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FRICTION-VIBRATION INTERACTIONS OF THE NUCLEAR FUEL RODS

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Abstract: *The paper deals with mathematical modelling of friction-vibration interactions of nuclear fuel rods in the hexagonal type nuclear fuel assembly. The fuel rods are transversally linked by three spacer grid cells to each other inside the fuel assembly skeleton in several vertical levels. The fuel assembly vibrations, caused by the fuel assembly support plates motion in the reactor core, generate variable contact forces between fuel rods cladding and spacer grid cells. Friction effects on stick-slip motion in contact surfaces are important to calculation of expected lifetime period of nuclear fuel assembly in terms of fuel rod cladding fretting wear.*

Keywords: Friction-vibration interactions, Nuclear fuel rod, Fretting wear

1. Introduction

Vibration of nuclear fuel assemblies (FA) was investigated in previous paper (Zeman & Hlaváč, 2012) and the monograph (Hlaváč & Zeman, 2013) as linear systems with proportional damping. Application of these mathematical models enables approximate calculation of the normal contact forces during slip motion without consideration possible stick phases and elastic deformations of spacer grid cells in tangential and vertical directions. Nevertheless, more exact computational friction-vibration analysis of fuel rod (FR) should have been based on more sophisticated computational stick-slip frictional model with force-slip velocity-displacement characteristic (Blau, 2014). The aim of this paper is presentation of the original approach to mathematical modelling of FR nonlinear vibration and fretting wear respecting the friction-vibration interactions in all contact surfaces between FR cladding and spacer grid cells.

2. Dynamic model of fuel rod

Fuel rods (Fig.1) in FA skeleton are fixed by means of the lower piece into mounting plate in reactor core barrel bottom (Hlaváč & Zeman, 2013). Each FR is surrounded by three spacer grid cells $j = 1,2,3$ (Fig.2) at several vertical levels $g = 1,\dots,8$ of the FA skeleton. FR mathematical model of beam type is derived by FEM for Euler-Bernoulli continua including mass forces on flexure (Raileigh theory) in space

$$\mathbf{q} = \left[\cdots, u_i, v_i, w_i, \varphi_i, \vartheta_i, \psi_i, \dots\right]^T, \; i = L_u^{(s)}, 1, 2, \dots, 16 \qquad (1)$$

where u_i, v_i, w_i are axial and two lateral displacements of the central nodal point i. Angular displacements $\varphi_i, \vartheta_i, \psi_i$ represent torsional and two bending angles of FR cross-sections. Even nods $i = 2g$, $g = 1,..,8$ are at the level of spacer grid g and odd nodes are located in the middles. The mathematical model of spatially vibrating FR kinematically excited in the node $L_u^{(s)}$ is derived in the decomposed block form

$$\begin{bmatrix} \mathbf{M}_L & \mathbf{M}_{L,F} \\ \mathbf{M}_{F,L} & \mathbf{M}_F \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_{L_u}^{(s)} \\ \ddot{\mathbf{q}}_F \end{bmatrix} + \begin{bmatrix} \mathbf{B}_L & \mathbf{B}_{L,F} \\ \mathbf{B}_{F,L} & \mathbf{B}_F \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_{L_u}^{(s)} \\ \dot{\mathbf{q}}_F \end{bmatrix} + \begin{bmatrix} \mathbf{K}_L & \mathbf{K}_{L,F} \\ \mathbf{K}_{F,L} & \mathbf{K}_F \end{bmatrix} \begin{bmatrix} \mathbf{q}_{L_u}^{(s)} \\ \mathbf{q}_F \end{bmatrix} = \begin{bmatrix} \mathbf{f}_L \\ \mathbf{f}_{C,FR} \end{bmatrix} \qquad (2)$$

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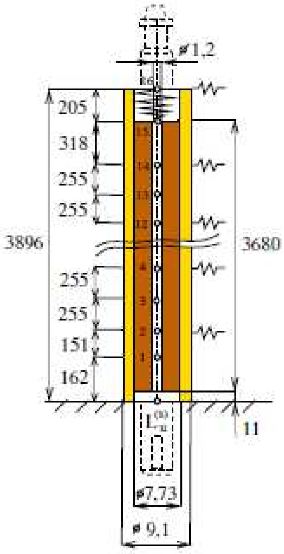

