22nd International Conference
ENGINEERING MECHANICS 2016
Svratka, Czech Republic, 9 – 12 May 2016

FIVE-BLADE BUNCH WITH DRY FRICTION INTER-CONNECTION FORCED BY RUNNING EXCITATION

L. Půst*, L. Pešek*, A. Radolfová*

Abstract: *The vibration of five-blade-bunch linked in the shroud by means dry-friction-elements is investigated. Quasi-stationary response curves are used for analysis of dynamic properties of isolated blade-bunches excited by running harmonic forces. This running wave excitation models the real excitation in the steam or gas turbine with different numbers of rotor and stator blades. Relation between the ratio of these blade-numbers and phase delay between neighboring blades is ascertained and the dynamic responses of isolated five-blade bunch are shown and analyzed in this paper. Gained theoretical results can be used for evaluation of data obtained from dynamic measurements on bladed disk.*

Keywords: Five-blade bunch, Coulomb dry friction damping, Delayed harmonic excitation, Response curves, Running wave.

1. Introduction

Reduction of resonance vibrations realized by dry friction between the blade-heads in the shroud is very often applied in technical practice [Sextro, 2007, Byrtus et al., 2013, Bruha & Zeman, 2014, Ding & Chen, 2008]. The dry friction contacts are strongly nonlinear and therefore the responses of such system are more complicated then responses of five-blade-bunch containing only linear viscous-elastic connections solved e.g. in [Pust et al., 2016].

Results of experimental or analytical investigation of vibration of blade-models with friction contact [Pesek & Pust., 2011] prove the efficiency of such devices for turbine blades vibrations improvement. In the Institute of Thermomechanics ASCR the detailed analysis and measurements of blade couple have been realized. Dynamics of the two blades model has been investigated with a lot of dry-friction-force characteristics, including also slip-stick models and possibility of instability and existence of dangerous self-excited vibrations. Because some laboratory experiments were carried out on the five-blade bunch [Pust & Pesek, 2013, Pust et al., 2013], the extension of two-blades-model investigation to the study of five-blades bunch is necessary.

According to real machines, the computational model of the turbine five-blades bunch is here excited by five delayed harmonic forces, with the same frequency and amplitude but with stepwise-enlarged delays Δt in time ($\Delta\varphi$ in phase). As opposed to [Pust et al., 2016], the presented contribution is focused only on analysis of response curves of separate five-blades bunch (first and fifth blade unconnected).

In this contribution only the simplest model of many dry friction characteristics - Coulomb model – is applied. The properties of systems with the more sophisticated models will be shown at presentation.

* Ing. Ladislav Půst, DrSc., Ing. Ludek Pešek, CSc, Ing. Mgr. Alena Radolfová, Institute of Thermomechanics AS CR, v.v.i., Dolejškova 5, 18200 Praha 8, e-mail: pust@it.cas.cz, pesek@it.cas.cz, radolfov@it.cas.cz

2. Dry friction connection and elastic micro-deformations

Dry friction is very complicated process, which has to be mathematically described by a strongly nonlinear characteristic. The generally used Coulomb model is only the first approximation of description of real properties. There are two main sets of dry friction characteristics:

2D — “force-velocity”

and 3D — “force-velocity-displacement” expressions.

The first 2D set of dry friction characteristics contains besides Coulomb model applicable in the ranges of great relative velocities also e.g. modified Coulomb model or arc-tangent model useful especially at vibrations with small amplitudes. However, including general dependence on friction velocity can further modify all these models (Půst at all, 2011). This 2D model enables an easy calculation for the majority of engineering problems, where the vibrating bodies are sufficiently stiff and only slip in contact surfaces exists.

In some friction couples especially at vibrations with small amplitudes and if friction surfaces are placed on some compliant parts of moving bodies, it is necessary to use more sophisticated 3D “stick-slip” computational model. There are also many kinds of this type; the simplest one consisting of an elastic linear spring connected with a dry friction element can be also applied.

