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MATHEMATICAL MODELING OF MICRO INDENTATION OF A TRANSVERSELY ISOTROPIC HALF-SPACE WITH FUNCTIONALLY GRADED COATING BY A CONICAL INDENTER

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Abstract: *The paper considers rigid punch with conical tip which is indented into a surface of an elastic transversely-isotropic half-space with a functionally-graded transversely-isotropic coating. Elastic moduli of the coating vary independently with depth according to arbitrary positive continuously differentiable functions. Integral transformation technique is used to construct a dual integral equation of the problem. Cases of free and fixed boundaries of the contact area are considered. Fixed boundaries of the contact correspond to the case when the cylindrical punch with conical tip is indented on a depth greater than height of the punch tip. Bilateral asymptotic method is used to construct the approximated analytical expressions for the contact stresses, indentation force and radius of the contact area (in case of free boundaries). Some aspects of modeling of micro- and nano- indentation experiments are discussed. Numerical examples are provided for a case of a hard homogeneous or functionally graded transversely isotropic coating.*

Keywords: **Indentation, Conical indenter, Functionally graded coating, Transversely isotropic material, Contact problem, Analytical solution.**

1. Introduction

Improvement of the service life of various structural elements is one of the most important tasks facing the modern industry. Various types of protective coatings which have a complex functionally graded or layered structure are used for this purpose. To characterize strength and elastic properties of these coatings nanoindentation experiments are widely used. Most of widely adopted nanoindentation analysis methods rely on solutions of classical contact problems for isotropic homogeneous materials (Oliver and Pharr, 2004). Increasing usage of complex functionally graded and composite anisotropic coatings requires development of more accurate mathematical models for analysis of experimental results. Recent results on mathematical simulation of deformation of functionally-graded materials can be found in (Alinia et al., 2016; Kudish et al., 2016a, 2016b, 2017; Su et al. 2016; Tokovyy and Ma, 2015).

Indenters of different sizes and shapes are used for indentation of samples. The most common shapes of the indenter's tip are spherical, pyramidal and conical. It is known that in mathematical simulation of the widely used Berkovich pyramid a conical punch with an appropriate opening angle can be successfully used. This paper is focused on mathematical simulation of indentation of an elastic transversely isotropic half-space with a functionally graded transversely isotropic coating by a conical punch.

2. Formulation of the problem

A nondeformable conical punch is pressed into a surface of an elastic transversely isotropic half-space consisting of a functionally graded layer (coating) of thickness *H*, and a homogeneous half-space (substrate). A cylindrical system of coordinates r', φ, z is chosen with the *z* axis being normal to the surface of the half-space and passing through the center of the punch. The *z* axis is the axis of symmetry

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for all elastic moduli. The punch is pressed into the surface $z = 0$ of the half-space by the action of a normal centrally applied force P and moves distance δ downward the z-axis. It is assumed that there are no friction forces between the punch and the half-space.

Cone angle – 2α is obtuse enough, i.e $\pi - 2\alpha << 1$; $\chi = a\,\mathrm{ctg}\,\alpha$ stands for a length of the projection of the segment connecting the edge of a contact area with the apex of the cone to z axis. The projection of the contact area is a circle of radius a. Elastic moduli $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ of the half-space vary with depth according to the following:

$$c_{kj} = \begin{cases} c_{kj}^{(c)}(z) & -H \le z \le 0, \\ c_{kj}^{(s)} = const & -\infty < z < -H, \end{cases} \quad (kj) = 11, 12, 13, 33, 44, \tag{1}$$

where $c_{kj}^{(c)}(z)$ are arbitrary continuously differentiable positive functions, $c_{kj}^{(s)}$ are positive constants. Hereafter, superscripts (c) and (s) correspond to the coating and to the substrate, respectively. The coating and the substrate are glued without sliding. Outside of the punch, the surface is traction-free.

3. Solution of the problem

Using the integral transformation technique the problem can be reduced to the following dual integral equation, see (Aizikovich et al., 2015) for details:

$$\begin{cases} \int\limits_0^\infty \overline{p}(\gamma) L(\lambda\gamma) J_0(r\gamma) d\gamma = E_{ef}^{(c)} \dfrac{(\delta - r\chi)}{2a}, \ r \le 1, \\ \int\limits_0^\infty \overline{p}(\gamma) J_0(r\gamma)\gamma d\gamma = 0, \ r > 1, \end{cases} \tag{2}$$

where $r = r'/a$ is the dimensionless radial coordinate; $\lambda = H/a$ is the relative thickness of the coating; J_0 is the Bessel's function of the first kind; $E_{ef}^{(c)}$ is the effective elastic modulus on the surface of the coating; $L(\lambda\gamma)$ is the kernel transform of the integral equation or the compliance function of the elastic half-space with functionally graded coating; $\overline{p}(\gamma)$ is the Hankel transform of the contact stresses $p(r) = -\sigma_z\big|_{z=0}, \ r \le 1$.

