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# A METHOD FOR DETERMINING A COMPLETE S-N CURVE USING MAXIMUM LIKELIHOOD

**P. Strzelecki*, T. Tomaszewski**, J. Sempruch*****

**Abstract:** *The study presents two methods to estimate the S-N curve. The first method is commonly known as the traditional approach and well documented in the normative sources. It consists in determining the curve using the least squares method in a limited fatigue life regime. A staircase method is used to determine the fatigue limit. The second approach assumes normal distribution of fatigue life and fatigue limit. A probability of failure is determined as a product of fatigue life probability distribution function and fatigue limit probability distribution function. The curve parameters are determined using the maximum likelihood estimation method. This is an alternative approach. The advantage of the latter approach is the ability to determine a complete S-N curve using less specimens than required in the traditional approach. The alternative method allows to use non-failed specimens for statistical calculations, which is not possible in the traditional approach. Comparison of those methods was presented using fatigue data for S355J2+C steel. The tests were carried out using the testing machine for rotating bending tests. The study also compares the accuracy of the alternative approach with the traditional approach, which is generally considered accurate.*

**Keywords: Fatigue, Steel, High-cycles, S-N curve, Accelerated methods.**

## 1. Introduction

Fatigue properties of materials within the high-cycle loading range are tested as scheduled based on standards, such as ISO-12107 (2003) or PN-EN-3987 (2010). For instance, offers a comparison between guidelines for such tests and analyses, as proposed by various standards. This scheduled testing based on standards is accurate but time-consuming and costly. Analytical methods, such as those proposed by Sempruch & Strzelecki (2011) or Strzelecki & Sempruch (2012a) can be used for initial computation. Unfortunately, following verification, these methods were proven as generating significant errors. This is why experiments are done where reliable machine components are required. The conventional approach to the determination of full S-N characteristics involves a number of tests (at least 14). Such tests are often done for comparing effects of process factors or geometry on fatigue strength, as described by Tomaszewski et al. (2014), for instance.

This paper proposes an alternative approach to determining full S-N characteristics, described in more detail in the following section. The paper aimed to demonstrate that the number of required tests can be reduced using another approach the processing of test results without compromising the acceptable estimation error.

## 2. Methods of determination S-N curve

ISO-12107 (2003) recommends at least 7 tests for the limited strength range and 15 tests with the up and down method for the unlimited strength range. This makes the total of 22 samples. Also, this document describes a full S-N characteristics determination procedure using 14 samples. Six samples are used for the fatigue limit range.

* Dr. Ing. Przemysław Strzelecki, Institute of Mechanical Engineering, University of Technology and Life Sciences, 85-789 Bydgoszcz, Poland; PL, e-mail:p.strzelecki@utp.edu.pl

** Dr. Ing. Tomasz Tomaszewski, Institute of Mechanical Engineering, University of Technology and Life Sciences, 85-789 Bydgoszcz, Poland; PL, e-mail:tomaszewski@utp.edu.pl

*** Prof. Ing. Janusz Sempruch, Institute of Mechanical Engineering, University of Technology and Life Sciences, 85-789 Bydgoszcz, Poland; PL, e-mail: semjan@utp.edu.pl

$$\hat{Z}_G = \frac{\sum_{i=2}^{n+1} S_i}{n} \tag{1}$$

where:
$n$ – number of specimen used for unlimited fatigue life,
$S_i$ – $i^{th}$ stress level.

Stress for $S_{n+1}$ is not tested; it is determined based on the last $n^{th}$ level. It is the level that would be applied if tests were to be continued. Before this procedure can be followed, characteristics for the limited strength range has to be determined.

The least squares method is used for estimating parameters of the characteristics within the limited strength range. See Lee et al. (2005) for a detailed description of a procedure for this method of testing. Fig. 1 a) shows a graphic representation of this method. This method is hereinafter referred to as method "I" (conventional).

