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LAMINATED GLASS STRUCTURES IN BENDING: TIME/TEMPERATURE-DEPENDENT FINITE ELEMENT MODELS

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Abstract: *The lamination of glass sheets with ductile interlayers significantly changes the post-fracture response of glass structures and increases their safety. The aspects important for the modeling of laminated glass structures are: (i) heterogeneity in material parameters of glass and polymer foil (the ratio of shear moduli exceeds 1,000), (ii) time/temperature-dependent behavior of polymer foil, and (iii) effects of geometric non-linearity as a result of slenderness of laminated glass. One of the modeling approaches is finite element formulation based on refined theories. In the proposed model, kinematics relations are formulated for each layer individually and the compatibility on the interfaces of layers is ensured via Lagrange multipliers with the meaning of forces holding the neighboring layers perfectly bonded. The comparison of models with different assumptions is performed in this contribution: formulation based on large deflection or finite strains theories for kinematics, and constitutive assumption of constant bulk modulus or constant Poisson's ratio in relations for time/temperature-dependent behavior of polymeric interlayer. The developed models were verified against the detailed finite element model in ADINA and compared with a simplified model assuming elastic behavior of polymer foil with the secant shear modulus set according given temperature and loading time.*

Keywords: Laminated glass, Finite element method, Lagrange multipliers, Generalized Maxwell model, Williams-Landel-Ferry equation.

1. Introduction

Through continuous improvements in production technologies over the last decades, glass elements have attained a more structural role. The lamination of glass sheets with ductile interlayers significantly changes the post-fracture response of glass structures and increases their safety. The interfacial adhesion between glass and the interlayer is ensured by heating in combination with the application of high pressures, resulting in high gluing forces of chemical nature. Several types of interlayers have found their use in practice, for example polyvinyl butyral initially used for automotive glass or ionoplast polymer providing increased safety and security in structural applications. From car industry applications, laminated glass has expanded into the building constructions, such as roof and floor systems, staircases, or pedestrian bridges, and the application area keeps expanding in response to the pursuit of ever greater transparency in modern architecture.

2. Overview and methods for modelling

An extensive overview of the current state-of-the-art in structural glass design and engineering can be found e.g. in (Louter et al., 2014).

2.1. Behavior of laminated glass

What makes the modeling of laminated glass structures nontrivial? The common denominator is their heterogeneity. Namely, (i) polymer foils are much more compliant than the glass layers, rendering the assumptions of conventional beam or shell theories for laminates inapplicable, (ii) due to the polymer foil incorporation, their mechanical response is sensitive to load duration, temperature, strain, and strain rate e.g. (Delincé, 2014), and (iii) laminated structures display effects of geometric non-linearity as a result of

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their slenderness. These phenomena have to be complemented with dynamic effects, arising from, e.g., impact of small projectiles in windstorms or fall of a human body – the so-called low-velocity impact that is the focus of our future research.

2.2. Overview of methods

Extensive research into the mechanical response of laminated glass structures has been undertaken to understand their unfractured behavior. Single-layer approaches, such as the concept of effective thickness, approximate the behavior of a unit by an equivalent homogeneous one. Their main advantage is their simplicity and satisfactory accuracy, but they are difficult to extend for the geometrically non-linear effects. The same arguments hold for the closed-form solutions for laminated glass plates. The most common approach is based on fully resolved 2D or 3D finite element models, which provide accurate response, but lead to expensive computations because of the large thickness-to-span ratio. The computational cost can be reduced by solid-shell elements, e.g. (Fröling and Persson, 2013), or refined beam and plate formulations, e.g. (Zemanová, 2014b). The latter approach builds on the variational formulation of refined plate theories, in which independent kinematics is considered for each layer and the inter-layer compatibility is enforced by the Lagrange multipliers when minimizing the total energy of the system. We adapted this concept to develop efficient and accurate finite element formulations for large-strain and large-deflection analyses of laminated beams and plates with temperature-dependent viscoelastic interlayer via time-incremental energy minimization. In this contribution, we focus on models for laminated glass beams.

3. Finite element formulations based on refined theories

3.1. Assumptions

The proposed geometrically nonlinear finite element models of laminated glass beams, derived from a refined plate theory by Mau (1973), assume planar cross sections of individual layers but not of the whole laminated glass unit. We treat each layer independently and enforce the compatibility by the Lagrange multipliers. This approach could capture possible delamination, however a perfect adhesion is supposed.

3.2. Kinematics

For kinematics equations, we used two approaches. The first of them, the Reissner finite-strain beam theory (FS), is more general, while the second one is based on von Kármán assumptions (VK) of large deflections and small in-plane displacements and rotations.