The kernel transform $L(\lambda\gamma)$ for arbitrary variation of elastic moduli can be calculated only numerically from the two point boundary value problem for a system of ordinary differential equations with variable coefficients, see (Aizikovich et al., 2015) and (Vasiliev et al., 2016a) for details.

Solution of the dual integral equation (2) is constructed using bilateral asymptotic method. The method is based on an idea of approximation of the kernel transform $L(\lambda\gamma)$ by the following expression:

$$L(\lambda\gamma) \approx L_N(\lambda\gamma) = \prod_{i=1}^N \left(\lambda^2\gamma^2 + A_i^2\right) / \left(\lambda^2\gamma^2 + B_i^2\right), \quad A_i, B_i \in C. \tag{3}$$

Replacing kernel transform L by its approximation L_N we get the approximated dual integral equation which can be solved in closed analytical form, see (Aizikovich et al., 2015) for details. Thus, the contact stresses has the form:

$$\begin{aligned} p(r) = \frac{E_{ef}^{(s)}\chi}{2a} \Bigg[& \ln\frac{1+\sqrt{1-r^2}}{r} + \left(\frac{2\delta}{\pi\chi} - 1\right)\frac{1}{\sqrt{1-r^2}} + \\ & - \sum_{i=1}^N \left(\frac{\lambda}{A_i} \frac{C_i \,\mathrm{ch}(A_i\lambda^{-1}) + D_i \,\mathrm{sh}(A_i\lambda^{-1})}{\sqrt{1-r^2}} - C_i \int\limits_r^1 \frac{\mathrm{sh}(A_i\lambda^{-1}t)dt}{\sqrt{t^2-r^2}} - D_i \int\limits_r^1 \frac{\mathrm{ch}(A_i\lambda^{-1}t)dt}{\sqrt{t^2-r^2}} \right) \Bigg] \end{aligned} \tag{4}$$

Here $E_{ef}^{(s)} = E_{ef}(-H-0)$ is the effective elastic modulus of the substrate. Expression (4) describes the contact pressure for the fixed contact boundaries, i.e. than the edges of the cylindrical punch with conical tip are cut into the material. For free boundaries we should satisfy the following condition: $p(1)=0$. Satisfying it we get contact stresses for the case of free contact boundaries:

$$p(r)=\frac{E_{ef}^{(s)}\chi}{2a}\left(\ln\frac{1+\sqrt{1-r^2}}{r}+\sum_{i=1}^{N}\left(C_i\int_r^1\frac{\mathrm{sh}(A_i\lambda^{-1}t)}{\sqrt{t^2-r^2}}dt+D_i\int_r^1\frac{\mathrm{ch}(A_i\lambda^{-1}t)}{\sqrt{t^2-r^2}}dt\right)\right). \quad (5)$$

Coefficients C_i, D_i can be obtained from a system of linear algebraic equations. For the case of free boundaries, this system has the form:

$$\begin{gathered}\sum_{i=1}^{N}\frac{D_i}{A_i^2-B_k^2}=B_k^{-2},\quad k=1,..,N;\\ \sum_{i=1}^{N}C_i\frac{A_i\,\mathrm{ch}(A_i\lambda^{-1})+B_k\,\mathrm{sh}(A_i\lambda^{-1})}{A_i^2-B_k^2}=B_k^{-1}-\sum_{i=1}^{N}D_i\frac{B_k\,\mathrm{ch}(A_i\lambda^{-1})+A_i\,\mathrm{sh}(A_i\lambda^{-1})}{A_i^2-B_k^2}.\end{gathered} \quad (6)$$

Radius of the contact area can be found from the equation:

$$\delta=\frac{\pi\chi}{2}\left(1+\lambda\sum_{i=1}^{N}\left[\frac{C_i}{A_i}\mathrm{ch}(A_i\lambda^{-1})+\frac{D_i}{A_i}\mathrm{sh}(A_i\lambda^{-1})\right]\right) \quad (7)$$

Using the conditions of the equilibrium of the punch $P=2\pi a^2\int_0^1 p(r)rdr$ we get the expression for the indentation force for the case of free boundaries:

$$P=\frac{\pi a E_{ef}^{(s)}}{2}\chi\left(1+2\lambda\sum_{i=1}^{N}\left(\frac{C_i}{A_i}\left[\mathrm{ch}\frac{A_i}{\lambda}-\frac{\lambda}{A_i}\mathrm{sh}\frac{A_i}{\lambda}\right]+\frac{D_i}{A_i}\left[\frac{\lambda}{A_i}-\frac{\lambda}{A_i}\mathrm{ch}\frac{A_i}{\lambda}+\mathrm{sh}\frac{A_i}{\lambda}\right]\right)\right). \quad (8)$$

For determination of mechanical properties of materials with coating from the results of nanoindentation it is convenient to consider the contact stiffness (Pharr et al., 1992): $S=dP/d\delta$. It can be obtained from (7) and (8) as a function of parameter λ.