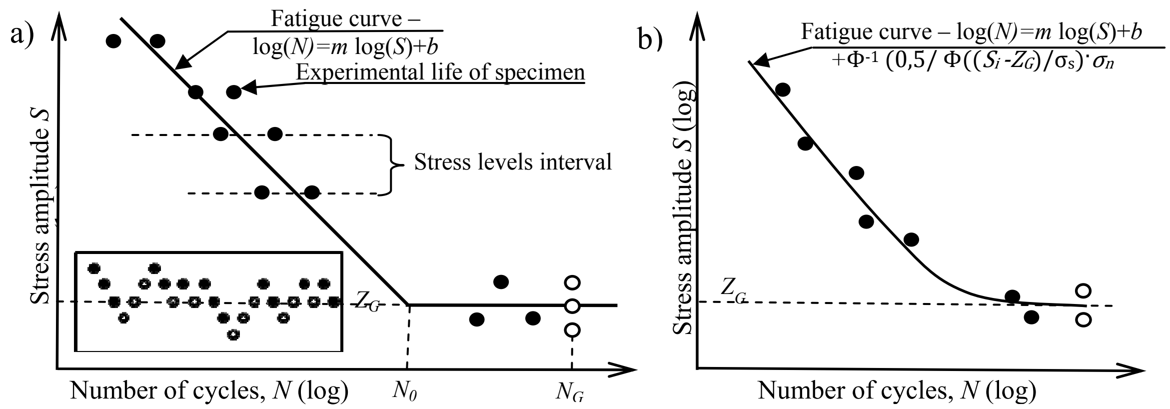


*Fig. 1. Schematic representation determination of the fatigue curve of the using a) model I and b) model II*

The method proposed by Pascual & Meeker (1999), shown schematically in Fig. 1 b), hereinafter referred to as method "II" (alternative), represents another approach. This method assumes that the distribution of strength in the limited strength range is normal. In addition, it assumes that also the distribution of fatigue limit is normal. See the following for equations describing these assumptions.

$$f(N) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(\frac{(\log(N) - (m\log(S) + b))^2}{\sigma_n^2}\right) \tag{2}$$

$$f(S) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(\frac{(S - Z_G)^2}{\sigma_s^2}\right) \tag{3}$$

where:
$\sigma_n$ – standard deviation for fatigue life in limited range,
$N$ – fatigue life,
$S$ – stress amplitude,
$m$ – slope of regression line,
$b$ – intercept of regression line,
$\sigma_s$ – standard deviation for fatigue limited,
$Z_G$ – fatigue limited.

The method of maximum likelihood method is used to estimate parameters of these distributions. The likelihood function used for this statistical method has the following form (Lorén & Lundström (2005)):

$$L(\theta)=\phi\left(\frac{\log N_i-(m\log S_i+b)}{\sigma_n}\right)^{\delta_i}\cdot\Phi\left(\frac{S_i-Z_G}{\sigma_s}\right)^{\delta_i}\cdot\left[1-\left(\Phi\left(\frac{\log N_i-(m\log S_i+b)}{\sigma_n}\right)\cdot\Phi\left(\frac{S_i-Z_G}{\sigma_s}\right)\right)\right]^{1-\delta_i},\quad(4)$$

where:
$\phi$ – density function of the normal distribution,
$\Phi$ – cumulative distribution function of the normal distribution,

$$\delta_i=\begin{cases}1\text{ if specemen failure}\\0\text{ if specimen runout}\end{cases}.\quad(5)$$

