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HEAT TRANSFER IN NON-CONTACTING FACE SEALS USING FRACTIONAL FOURIER LAW

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Abstract: *The paper presents a mathematical model of the heat transfer process in non-contacting face seals. A time fractional Fourier law, obtained from fractional calculus, was applied for a slewing ring (rotor). Heat transfer in a stator was described by Laplace's equation. The fractional heat conduction equation (for the rotor) was considered in the case* $0 < \alpha \le 2$.

Keywords: Time Fractional Fourier Law, Mechanical Seal, Non-Contacting Face Seal, Heat Transfer, Integral Transform.

1. Introduction

The paper presents a model based on time fractional Fourier law, where a derivative of temperature becomes fractional over time. Most of the scientific papers relating to the transfer of heat in non-contacting face seals are based on the classical Fourier law. One of the first studies illustrate mathematical models which contain an explicit reference to heat transfer in one direction (Pascovici and Etsion, 1992) and do not include changes in temperature over time. Other works include a much more advanced thermo-hydrodynamic and thermo-elasto-hydrodynamic models of non-contacting face seals (Blasiak, S., 2015; Blasiak, S. et al., 2014). In the field of non-contacting face seals one can see a broader aspect of research. It concerns vibration analysis of sealing rings based on models presented in the works (Blasiak, M., 2016; Blasiak, S. and Zahorulko, 2016; Koruba and Krzysztofik, 2013; Krzysztofik and Koruba, 2014), but also experimental studies including measurements of geometrical dimensions and surface structure, materials (Adamczak and Bochnia, 2016; Bochnia, 2012; Koruba and Krzysztofik, 2013; Krzysztofik and Koruba, 2014) and protective layers (Nowakowski, Miesikowska, et al., 2016; Nowakowski, Miko, et al., 2016; Nowakowski and Wijas, 2016), or studies aimed at obtaining temperature measurements on different test rigs. Another aspect of ongoing research work, which has become extremely popular in the last thirty years, is the differential and integral calculus conducted on derivatives of fractional order. An example may be the new rheological model, heat transfer and mass flow (Blasiak, S., 2016), dynamic processes (Kaczorek, 2013), phenomena taking place in electrical networks, robotics (Janecki et al., 2015; Janecki and Zwierzchowski, 2009, 2015; Laski, 2016; Laski et al., 2014; Laski Pawel et al., 2015; Pietrala, 2016; Takosoglu, J E et al., 2014; Takosoglu, J E, 2016) or properties of viscoelastic materials. This paper presents a model of heat transfer in non-contacting face seals. Integral transform was used to solve differential equations involving partial derivatives. Results concerning the solution of time fractional Fourier law were presented and discussed.

2. Theoretical Model

A part of the theoretical model was presented and solved in previous work (Blasiak, S., 2016), while describing the stationary issues of heat transfer in the following system: fluid film – sealing rings for non-contacting face seals. A conduction equation (9) for a non-rotating ring will be fixed (steady state), while the rotor will take into account the changes in temperature over time (10) and it will be a fractional equation with respect to time.

$$\frac{1}{r}\frac{\partial \theta^s}{\partial r}+\frac{\partial^2 \theta^s}{\partial r^2}+\frac{\partial^2 \theta^s}{\partial z^2}=0 \text{ for } r_i \le r \le r_o;\ 0 \le z \le L^s, \qquad (1)$$

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$$\frac{\partial^\alpha \theta^r}{\partial t^\alpha} = \kappa\left(\frac{1}{r}\frac{\partial \theta^r}{\partial r} + \frac{\partial^2 \theta^r}{\partial r^2} + \frac{\partial^2 \theta^r}{\partial z^2}\right), \text{ for } r_i \le r \le r_o;\ 0 \le z \le L^r;\ t > 0. \quad (2)$$

