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INVARIANTS OF SIMPLE NOZZLES

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Abstract: *Paper presents a new concept: invariants of second order. The idea bypasses the necessity of geometric similarity (allowing only scaling up or down) in working with dimensionless criteria parameters. Practical aspects of the approach are demonstrated on family of simple nozzles, with different lengths of exit channels. While fluid flow in each nozzle is characterised by its primary invariant, the family as a whole is characterised by additional two secondary invariants.*

Keywords: Invariants, Secondary invariants, Nozzles, Single-parameter nozzle family

1. Introduction

Even though researchers themselves may be not aware of the fact, the ultimate goal of all research activities in physical sciences is to identify ***invariants*** of the investigated problem. In fluid mechanics, this usually means identifying the expressions (usually dimensionless combination of parameters) that remain constant (Fig.1) when magnitude of fluid flow rate passing through the investigated domain is varied. Generally believed to be sine-qua-non condition for the application of the similarity approach is *geometric similarity of* individual cases, varying only in their size. Author now extends the concept of invariants to quasi-similar geometries, demonstrated on family of shapes characterised by two secondary invariants, Fig. 2. It is demonstrated on the case of single-parameter family of simply shaped nozzles having different exit channel lengths There is an analogy with the quasi-similarity of turbulent jets discussed in Tesař and Kordík (2009). The two secondary invariants are a starting point: the number of the invariants increases with each added geometric degree of freedom.

2. Methods

Nozzles are not used alone. They are, instead, components of a hydraulic or aerodynamic system. For proper operation in a system, behaviour of any nozzle should be fully characterised by a numerical parameter - together with some size factor, usually exit diameter d. Until now is in this characterisation role used discharge coefficient (or Euler number Eu, with which it is uniquely related). This is not satisfactory: the coefficient varies with varying fluid flow rate. A really universal validity was demonstrated for the newly introduced coefficient c_T, derived in Tesař (2008). The idea of c_T is based on two hypotheses:

(1) Hydraulic loss in a nozzle is uniquely dependent on displacement thickness δ_* of the boundary layer that forms on internal walls exposed the to fluid flow. Euler number is evaluated from δ_* as $Eu = (1 - 2\delta_*/d)^{-4}$.

(2) The displacement thickness δ_* varies with nozzle exit Reynolds number as $\delta_* = c_T\, d/\sqrt{Re}$. This actually assumes laminar character, an acceptable assumption because the favourable pressure gradient in nozzles delays transition into turbulence.

The first hypothesis ceases to be valid at extremely low Reynolds numbers, due to mutual interference of the too thick boundary layer on opposite sides of the nozzle exit. Another limit of validity is at extremely large Reynolds numbers, where the flow in the exit channel ceases to be laminar.

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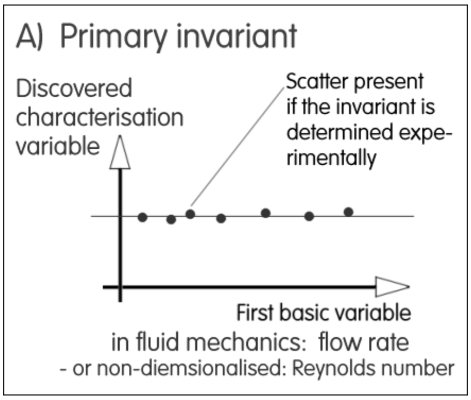

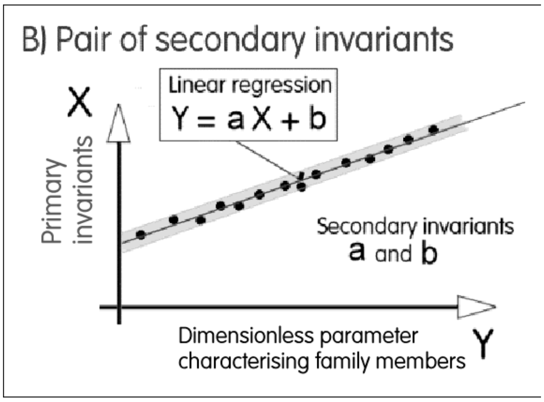

Fig. 1 (left): What all research activities should achieve as their final result is discovering – or deriving, using a mathematical model – the primary invariants of the solved problem.