3. Delayed harmonic excitation

The main part of excitation forces acting on blades in steam or gas turbines are aerodynamic forces generating by revolution of rotor bladed disk in the non-homogenous stationery gas flow through the stator blades cascade. The phase delay $\Delta\varphi$ of harmonic excitation depends on number l_r of blades of rotating disk and on number ls of stator blades according to the following relations:

$$\Delta\varphi = 2\pi * (1 - \frac{l_s}{l_r}) \ . \tag{1}$$

In technical practice there are the numbers l_s, l_r in the ratio l_s / l_r usually selected in such a way, that their least common multiple is very high. In order such a system to be sufficiently resistant against resonance excited by the stator flow irregularity. However, in the case of isolated five-blade bunch, any arbitrary phase delay $\Delta\varphi$ (and corresponding arbitrary blades ratio l_s/l_r) can be selected.

4. Response curves of five-blade bunch with Coulomb 2D friction connection

Dry friction connections of blades are in the practice very often applied for introducing additional damping into mechanical systems.

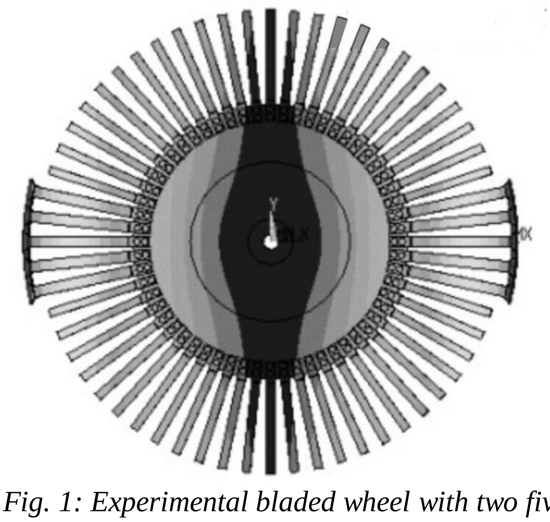

Fig. 1: Experimental bladed wheel with two five-blade bunches

An experimental bladed wheel with 60 models of blades has been investigated in the dynamic laboratory of the Institute of Thermomechanics ASCR.It contains two five-blade bunches situated on opposite sides of the wheel – see Fig. 1.

The boundary blades of these bunches are free on their external sides and each of the measured five-blade bunches is isolated from the other blades. The five-blade bunch can be modeled by a five masses system with blades replaced by 1 DOF systems, the eigenfrequencies of which correspond to the first bending eigenfrequency of real blade (mass *m*, stiffness *k*, damping coefficient *b*).

Motion of the isolated blade bunch shown in Fig. 2 and excited by delayed harmonic forces $F_i(t) = F_{0i} \cos(\omega t - (i-1)\Delta\varphi)$, is described by a set of equations

$$
\begin{aligned}
m\ddot{y}_1 + b\dot{y}_1 + ky_1 + g_1 &= F_0 \cos(\omega t), \\
m\ddot{y}_2 + b\dot{y}_2 + ky_2 + g_2 - g_1 &= F_0 \cos(\omega t - \Delta\varphi), \\
m\ddot{y}_3 + b\dot{y}_3 + ky_3 + g_3 - g_2 &= F_0 \cos(\omega t - 2\Delta\varphi), \\
m\ddot{y}_4 + b\dot{y}_4 + ky_4 + g_4 - g_3 &= F_0 \cos(\omega t - 3\Delta\varphi), \\
m\ddot{y}_5 + b\dot{y}_5 + ky_5 - g_4 &= F_0 \cos(\omega t - 4\Delta\varphi),
\end{aligned}
\tag{2}
$$

where functions g_i are given for Coulomb dry friction connections by

$$
g_i = F_t sign(\dot{y}_i - \dot{y}_{i+1}), \qquad i = 1,...,4. \tag{3}
$$

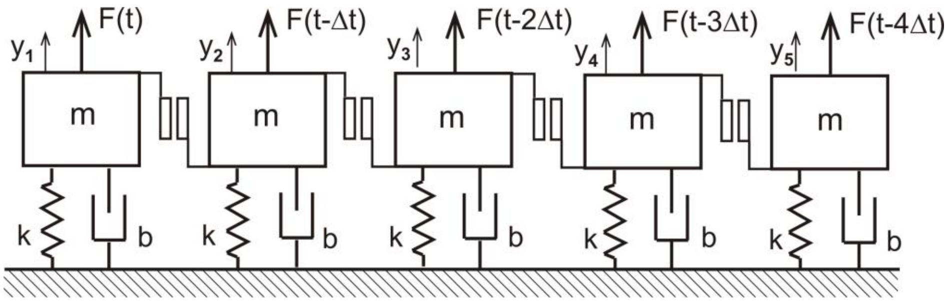

Fig. 2: Computational model of isolated five-blade bunch with dry friction connections