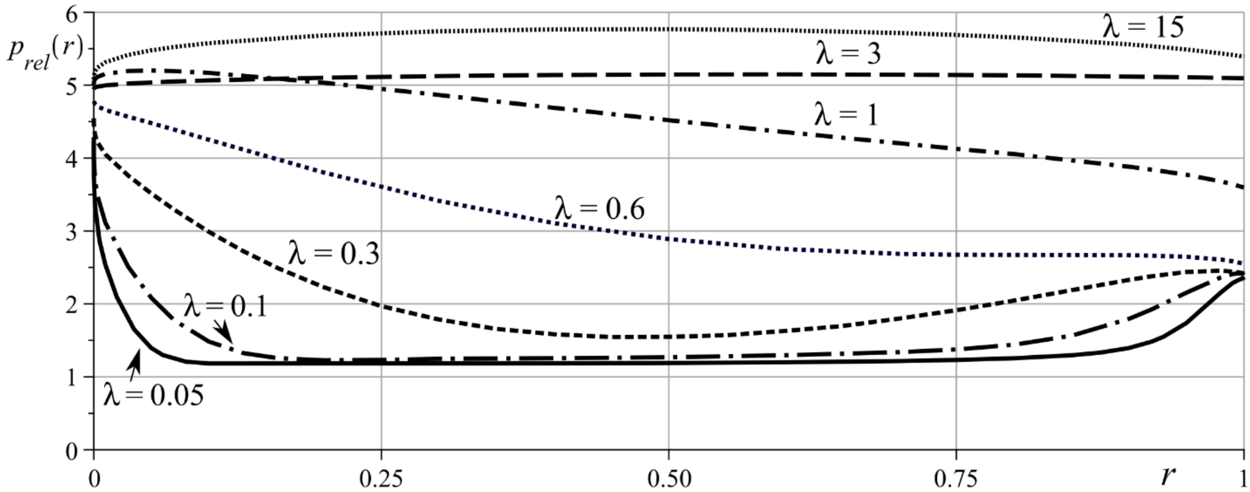

Fig. 1: Relative contact stresses p_{rel} on surface of the functionally graded coating.

As an illustration of the obtained results let us consider an isotropic glass fiber as a substrate material, Young's modulus and Poisson's ratio are $E^{(s)}$ = 21 GPa and $\nu^{(s)}$ = 0.15. Coating is assumed to be transversely isotropic with elastic moduli on its surface: c_{11} = 213.5 GPa, c_{13} = 138.67 GPa, c_{33} = 222.67 GPa and c_{44} = 23.67 GPa corresponding to the single crystal of gold with all the crystallites having a <111> axis parallel to each other and oriented normal to the substrate surface. We consider a coating with linearly varying elastic moduli to values corresponding to pure platinum $c_{11}=c_{33}$ = 314.5 GPa, c_{13} = 192.75 GPa, c_{44} = 60.87 GPa. To illustrate the behavior of the contact normal pressure, let us designate the dimensionless relative function $p_{rel}^{(c)}(r)=p(r)/p_{\mathrm{hom}}^{(c)}(r)$, where

$$p_{\text{hom}}(r) = E_{ef}^{(s)} \chi \big/ (2a) \ln\left(\left(1 + \sqrt{1-r^2}\right)\big/ r\right) \tag{9}$$

Relative contact pressure p_{rel} for various values of λ are presented in Fig. 1.

4. Conclusions

The present research continues the study of contact mechanics for functionally-graded (FG) materials made by authors earlier. Approximated analytical solutions was previously constructed for: plane contact problem on indentation of an isotropic (Kudish et al., 2016a) and transversely isotropic (Vasiliev et al., 2017) FG half-plane; axisymmetric torsion of a transversely isotropic FG half-space (Vasiliev et al., 2016b) and axisymmetric indentation of an isotropic FG half-space by a conical punch (Aizikovich et al., 2015), etc. Analytical expression for the contact stiffness which takes into account the inhomogeneity of the elastic material can be used in the analysis of nano- or micro-indentation experiments.

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