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STABILITY OF A DAMAGED SLENDER STRUCTURE LOADED BY A FORCE DIRECTED TOWARDS A POSITIVE POLE

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Abstract: *This paper presents the results of numerical simulations and theoretical studies on the natural vibration frequency and instability of a slender system, which is loaded by a force directed towards a positive pole. It is assumed that the column has a defect in the form of crack. The Hamilton's principle is used in the formulation of the boundary problem. The results focus on the shape of characteristic curves in the external load – natural vibration frequency plane as well as on the loading capacity.*

Keywords: column, crack, natural vibration, instability, positive pole.

1. Introduction

The column presented in this manuscript is classified as a slender supporting system due to its geometrical features (relation of total length to cross-sectional area). It is loaded by a force directed towards a positive pole (Tomski 2004). The pole in this case is a point located (below the loaded end) on the undeformed axis of the column. The line of action of the external load is defined by two points: the pole and the loaded end of the column. The implementation of the investigated type of external load has great influence on the natural vibration of the system (especially on the shape of the characteristic curves). By appropriately selecting the distance between the points, which define the line of action of the load, the divergence – pseudo-flutter instability can be obtained. A similar phenomena is present when Tomski's load is introduced (specific load). Both Tomski's load and the one investigated in this paper are fundamentally different from the one proposed by Beck in 1952. Beck's load (a follower load) is a non-conservative load while the ones presented here are conservative. The condition of conservation can be found in the paper (Tomski 2012). The load induced by the force directed towards the positive pole is a real life load which can be realized by means of the rigid rod used to transfer the load to the column from the rigid beam.

The presence of a crack is very undesirable defect in the supporting structure especially when its length is much greater than the cross-sectional area. A reduction in the cross-sectional area leads to a reduction in the loading capacity and a change in the dynamic properties of the structure. The supporting elements must be monitored in order to prevent the destruction of the supported construction. In the literature cracks are simulated by means of FEM packages like Abaqus or by using discrete elements such as rotational springs (Chondros 1998; Arif Gurel 2007; Sokół 2014, 2015). Moreover, the reduced cross-sectional area is also used. The type of simulation is selected according to the categorization of the crack (always open, always close or breathing). Depending on the type of model used, linear or non-linear phenomena can be studied.

In this paper the influence of the parameters of the introduced load, such as rigid rod length as well as crack size and crack location, on the natural vibrations and instability of a column with a defect are discussed.

2. Formulation of the boundary problem

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The column is shown in Figure 1. The slender structure is loaded by an external force P (force directed towards the positive pole) which is located on the upper end of the column. The defect in the form of crack is modelled by means of a rotational spring. The crack is considered to be an open one which divides the column into two elements (the natural boundary conditions at the common point facilitate the continuity of transversal and longitudinal displacements, bending moments and shear forces). A stiff rod of length l_c is installed on the loaded end in order to control the transversal displacements. The additional symbols in Figure 1 are as follows: E_i – Young's modulus, J_i – moment of inertia, A_i – cross-sectional area, ρ – material density, C – stiffness of rotational spring (crack size), P – external load, l_c – rod length, m – mass of loading head.

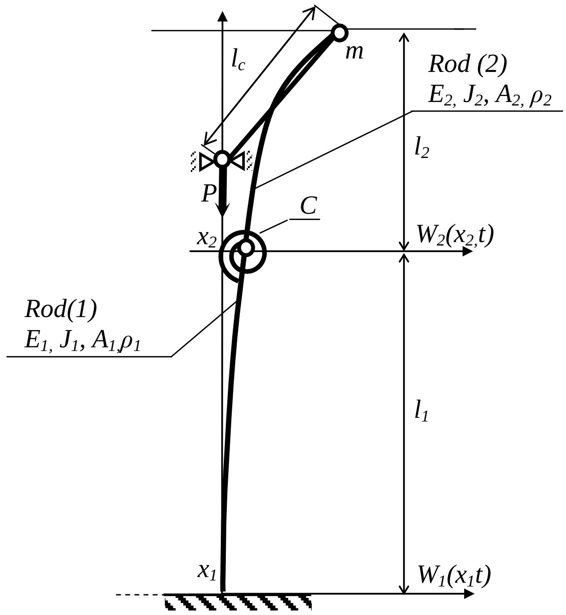

Figure 1. An investigated system

The Hamilton's principle $\delta\int_{t_1}^{t_2}(\mathrm{T}-V)dt=0$ is used in the formulation of the boundary problem, where kinetic T and potential V energies are expressed as follows:

$$T=\frac{1}{2}\sum_{i=1}^{2}\rho A_i\int_0^{l_i}\left(\frac{\partial W_i(x_i,t)}{\partial t}\right)^2 dx+\frac{1}{2}m\left(\frac{\partial W_2(x_2,t)}{\partial t}\bigg|^{x_2=l_2}\right)^2 \tag{1}$$

$$V=\frac{1}{2}\sum_{i=1}^{2}EJ_i\int_0^{l_i}\left(\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right)^2 dx_i+\frac{1}{2}C\left(\frac{\partial W_1(x_1,t)}{\partial x_1}\bigg|^{x_1=l_1}-\frac{\partial W_2(x_2,t)}{\partial x_2}\bigg|_{x_2=0}\right)^2+$$
$$-P\frac{1}{2}\sum_{i=1}^{2}\int_0^{l_i}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2 dx_i+\frac{1}{2}P\frac{1}{l_c}\left(W_2(l_2,t)\right)^2 \tag{2}$$