3.3. Constitutive relations

The main engineering property relevant to the composite behavior of the units is the shear-stress versus shear-strain characteristics of the soft interlayer. The shear modulus of the polymer interlayer is experimentally determined as a function of duration of loading and temperature, see (Pelayo et al., 2013). For that reason, the behavior of polymer foil is linearly viscoelastic, while glass is an elastic material in our models. Two different formulations assuming constant value of bulk modulus K (FS_K and VK_K) or Poisson's ratio υ (FS_υ and VK_υ) were proposed. The temperature dependence is taken into account by the time-temperature superposition principle; we employ the Williams-Landel-Ferry equation. These viscoelastic approaches are compared with a simplified model assuming elastic behavior of polymer foil with the value of the shear modulus set according given temperature and loading time.

3.4. Solution procedure

The geometrically nonlinear solver from (Zemanová et al., 2014a) was extended to the incremental viscoelasticity formulation. The sought displacement increment ${}^{k+1}\delta r$ and the vector of Lagrange multipliers ${}^{k+1}\lambda$ are determined from the linearized system

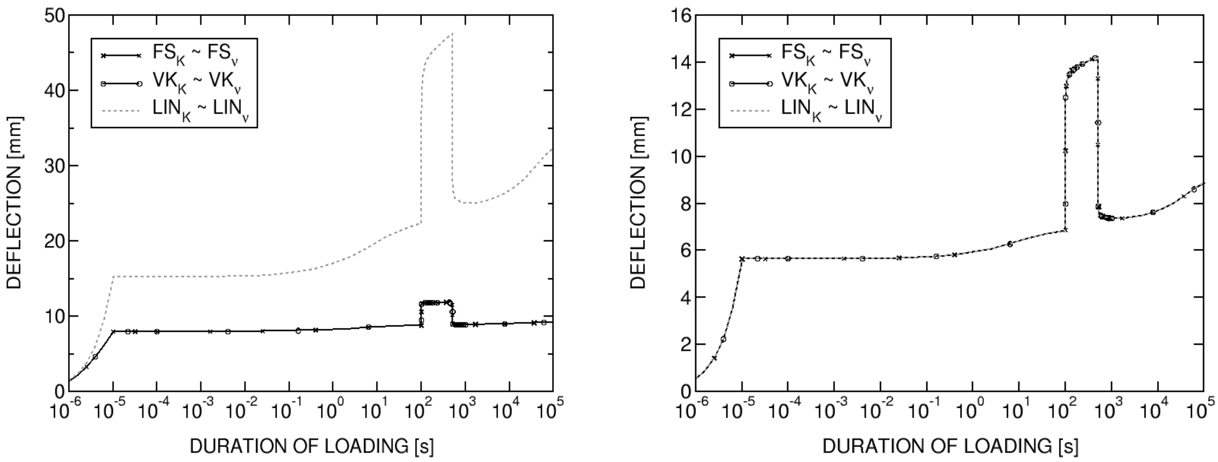
$$\begin{bmatrix} {}^{k}\hat{K} & {}^{k}C^{\mathrm{T}} \\ {}^{k}C & 0 \end{bmatrix} \begin{bmatrix} {}^{k+1}\delta r \\ {}^{k+1}\lambda \end{bmatrix} = -\begin{bmatrix} {}^{k}\hat{f}_{\mathrm{int}} - f_{\mathrm{ext}} \\ {}^{k}c \end{bmatrix}. \qquad (1)$$

The stiffness matrix for k-th iterative step ${}^{k}\hat{K}$ is composed of independent stiffness matrices of layers and therefore exhibits block structure. Matrix ${}^{k}C$ and vector ${}^{k}c$ implement the compatibility conditions for displacements on the interface of neighboring layers. For finite-strain formulation, these matrices have to be calculated for each iterative step, whereas for von Kárman assumptions, C is still the same matrix composed of constant values and ${}^{k}c$ is the zero vector. External nodal forces f_{ext} correspond to the loading of structure for given time step and ${}^{k}\hat{f}_{\text{int}}$ has the meaning of internal nodal forces for k-th iterative step. The notation ${}^{k}\hat{K}$ and ${}^{k}\hat{f}_{\text{int}}$ is used to emphasize that these quantities are determined for interlayer using the effective values of Young's modulus (or shear modulus) and include the additional terms due to relaxation effects.

4. Results, comparisons and verification

The most common laminated beams with three layers (glass/PVB/glass) are considered in this section. We compare the proposed models for two examples: the fixed-end beam and the simply-supported beam. In practical applications, the laminated glass elements are not perfectly fixed or simply-supported; therefore these two cases are limits of the real support conditions. The beams are loaded by uniformly distributed transverse pressure with a constant magnitude during the loading duration or with two-load history (with a jump in magnitude of loading).