Fig. 2 (right): Secondary invariants for single-parameter geometries exist in pairs. They are evaluated from dependence between variables which, with some good luck, may be transformed into a linear function with the secondary invariants a *and* b*, evaluated by the east-squares regression.*

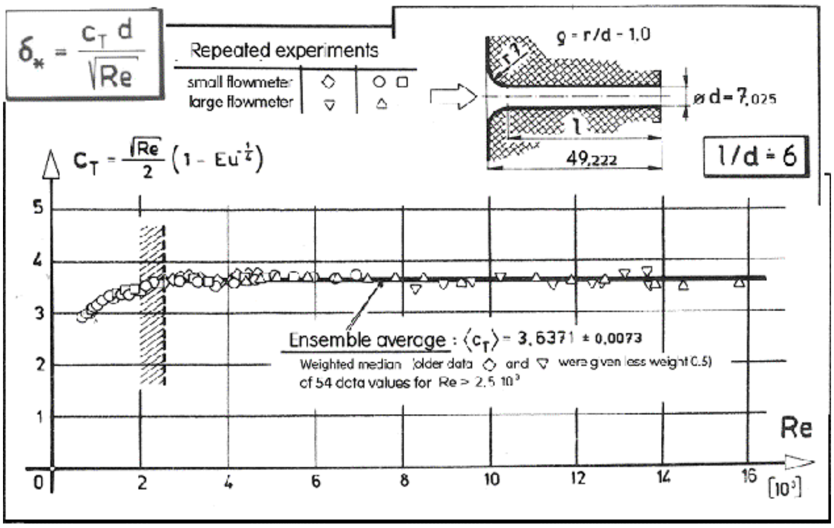

Fig. 3: Demonstrated invariance of the shape coefficient c_T *for a nozzle with quadrant-shaped inlet and constant-diameter exit channel. The deviation at small Reynolds numbers (roughly below* Re ~2500*) is due to mutual interaction of too thick boundary layers on the opposite sides of the nozzle exit, making one of the hypotheses invalid.*

3. The pair of secondary invariants

Derivation of the two hypotheses was based on the idea of nozzle exit blockage by the boundary layer. Validity is fully supported by constant c_T values obtained experimentally for a large number of various nozzles, with examples shown in Figs. 3 and 4. Shapes of the two nozzles there are different and classical hydrodynamics would therefore consider comparing these two cases impossible. Of course, the boundary layer in the shorter nozzle is thinner and with this agrees the lower value c_T in Fig. 4 — but going beyond this fact would similarity approach consider unreasonable**.** Author has, however, performed repeated measurements of pressure losses in altogether 7 quadrant-entrance nozzles (similar to shown in Fig, 3 but of various lengths). The data of primary invariants c_T are then presented in Fig. 5 as a function of the relative exit channel lengths. Transformation of the co-ordinates in the diagram have also made possible placing the data points on a common straight line, the Gauss' linear regression (Seal, 1967) thus enabling an easy evaluation of the two secondary invariants.

This procedure may seem to be simple and perhaps obvious. It should be, however, emphasised that what was achieved is actually breaking an old dogma of impossible comparison of flows in different domain shapes. Within reasonable limits of Reynolds number range, it is possible to predict with reasonable accuracy nozzle behaviour, thus marking the road to possible procedures as, e.g., optimisation of hydraulic circuits.

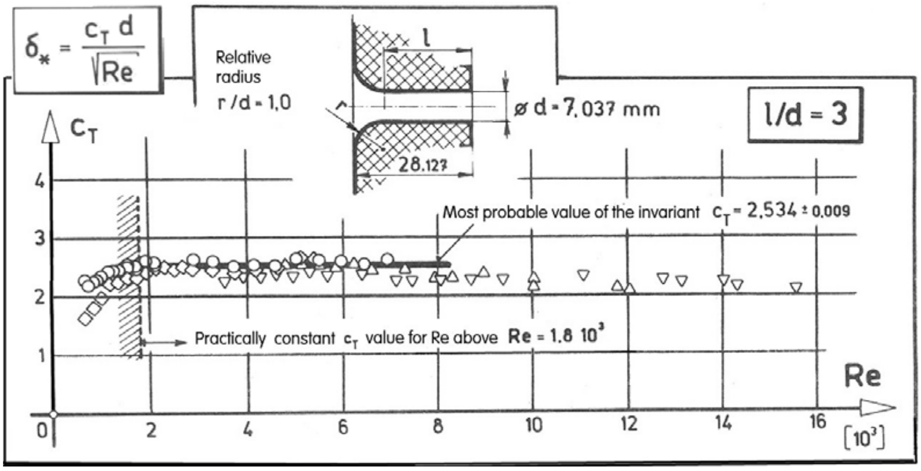

Fig. 4: Invariant c_T *of the short* l/d = 3 *nozzle. The scatter is larger than in Fig. 3 but the behaviour is fully characterised with accuracy needed in engineering calculations. Also here the boundary layer thickness becomes comparable with the nozzle exit diameter* d *at low Reynolds numbers, here below* Re ~ 1.8 10^3.