If $\Delta\varphi = 0$, all blades vibrate in the same phase and no deformations in dry friction connections occur. If the blades are excited with non-zero phase delay e.g. $\Delta\varphi = 2\pi/5$, all the joining elements are deformed and response curves of individual blades differ. This property is shown in Fig. 3, computed for parameters m = 0.182 kg, k = 105000 kgs^{-2}, b = 2 kgs^{-1}, F_0 = 1 N and several dry friction force F_f = 0.9 – 1.3 N. The height of resonance peaks decreases with increase of dry friction forces in the inter-connections.

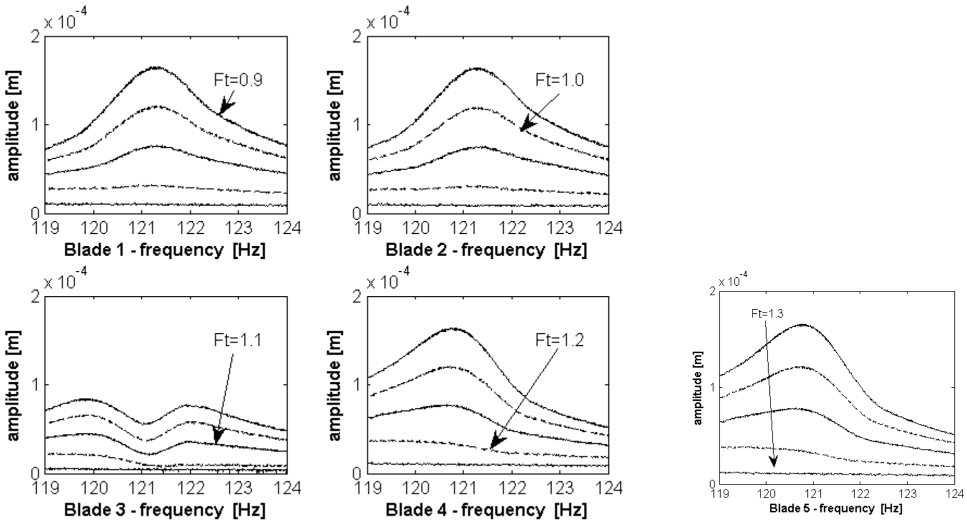

Fig. 3 : Response curves for phase shift Δφ =2π/5 and various Coulomb dry friction joining elements with F_f = 0.9 – 1.3 N.

No motion occurs for higher friction (approx. $F_t > 4/\pi * F_0$). Remarkable is the property of the central blade 3, which vibrates with minimum amplitudes and response curves have two maxims.

5. Influence of phase shift of excitation forces

The height of response curves of five-blade bunch is influenced not only by the intensity of damping forces but also by the phase shifts of the exciting external forces. The influence of friction forces F_t is analyzed in the previous chapter for phase shift $\Delta\varphi = 2\pi/5$, which corresponds according to the equation (1) the ratio of number l_r of blades of rotating disk and the number l_s of stator blades, equals 5/4. However if the five-blade bunch is included into a bladed disk with many other blades, as shown in Fig. 1, then the ratio of number l_r of blades of rotating disk and on number l_s of stator blades can be very different with the value in interval $\Delta\varphi = (0,\ 2\pi/5)$. In addition, if the number l_s of stator blades is higher than number l_r of rotor blades the phase shift is negative according to the formula (1). According to the sign of phase shift, the force excitation runs on the rotor bladed wheel either forwards or backwards.

The results of computing the set of equations (2), (3) for the same dynamical system as in the previous chapter i.e. for parameters m = 0.182 kg, k = 105000 kgs^{-2}, b = 2 kgs^{-1}, F_0 = 1 N, and for the friction force F_t= 0.9 N are shown in Fig. 4. Response curves are plotted for the different phase shifts from the set of values $\Delta\varphi = (-2\pi/5,\ -\pi/5,\ 0,\ +\pi/5,\ +2\pi/5)$. Response curves for $\Delta\varphi = -2\pi/5,\ 0,\ +2\pi/5$ are drawn in full lines, curves for $\Delta\varphi = -\pi/5,\ +\pi/5$ are in dashed lines. In the case of the same numbers of stator and rotor blades ($l_s = l_r$, $\Delta\varphi = 0.$), all blades are excited by the same forces without any shifts and vibrate also without any shifts. No energy is therefore lost in the dry friction connection between blades, which are damped only by their small material damping (b = 2 kgs^{-1}). Response curves of all five blades are the highest with the equal peek value and are drawn by a full thin lines.