Integration and variation lead, inter alia, to the differential equations of motion in the transversal direction (3):

$$EJ_iW_i^{''''}(x_i,t)+PW_i^{''}(x_i,t)+\rho A_i\ddot{W}_i(x_i,t)=0 \quad i=1,2 \tag{3}$$

as well as the natural boundary conditions which are obtained using the geometrical one 4(a-h):

$$W_1(0,t)=W_1^{'}(0,t)=0 \qquad W_1(l_1,t)=W_2(0,t) \qquad W_2^{''}(l_2,t)=0$$
$$EJ_1W_1^{'''}(l_1,t)+PW_1^{'}(l_1,t)-EJ_2W_2^{'''}(0,t)+PW_2^{'}(0,t)=0$$
$$-EJ_2W_2^{''}(0,t)+C\left[W_2^{'}(0,t)-W_1^{'}(l_1,t)\right]=0 \qquad EJ_1W_1^{''}(l_1,t)-C\left[W_2^{'}(0,t)-W_1^{'}(l_1,t)\right]=0$$

$$EJ_2 W_2'''(l_2,t) + P\left[W_2'(l_2,t) - \frac{1}{l_c} W_2(l_2,t)\right] - m\ddot{W}_2(l_2,t) = 0 \qquad \text{(4a-h)}$$

The problem is solved numerically by the introduction of (3) into the boundary conditions (4a-h) which leads to a set of homogenous equations, on the basis of which the obtained matrix determinant, equated to zero, creates the transcendental equation used to find, inter alia, the external load – natural vibration frequency relationship.

3. The results of numerical simulations

The results are discussed using the non-dimensional parameters:

$$p = \frac{Pl^2}{EJ_1}, c = \frac{Cl}{EJ_1}, d = \frac{l_1}{l}, \mu = \frac{EJ_2}{EJ_1}, m_b = m\rho A_1 l, l_{CB} = \frac{l_c}{l}, \omega = \sqrt{\Omega^2 \frac{\rho A_1 l^4}{EJ_1}} \qquad \text{6(a-f)}$$