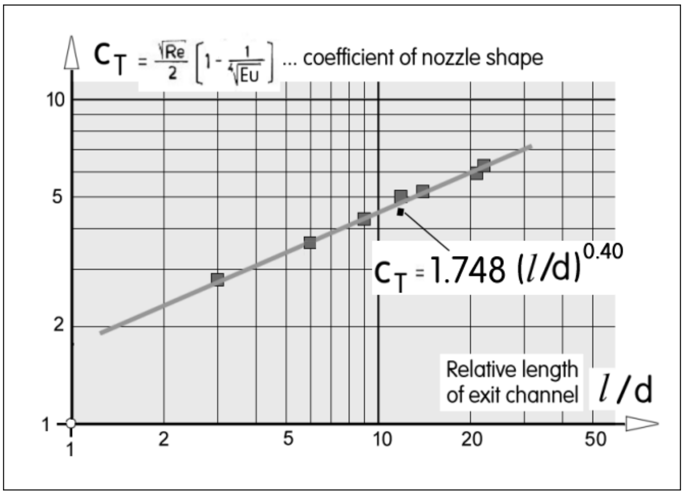

Fig. 5: Two secondary invariants a = 1.748 *and* b = 0.40 *found for the family of nozzles shaped as shown in Figs. 3 and 4, with various exit channel lengths* l/d.

4. Extension to very low Re

Recent development of low Reynolds number microfluidics, as discussed e.g. in Tesař (2007), has shifted interest in fluid mechanics towards the flows that were earlier considered negligible – as is the case of the very small Re values in Figs. 3 and 4, where the above described search for invariants fails because of the violation of the displacement thickness model. At low Reynolds numbers the boundary layer thickness grows extremely rapidly with decreasing Re so that he parts of the layer on the mutually opposed positions inside the exit channel begin to interact. The Reynolds number alone then ceases to be the determining parameter for the character of the flow – it is replaced in this role by Boussinesq number Bo = Re/(l/d). Below the approximate limiting value Bo ~ 50 the whole concept of boundary layer actually loses sense. If the flow rate is decreased further, then in the asymptotic limit the fully developed Hagen-Poiseuille flow takes place for which Eu = 64 / Bo. This is a useful starting point for the alternative approach to the search for invariants of the flow in the nozzle exit channels. The complicating factor

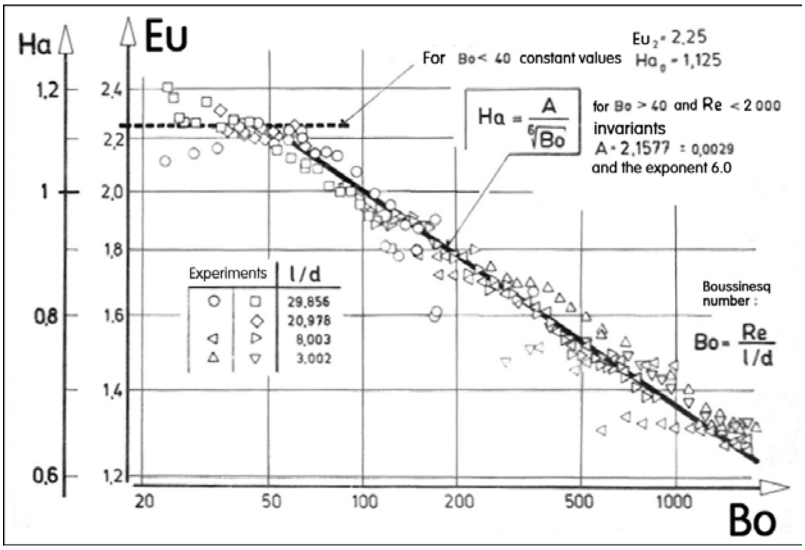

Fig. 5: The law of local quasi-similarity of discussed nozzles at very low Reynolds numbers, where the cT invariance is lost. The two secondary invariants for this regime, the Boussinesq number Bo and Euler number Eu, were derived from a pipe flow model with initial development characterised by Hagenbach number Ha.

in real laminar pipe flows is the behaviour of the flow at the entrance part of the exit channel. Historically, this region of flow development caused trouble in capillary viscometry, where Hagenbach (1860) found good correspondence with experimental data if the classical law is supplemented by an additive correction term, so that: $Eu = 64 / Bo + 2\ Ha$. Inadequacy of this approach was caused the wrong idea of Ha being constant. If it is, instead, taken to be a variable dependent on the Boussinesq number Bo, the Hagenbach's approach becomes very useful. Author's experimental data plotted in Fig. 5 show a very good correlation - with the fitted power law indicating another pair of the secondary invariants.

5. Conclusions

Author's experimental data demonstrate constant value of the characterisation invariant c_T of tested nozzles. Geometric quasi-similarity, with characterisation by secondary invariants, was then demonstrated for family of nozzles having different exit channel lengths and hence not similar. At very low Re, of interest in microfluidics, one of the underlying hypotheses is not valid and this required another approach to secondary invariance.

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