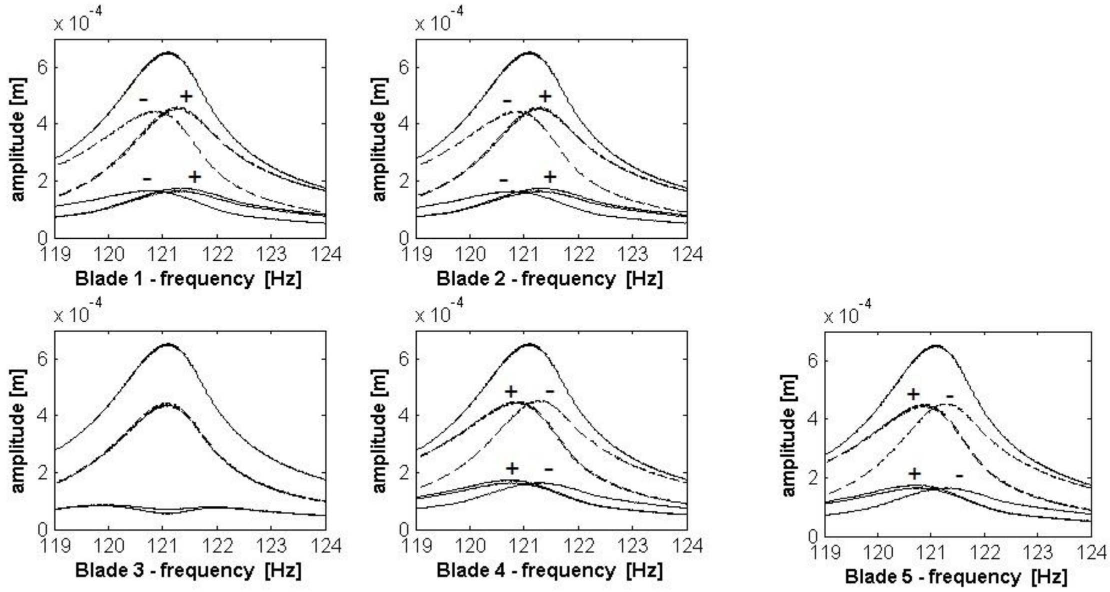

Fig. 4: Response curves for Coulomb dry friction F_f = 0.9 N and various phase shift $\Delta\varphi$ =-2π/5, -π/5, 0, π/5, 2π/5.

Increase of phase shifts on $\Delta\varphi = \pi/5$ causes relative motion between neighboring blades increase of damping level and following degrease of resonance peaks in the response curves subfigures of all blades 1 to 5. Frequency positions of the resonance peek of blade 1 is a little higher than the resonance frequency of the response curve for $\Delta\varphi = 0$, see dashed thick line in the first subfigure for blade 1 marked +. Such a resonance frequency shift occurs also in the second subfigure for blade 2 labeled again +.

Very interesting is behavior of the resonance curve of the middle third blade 3 – see left bottom subfigure – where no frequency shift between resonance peeks of response curves for $\Delta\varphi = 0$ and $\Delta\varphi = \pi/5$ occurs. In additional, the response curves for $\Delta\varphi = \pi/5$ and $\Delta\varphi = -\pi/5$ merge and only one curve of this middle blade exists for positive and negative values of phase shifts.

The response curves for resting two blades 4 and 5 at phase shift $\Delta\varphi = \pi / 5$ have again the same height of resonance peeks, but the frequency shifts compared to the response curve for $\Delta\varphi = 0$ is now negative, as it is seen from the dashed thick curves + in the right bottom subfigures.