Fig. 1: Fuel rod model

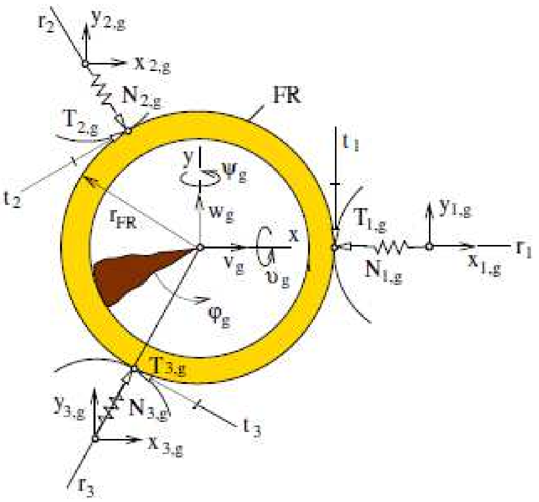

Fig. 2: Contact forces between FR and spacer grid cells

where displacements of the node $L_u^{(s)}$ (coupled with the mounting plate) are integrated in subvector $\mathbf{q}_{L_u}^{(s)}$ and displacements of the nodes $i = 1,\dots 16$ are integrated in subvector $\mathbf{q}_F \in R^{96}$. The force subvector $\mathbf{f}_L$ expresses forces acting in the kinematically excited node $L_u^{(s)}$. The subvector $\mathbf{f}_{C,FR} \in R^{96}$ expresses the coupling forces between surrounding spacer grid cells and FR at the level of all spacer grids $g = 1,\dots,8$ (level of the even nodes). The second set of equations extracted from FR model in decomposed form (2) can be written as

$$\mathbf{M}_F\,\ddot{\mathbf{q}}_F+\mathbf{B}_F\,\dot{\mathbf{q}}_F+\mathbf{K}_F\,\mathbf{q}_F=-\mathbf{M}_{F,L}\,\ddot{\mathbf{q}}_{L_u}^{(s)}(t)-\mathbf{B}_{F,L}\,\dot{\mathbf{q}}_{L_u}^{(s)}(t)-\mathbf{K}_{F,L}\,\mathbf{q}_{L_u}^{(s)}(t)+\mathbf{f}_{C,FR}\ . \tag{3}$$

The first three members on the right side express the kinematic excitation by the lower FA mouting plate motion calculated from the reactor global model (Zeman & Hlaváč, 2008).

3. Contact forces and prediction of the FR fretting wear

Contact forces between FR and spacer grid cells (see Fig. 2) can be expressed by normal $N_{j,g}$, tangential $T_{j,g}$ and axial $A_{j,g}$ components in all contact points $j = 1,2,3$ at the level of the spacer grids $g = 1,\dots,8$. Friction-vibration interactions generally respect three states – ***stick, slip*** and ***separation*** – depending on the slip velocity components $c_{j,t,g}$ and $c_{j,z,g}$ and relative FR shifts $d_{j,r,g}$, $d_{j,t,g}$, $d_{j,z,g}$ in radial (r), tangential (t) and axial (z) directions compared with cell centres. Contact forces components in each contact points including all three states can be approximated by functions

$$N_{j,g}=\left(N_O+k_r d_{j,r,g}\right)H\left(N_O+k_r\,d_{j,r,g}\right), \tag{4}$$

$$T_{j,g}=f\left(c_{j,g}\right)N_{j,g}\,\frac{c_{j,t,g}}{c_{j,g}}+k_t\left(c_{j,g}\right)d_{j,t,g},\quad A_{j,g}=f\left(c_{j,g}\right)N_{j,g}\,\frac{c_{j,z,g}}{c_{j,g}}+k_z\left(c_{j,g}\right)d_{j,z,g} \tag{5}$$