## 3. Experimental results

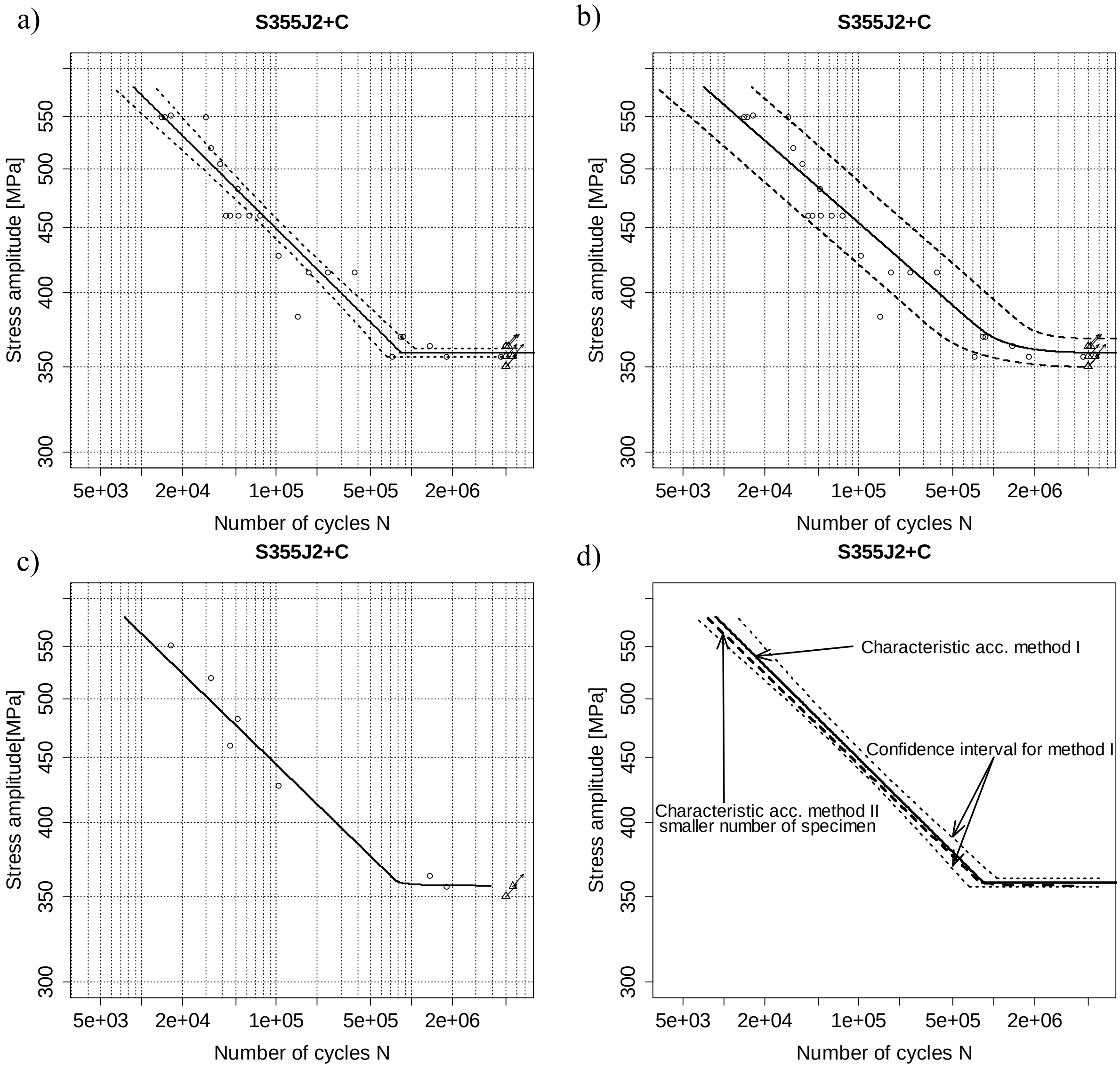


*Fig. 2. Fatigue characteristics for S355J2+C steel acc. a) method I, b) method II with complete data, c) method II with smaller number of specimen, d) comparison of characteristic acc. Method I and II with smaller number of specimen*

Tests were done for steel S355J2+C, using a rotary bending as the fatigue testing machine. See Strzelecki & Sempruch (2012b) for descriptions of the verification, work station and samples. Fig. 2 shows the resulting fatigue diagrams and Tab. 1 presents parameters estimated based on the proposed methods. In addition, the paper presents separately parameters estimated using the alternative method based on the data set contained in the graduation paper of Zawadzki (2015), constituting a part (9 tests) of the whole set (33 tests) used for analysis. Fig. 2 d) shows characteristics based on the conventional method (with the confidence interval) and the alternative method, for a smaller number of tests.

*Tab. 1: Estimated parameters fo method I i II for steel S355J2+C.*

| Model | $m$ | $c$ | $\sigma_v$ | $Z_g$ | $\sigma_z$ |
|---|---|---|---|---|---|
| I | -9.92 | 31.33 | 0.175 | 358.9 | 7.534 |
| II – complete data | -11.14 | 34.62 | 0.244 | 359.0 | 7.245 |
| II – first part data | -10.76 | 33.57 | 0.171 | 356.6 | 1.442 |

## 4. Conclusions

The conclusion from comparing the characteristics obtained by method "I" for 33 tests to the characteristics obtained by method "II" for 9 tests is that the diagram for the alternative method fits within the confidence interval obtained for the conventional method. This means that it is possible to obtain satisfactory fatigue characteristics based on 9 tests. This conclusion requires verification for other construction materials.

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