The response curves peeks of the five-blade bunch excited by forces with twice greater phase shift $\Delta\varphi = 2\pi / 5$ are again lower due to the greater relative motion between blades. The resonance frequency shifts have approximately the same value as in the case of half-phase-shift $\Delta\varphi = \pi / 5$, but the sense if frequency shifts is similar: Resonance peeks of blades 1 and 2 are a little higher than the resonance frequency of the response curve for $\Delta\varphi = 0$, peeks of blades 4 and 5 are a little lower. Resonance curve of the middle third blade 3 has no frequency shift and is again common for $\Delta\varphi = +2\pi / 5$ and $-2\pi / 5$. It corresponds to the curve in the left bottom subfigure in Fig. 3.

Let us see now on the above mentioned case in which the number l_s of stator blades is higher than number l_r of rotor blades and the phase shift between excitation forces is negative $\Delta\varphi < 0$. Corresponding response curves are in Fig. 4 labeled by sign "-"and are drawn by thin lines. The heights of these response curves are approximately the same as the curves calculated for positive $\Delta\varphi$, but the frequency shifts of the response peeks are in this case opposite. Frequency positions of the resonance peeks of blades 1 and 2 are a little lower than the resonance frequency of the response curve for $\Delta\varphi = 0$. Blades 4 and 5 have peeks that have compared with the position of peek for $\Delta\varphi = 0$ higher resonance frequency.

Resonance curves of the middle third blade 3 are identical both for positive or negative phase shifts $\Delta\varphi$.

6. Conclusions

Dynamic properties of an isolated five-blade-bunch linked in the shroud by means of Coulomb dry friction connections and excited by running harmonic forces were investigated. The developed method of solution has been applied for computing of sets of response curves at different values of fry friction forces at running harmonic excitation with given phase angle shift produced by non-homogenous stationary gas flow through the stator blades cascade. Analysis of this set of curves reveals differences in dynamic responses of individual blades and creates base for evaluation of experimentally gained data from dynamic measurements on bladed disk.

Influence of two parameters dry friction force F_t and phase shift $\Delta\varphi$ of the external forces characterizing the running excitation of five-blade-bunch damped by dry friction are studied in detail. It is shown that the dynamic properties of the middle blade differ from the dynamic properties of other blades in the bunch.

This method of solution can be used also for investigation of dynamic properties of blade bunches with other types of inter-blades connections e.g. modified Coulomb, slip-stick contact, etc.

Acknowledgement

This work was supported by the research project of the Czech Science Foundation No. 16-04546S "Aero-elastic couplings and dynamic behaviour of rotational periodic bodies".

References

Bruha, J. & .Zeman, V. (2014) Vibration of bladed disk with friction elements, In Proc. Dynamics of Machines 2014, (L. Pesek),Prague, IT AS CR, pp.15-22

Byrtus, M., Hajzman, M. & Zeman, V., (2013) Linearization of friction effects in vibration of two rotating blades, *Applied and Computational Mechanics, Vol.*. 7, No. 1, pp. 5-22

Ding Q. & Chen Y., (2008) Analyzing Resonance Response of a System with Dry Friction Damper Using an Analytical Method.. Journal of Vibration and Control, 14(8), pp. 1111-1123

Pešek L. & Půst L. (2011) Mathematical model of a blade couple connected by damping element, in Proc. 8th International Conference on Structural Dynamics, EURODYN2011, Leuven, Katolieke Universiteit Leuven, (Roeck G. De, ed.), pp. 2006-2011.

Pust L., Pesek L. & Radolfova A. (2011) Various Type of Dry Friction Characteristics for Vibration Damping, Engineering Mechanics, 18, , No. ¾, pp. 203-224.

Pust L. & Pesek L.(2013) Model of Turbine Blades Bundles, in. ROM Proceedings Engineering Mechanics, (J. Fuis) VUT Brno, Svratka, CR, pp. 467-477

Pust L., Pesek L., Cibulka J. & Bula V.(2013) Mathematical Model of Blades Bundle with Damping Connections, Eastern-European Journal of Enterprise Technologies, 3/12(63), pp. 43-46

Pust L., Pesek L. & Radolfova A. (2016) Vibration of blade bunch forced by delayed excitation, in Proc. Colloquium Dynamesi 2016 (L.Pesek & I. Zolotarev), IT AVCR, Prague, pp. 12.

Sextro W.(2007) Dynamical Contact Problems with Friction, Springer-Verlag Berlin Heidelberg,