The exact transition to time fractional heat conduction equation was presented in the work of (Povstenko, 2015). The left side of the equation (2) is the Caputo fractional derivative in the general form:

$$\frac{\partial^\alpha}{\partial t^\alpha} f(t) \equiv D_C^\alpha f(t) = I^{n-\alpha} D^n f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n} d\tau \qquad (3)$$
$$t > 0, \qquad n-1 < \alpha < n$$

with initial conditions:

$$t = 0 \quad \theta = 0, \quad 0 < \alpha \le 2,$$
$$t = 0 \quad \frac{\partial \theta}{\partial t} = 0,\ 1 < \alpha \le 2. \qquad (4)$$

The solution of equations (1) and (2) for the sealing ring – fluid film system is possible using the analytical methods including boundary conditions specified in the paper (Blasiak, S., 2016).

3. Analytical solution of the model

Marchi–Zgrablich integral transform, Fourier finite cosine transform and Laplace transform for time-fractional differentia equations were used to solve the (2) equation for a rotor. After performing the algebraic transformations and applying three integral transforms, the equation (2) has the following form:

$$\hat{\bar{\theta}}^{r*}(s) = -\frac{\kappa \bar{q}}{\lambda^r}\frac{1}{\left(s^\alpha + \omega^2\right)}. \qquad (5)$$

Temperature of the rotor is described by the following equation:

$$T^r = T_o - \frac{\kappa}{\lambda^r L^r}\sum_{n=1}^{\infty}\frac{\bar{q}\cdot S_p\left(\lambda^r, \alpha^f, k_n \cdot r\right)}{\int_{r_i}^{r_o} r\left[S_p\left(\lambda^r, \alpha^f, k_n \cdot r\right)\right]^2 dr}\sum_{m=1}^{\infty} t^{\alpha-1}E_{\alpha,\alpha}\left(-\omega_0^{\,2} t^\alpha\right) +$$
$$-\frac{2\kappa}{\lambda^r L^r}\sum_{n=1}^{\infty}\frac{\bar{q}\cdot S_p\left(\lambda^r, \alpha^f, k_n \cdot r\right)}{\int_{r_i}^{r_o} r\left[S_p\left(\lambda^r, \alpha^f, k_n \cdot r\right)\right]^2 dr}\sum_{m=1}^{\infty} t^{\alpha-1}E_{\alpha,\alpha}\left(-\omega^2 t^\alpha\right)\cdot\cos\left(\frac{m\pi z}{L^r}\right). \quad (6)$$

A similar calculation procedure aimed at solving equation (1) was carried out for a stationary ring.

$$T^s = T_0 + \sum_{n=1}^{\infty}\frac{\cosh(k_n z)\int_{r_i}^{r_o} r\cdot\theta^f\cdot S_p\left(\lambda^s, \alpha^f, k_n\cdot r\right)dr}{\cosh\left(k_n L^s\right)\int_{r_i}^{r_o} r\left[S_p\left(\lambda^s, \alpha^f, k_n\cdot r\right)\right]^2 dr}\cdot S_p\left(\lambda^s, \alpha^f, k_n\cdot r\right). \qquad (7)$$

4. Results and discussion

Numerical calculations were performed based on physico-chemical properties of the materials used for sealing rings and the medium to be sealed, gathered in the table below.

Tab. 1:The properties of materials used for rings.

	Young's modulus E	**Poisson's coefficient ν**	**Thermal conductivity λ**	**Thermal Expansion τ**
Silicon carbide	380 (GPa)	0.18	130 (*W/mK*)	*$5x10^{-6}$ (°C^{-1})*
Carbon	25 (*GPa*)	0.20	15 (*W/mK*)	*$4x10^{-6}$ (°C^{-1})*

The calculations also assume, as in the work:(Pascovici and Etsion, 1992), that the height of the radial slot is $1\,\mu m$.

Tab. 2:Geometric and operating reference parameters.