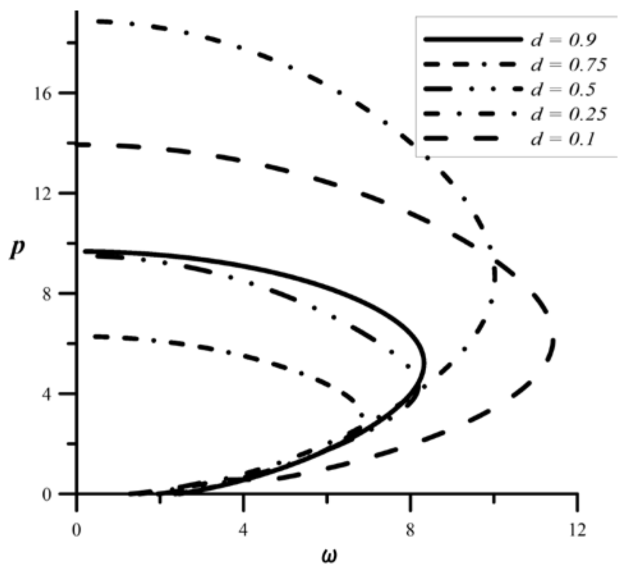

Figure 2. The characteristic curves at c = 1, μ = 1, m_b = 0.15, l_{CB} = 0.1

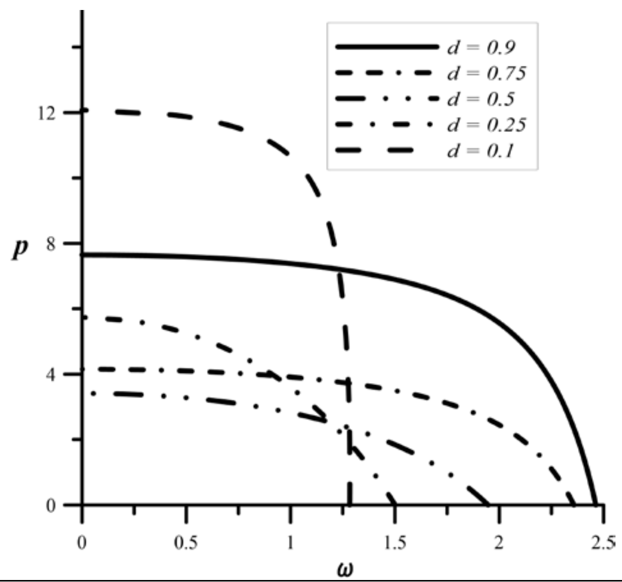

Figure 3. The characteristic curves at c = 1, μ = 1, m_b = 0.15, l_{CB} = 0.9

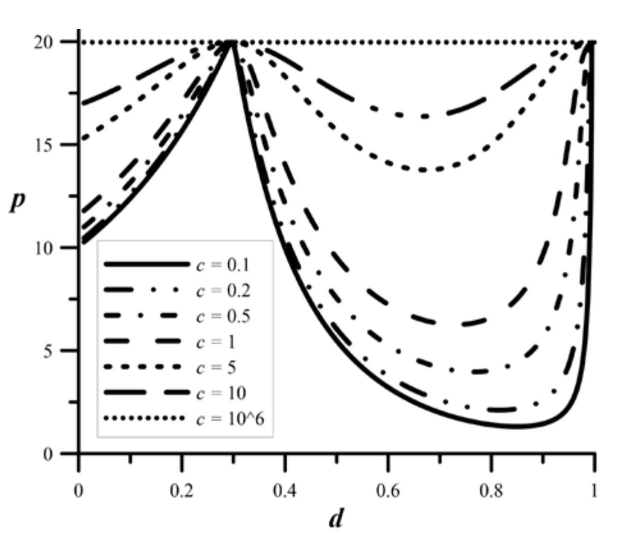

Figure 4. The influence of crack location on loading capacity at μ = 1, m_b = 0.15, l_{CB} = 0.1

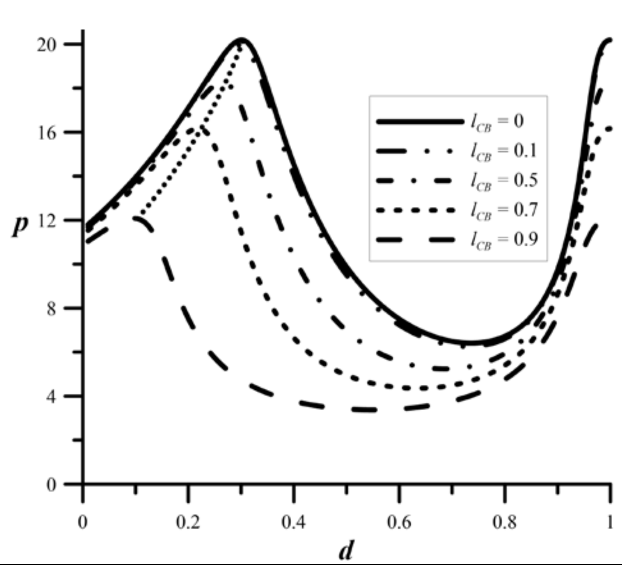

Figure 5. The influence of rod length on loading capacity at μ = 1, m_b = 0.15, c= 1

As space is limited only small samples of the results are presented here. The studies start from calculations on the basis of which the external load – natural vibration frequency relationship was obtained. As shown in Figures 2 and 3, regardless of the rod length, the change in the crack location described by *d* parameter shows that the investigated column is very sensitive to this type of change (the shape modification of the characteristic curves). As shown, the change in the crack location does not affect the instability type but only the shape of the characteristic curves. Furthermore, the divergence – pseudo - flutter system (Figure 2) has low initial sensitivity of vibration frequency to the crack location in relation to the divergence system (Figure 3).

The initial shift of the crack location (Figure 4) from the fixed end in the direction of the loaded end results in an increase in the loading capacity up to a point at which the critical forces of both rods are identical (the location of this point largely depends on the rod length – see Figure 5). A further increase in parameter d causes a reduction in the maximum load and after reaching the lowest level an increase in the capacity can be observed. With a very short rod, the presented distribution of the loading capacity is very similar to the one that can be obtained for the fixed – pinned column. An increase in the rod length regardless of the crack size allows one to find the two points at which the column is insensitive to the crack size. An analysis of the vibration modes (not shown in this paper) leads to easy crack detection, especially in the divergence – pseudo - flutter system in which the change in the vibration modes is present along the characteristic curve.

4. Conclusions

The following conclusions can be drawn on the basis of the results of the numerical simulations:

– the location of the crack greatly affects the shape of the characteristic curves and the loading capacity,

– the character of those changes also depends on the rod length,

– two specific points can be found where the loading capacity does not depend on the crack size,

– the specific points are located as follows: the first on the loaded end, the second in relation to rod length shifts from $d = 0.3$ towards the fixed end,

– the crack does not change the type of instability (divergence or divergence – pseudo - flutter).

Future studies should be carried out on the higher components of natural vibration frequency in relation to parameters such as: rod length, crack size/location, bending rigidity factor. In addition different methods of crack simulation should be compared to the experimental studies in the one paper.

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