where $c_{j,g}$ are resulting slip velocities. The Heaviside function H in (4) is zero for negative normal contact force $N_0+k_r d_{j,r,g}<0$ (separation state), where N_0 is static preloading of spacer grid cells about radial stiffness k_r. The approximative function for friction coefficient have to satisfy conditions $f(c_{j,g})<f_{st}$ for $0<|c_{j,g}|<c_{krit}$ and $f_d<f(c_{j,g})<f_{st}$ for $|c_{j,g}|>c_{krit}$, where c_{krit} is critical sliding velocity separating the stick-state (phase of microslip) from slip-state (phase of macroslip) and f_{st} is static and f_d dynamic friction coefficient. The approximative functions for cell stiffnesses in tangential and axial directions satisfy conditions $k_t(c_{j,g})\approx k_t$, $k_z(c_{j,g})\approx k_z$ for $0<|c_{j,g}|<c_{krit}$ and $k_t(c_{j,g})<<k_t$, $k_z(c_{j,g})<<k_z$ for $|c_{j,g}|>c_{krit}$. The smooth functions

$$f\left(c_{j,g}\right)=\frac{2}{\pi}arctg\left(\varepsilon_f\,c_{j,g}\right)\left[f_d+\left(f_{st}-f_d\right)e^{-d\,c_{j,g}}\right] \tag{6}$$

$$k_t\left(c_{j,g}\right)=k_t\,e^{-\left(\varepsilon_k c_{j,g}\right)^2}\;,k_z\left(c_{j,g}\right)=k_z\,e^{-\left(\varepsilon_k c_{j,g}\right)^2} \tag{7}$$

satisfy approximately the required conditions for appropriately selected parameters f_{st}, f_d, ε_f, ε_k, d. The components of the contact forces are transformed into FR even nodes and produce the nonlinear vector $\mathbf{f}_{C,FR}\left(\mathbf{q},\dot{\mathbf{q}},t\right)$ in (3). The dynamic response is investigated by integration of motion equations (3) in time domain using standard Runge-Kutta integration scheme in Matlab. Because of a large number of nonlinearities (fully 3x8=24), the calculation is very time-consuming.

The fretting wear of the FR cladding is a particular type of FR wear that is expected in nuclear FA (Pečínka et al., 2014). The criterion of the fretting wear can be expressed using the work of friction forces during the representative time interval $< t_1 , t_2 >$ calculated for normal component contact force $N_{j,g}(d_{j,r,g})$ and friction coefficient $f(c_{j,g})$.

The ***friction work*** in contact points can be written as

$$W_{j,g}=\int_{t_1}^{t_2} f\left(c_{j,g}\right)\left|N_{j,g}\left(d_{j,r,g}\right)\cdot c_{j,g}\right| dt,\; j=1,2,3,\; g=1,2,\ldots,8. \tag{8}$$

The ***fretting wear*** in grams of the FR cladding in particular contact points during the interval $< t_1 ,t_2 >$ can be expressed as

$$m_{j,g}=\mu W_{j,g}\;,\; j=1,2,3;\; g=1,2,\ldots,8, \tag{9}$$

where μ [g J^{-1}] is experimentally obtained fretting wear in grams (i.e. loss of FR cladding mass generated by the friction work W=1[J]) at the middle excitation frequency (Pečínka et al., 2014).

4. Application

The presented method has been applied to vibration analysis and fretting wear calculation of the Russian FA. The FA mounting plates motion and displacements of spacer grid cells centres were precalculated using FA global linearized model in the VVER-1000 type reactor for excitation by pressure pulsations generated by main circulation pumps (Zeman & Hlaváč, 2012). The basic mean excitation frequency corresponding to rotational speed of pumps is f=16.6 [Hz] , project values of cell stiffnesses are k_r=0.537 * 10^6 , k_t=k_z=10^6[N/m] and static contact force expressing preloading between FR and particular spacer grid cells is N_0=10 [N]. The friction-vibration characteristics of contact forces between FR and spacer grid cells in the form (5) are approximated by smooth function (6) and (7) for parameters $f_{st}=0.2, f_d=0.065, \varepsilon_f=10^4, \varepsilon_k=5000\,[s/m]$ and $d=100[s/m]$. The orbit of FR centre in nodal point i=10 (approximately in the FR middle) in time interval < 19.9, 20.1> [s] about the moment t_0=20 [s] of the maximal FA beating vibration is presented in the Fig. 3. Three orbit loops correspond to three harmonic components of the coolant pressure pulsations considered in the kinematic excitation in (3).