Geometric parameters		**Operating parameters**	
Inner radius r_i	0.040 (m)	Angular velocity ω	500 (rad/s)
Outer radius r_o	0.045 (m)	Nominal slot h_o	$1x10^{-6}$ (m)
Thickness of rings L^s and L^r	0.010 (m)	Medium temperature T_o	20 (^oC)
Thermal conductivity λ^f	0.65 (W/mK)	Heat transfer coefficient (water) α^f	18000 $(W/m^2\,K)$

Fig 1. shows temperature distribution for $t = 0.25(s)$ and the order of fractional derivative α in the range of $\alpha = 0.5 - 2$.

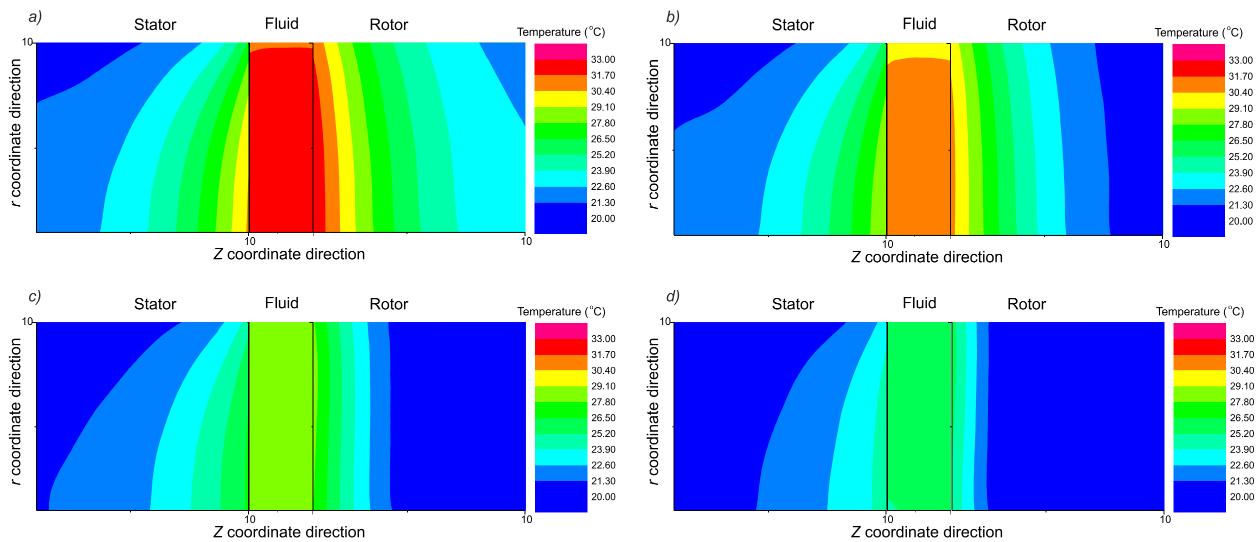

Fig. 1: Temperature distribution in the fluid film and sealing rings for: a) $\alpha = 0.50$, b) $\alpha = 1.00$, c) $\alpha = 1.50$, d) $\alpha = 2.00$.

The use of fractional heat conduction equation reflects the whole spectrum of heat conduction. If we assume $\alpha = 1$, we will receive the classical form of Fourier's equation. In case of $\alpha = 2$, it will be the ballistic heat conduction.

5. Conclusion

The use of differential equations of fractional order has expanded the possibilities for the analysis of thermal phenomena occurring in the slot, on the surface and subsurface layer of the stator and rotor. The friction heat caused by shear stresses occurring in the medium layer affects the local changes in temperature of the seal. After exceeding the boiling point in fluid film, the working medium is discharged, which in turn leads to dry friction and drastic wear of sealing rings. In addition, it may also result in the allowable temperature being exceeded for the used materials, lead to their destruction due to the occurrence of defects related to cracking or obliterating of rings resulting from thermal stresses.

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