It follows from the comparisons of results and from Fig. 1 that the viscoelastic approaches based on the assumption of constant value of bulk modulus K (FS_K, VK_K), or Poisson's ration υ (FS_υ, VK_υ) provide the same results for all examples. The errors in deflections and stresses are much smaller than 0.1%. The geometrically nonlinear approaches based on the assumptions of large deflections (VK_K, VK_υ) or finite strains (FS_K, FS_υ) give comparable results for tested examples. The errors are about 0.1%. For statically determinate example, the geometrically linear (LIN_K , LIN_υ) and nonlinear solvers provide the same results, whereas for statically indeterminate examples, the error of linear approach can be about 100% or up to 300%.



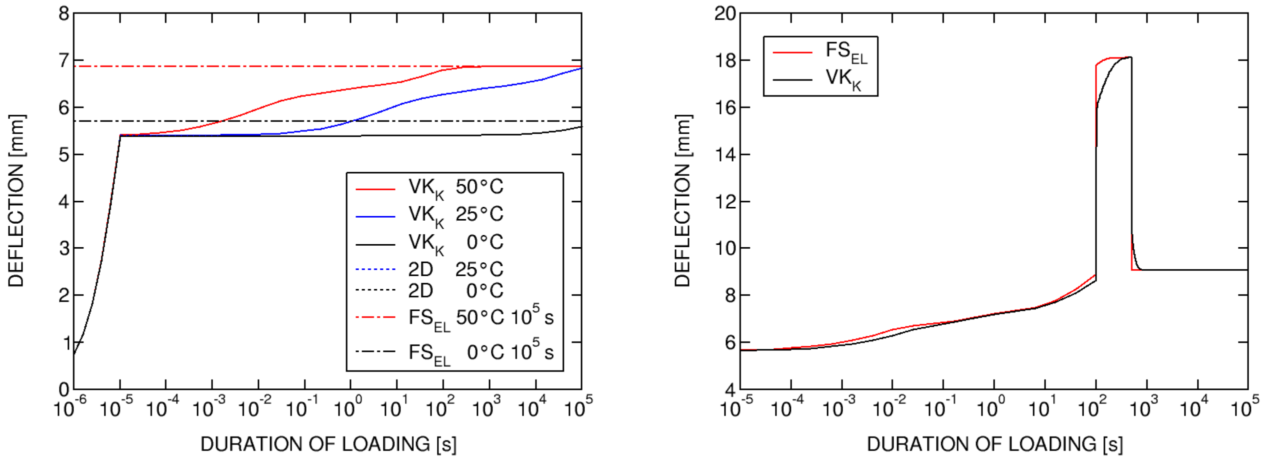
(a) *(b)*

Fig. 1: Comparison of deflections at the mid-point of (a) a fixed-end beam and (b) a simply-supported beam for two-load history at the temperature 25°C: response of proposed finite element model under finite strains assumptions ($FS_K \sim FS_\upsilon$), large deflections assumptions ($VK_K \sim VK_\upsilon$), and geometrically linear case ($LIN_K \sim LIN_\upsilon$).

The response of proposed multi-layered model for both, deflection (Fig. 2 (a)) and stresses, is in a full agreement with the results of 2D analysis in ADINA. The error in values is under 0.5%. The results were compared for temperatures 0°C and 25°C; ADINA solver had problems with convergence for 50°C due to the low values of shear modulus of PVB.

The results obtained with and without accounting for the viscoelastic behavior of the interlayer are compared for a simply-supported beam in Fig. 2 (b). The elastic model gives good prediction for behavior of laminated glass beams, especially for small temperatures (error around 2% for maximum deflections).

For higher temperatures, the error can be significant (up to 27%) after the change of loading level. There are cases and zones where the elastic solution is not on the side of safety. The viscoelastic effects could be important for dynamic, impulsive, and reverse loading.



(a) *(b)*

Fig. 2: Deflections at the mid-point (a) of a fixed-end laminated glass beam under distributed loading at the temperature 0°C, 25°C, and 50°C, (b) of a simply-supported laminated glass beam with two-load history at the temperature 50° C. Response of geometrically nonlinear proposed model (VK_K) is compared with model in ADINA (2D), model for the elastic behavior of interlayer (FS_{EL}) with material parameters (a) for 0°C or 50°C and duration of loading 10^5 s or (b) for given loading duration.

5. Conclusions

(1) The viscoelastic approaches based on the assumption of constant value of bulk modulus or Poisson's ration provide the same results for all examples. (2) The geometrically nonlinear approaches based on the assumptions of large deflections or finite strains give comparable results for tested examples. (3) Effects of geometric nonlinearity can be significant for behavior of statically indeterminate laminated glass beams and plates; however the geometric non-linearity can be neglected for simply-supported beams. (4) Temperature affects the behavior of laminated glass significantly. (5) The simplified model assuming elastic behavior of polymer foil gives good prediction for behavior of laminated glass beams under static loading, especially for low temperatures. The viscoelastic effects could be important for dynamic, impulsive, and reverse loading under room or higher temperatures.

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