As an illustration, time behaviour of slip velocities $c_{1,g}$ in contact points of FR with the first cell at the level of spacer grids g = 1,4,8 is presented in the Fig. 4. As it is evident, maximal slip velocities increase with growing distance of spacer grid from lower FA mounting plate. The strong stick states are repeated in contact points with upper-most spacer grid g = 8.

5. Conclusion

The main objective of this contribution is to present the new basic method of friction-vibration interactions of the fuel rods in the nuclear fuel assembly. The method is based on mathematical modelling and computer simulation of nonlinear vibrations of the fuel rod in interactions with spacer grid cells inside the fuel assembly skeleton. The contact forces include three possible states depending on deformations of cells and slip velocities. The developed software in Matlab code is conceived in such a way that it enables to calculate fuel rod deformations, slip velocity and friction work in all fuel rod contact points with spacer grid cells. The fuel rod and spacer grid vibrations are kinematically excited by the fuel assembly support plates motion caused by pressure pulsations generated by main circulation pumps. The friction works can be used for fretting wear prediction of the Zr fuel rod cladding.

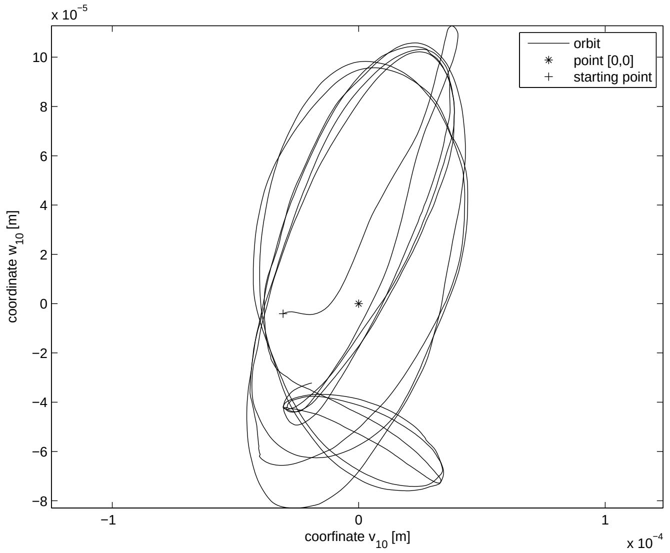

Fig. 3: Orbit of the fuel rod centre in the nodal point i=10 in time interval <19.9,20.1> [s]

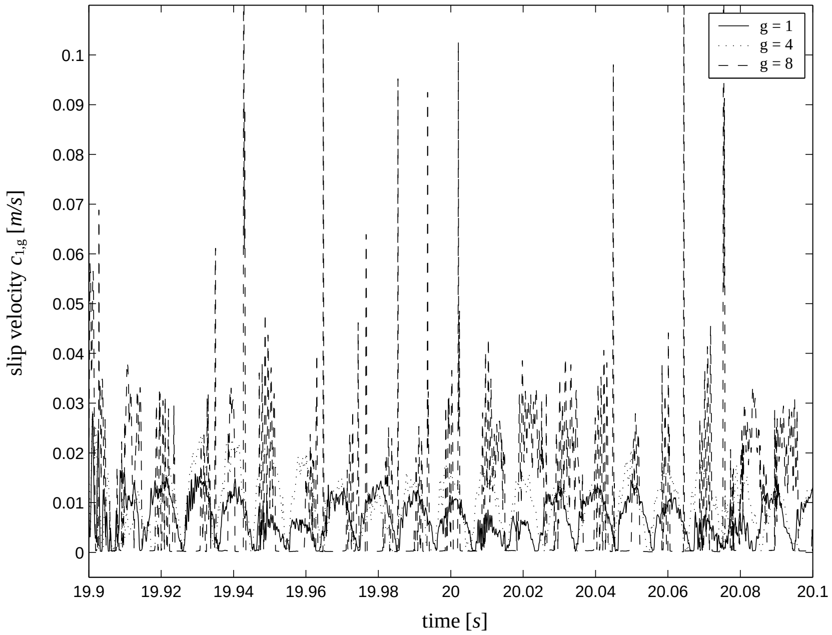

Fig. 4: Fuel rod slip velocities in contact points with the first cell at the level of spacer grid g=1,4,8

The presented method was applied for the fuel rods in Russian TVSA-T fuel assembly in the VVER 1000 type reactor in Czech NPP Temelín.

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