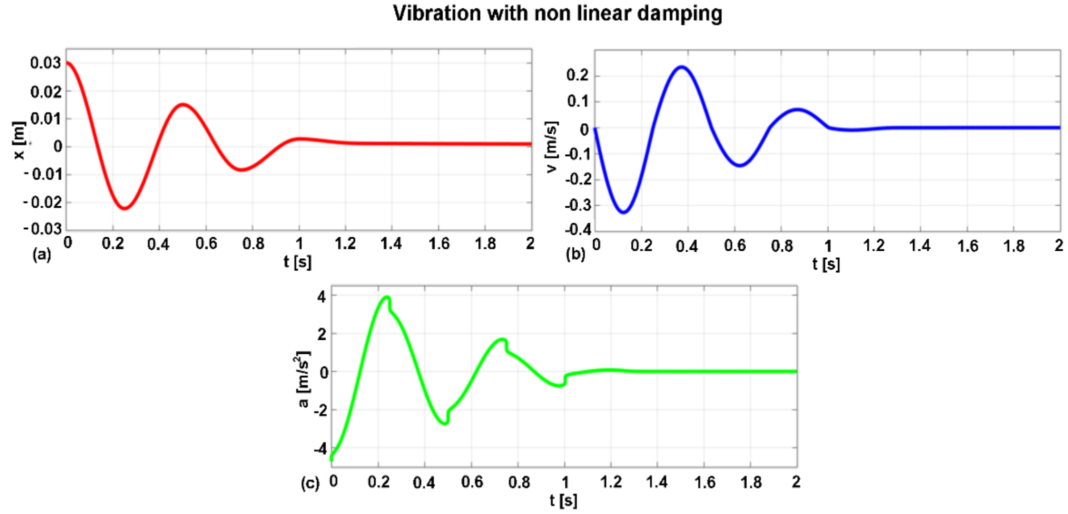


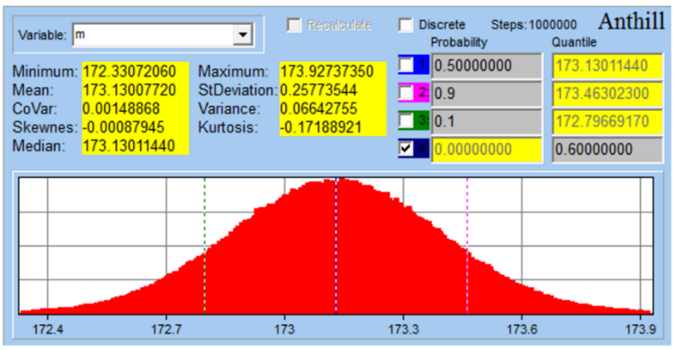
Fig. 3: Vibration with nonlinear damping: a) trajectory; b) velocity; c) acceleration.

3. Stochastic inputs

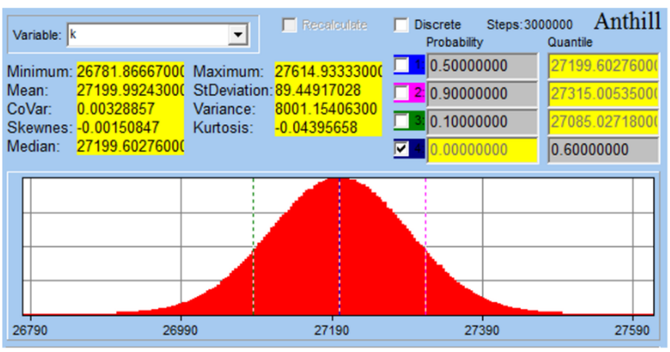
The stochastic approach (Monte Carlo method) is explained in the references Frydrýšek (2010), Frydrýšek (2011) and Frydrýšek (2012).

In order to solve the vibration of a body with a certain probability value (the stochastic approach), the individual inputs were assigned probability distributions. This corresponds sufficiently with reality. Three basic parameters were used with the specified variance (m_t = 173.13 ± 0.8 kg, k_c = 27199.603 ± 4 5.331 N/m, x_0 =0.03 ± 0.006 m).

Probability distributions of the individual parameters can be depicted using histograms. For example, the histogram for the probability distribution of the total mass of the body is shown in Fig. 4a, and the histogram for suspension spring stiffness is shown in Fig. 4b.



(a)

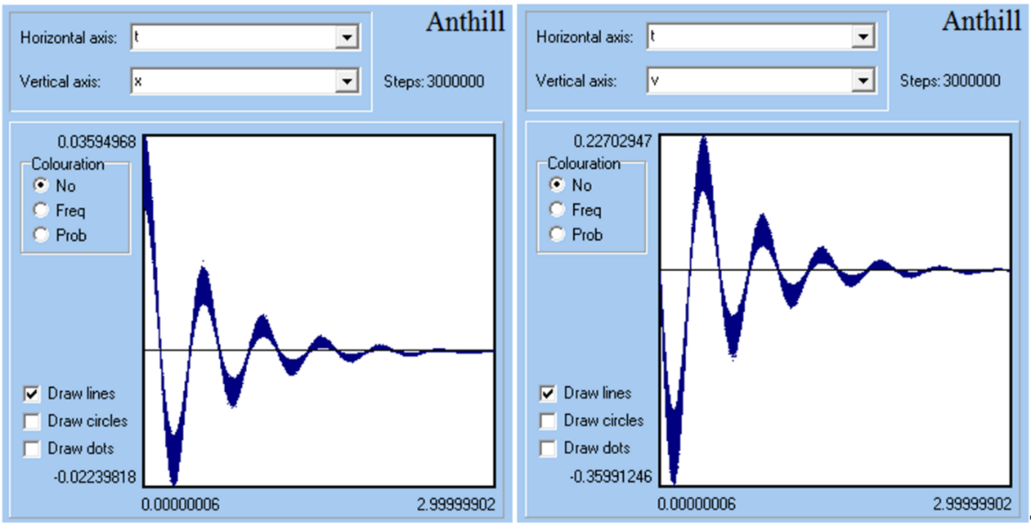


(b)

Fig. 4: Probability distribution histogram for: a) total mass m_t= 173.13 ± 0.8 kg; b) suspension spring stiffness k_c= 27199.603 ± 415.331 N/m.

4. Stochastic outputs

The resulting vibration curves for 3×10^6 Monte Carlo simulations show a variance that is best illustrated in graphic form, e.g. depicting the time dependence of trajectory and velocity; see Fig. 5.



(a) **(b)**

Fig. 5: a) time dependence of trajectory; b) time dependence of velocity.

5. Conclusion

The data characterizing the damping of a body in motion were interpolated by means of a nonlinear function that was incorporated into the vibration calculation. As it was not possible to find a closed-form solution, the problem was solved numerically on a simple 1D model. The calculation used stochastic inputs (histograms) in the Monte Carlo method. The individual input parameters were assigned probability distributions corresponding with reality; the solution thus describes real variance. The advantage of using stochastic calculations is the ability to determine the interval of values in which an observed parameter occurs with a specified level of probability. In the case of the vibration of a body with rotating unbalance, this method brings significant benefits especially in determining the resonance area which must be overcome as rapidly as possible when the machine is starting or finishing its rotary motion. The calculations using Anthill software (Monte Carlo method) were conducted with 3×10^6 random simulations. The stochastic approach is a modern trend in science and technology which enables engineers to respect the reality of random parameters that are typical of random vibration.

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