22nd International Conference
ENGINEERING MECHANICS 2016
Svratka, Czech Republic, 9 – 12 May 2016

MATHEMATICAL MODEL OF SELECTED OBJECT THERMAL PROPERTIES

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Abstract: *The paper presents the mathematical model of temperature changes of selected point in the home sauna obtained by the experimental identification. The basic simplifying assumptions, which lead to a linear model with constant coefficients in the form of the transfer function, are presented. The model parameters which are missing and impossible to measure have been determined by simulations conducted in ANSYS-FLUENT software. A qualitative assessment of compliance measurement results of the tested object and its model has been performed after a simulation in the Scilab-Xcos computing environment.*

Keywords: thermal properties, ANSYS-FLUENT, SCILAB-Xcox, on-off control, mathematical model.

1. Introduction

Thermal objects, for example: furnaces, heating chambers, heat exchangers, rooms, greenhouses are characterized by large time constants and significant delays. This is the reason why the on-off control is usually used for their control in case when there is no necessity of very precise control process, e.g. domestic equipment. However, there are processes for which this type of control in not precise enough, e.g. air conditioning of an operating room, technical devices in the pharmaceutical or bioengineering industry. Then, the on-off control with correction or the control by constant or discrete controllers (PLC – Programmable Logic Controller) is applied.

The task is more complicated when it is necessary to apply programme changes of set temperature, e.g. drying rooms. Then, the control process requires PLC. For the proper selection of a controller and its preliminary setting, there is the need for knowledge of the mathematical model of controlled object and the processes it undergoes. The mathematical model in the form of motion equations can be obtained as a result of thermal processes analysis occurring in a given technical object. In practice, however, the preparation of energy balance requires significant simplification, which affects the quality of the final model. Due to the fact that we very often have to deal with modernisation of existing buildings, it is useful to determine the model by using actual course analysis of selected thermal object.

2. Experimental studies

The home sauna presented in Fig. 1 has been investigated. Fig. 1a shows the heater with power of 2 kW. The characteristic dimensions and the measurement point arrangement are presented in Fig. 1b. The left bottom corner of the front wall of the sauna, where the heater is located nearby, was adopted as the beginning of coordinate system.

The measuring system (Szews M., et al., 2014) contains of five temperature sensors, which measure its value in different points of the sauna. Ambient temperature has been measured by a mercury

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*Assist. Prof. Daniel Perczyński, PhD.: UTP University of Science and Technology, Faculty of Mechanical Engineering, Al. prof. S. Kaliskiego 7, 85-796 Bydgoszcz; PL, daniel.perczynski@utp.edu.pl thermometer and it has been taken as a constant, because its changes during the measurements did not exceed the temperature of $0{,}5°\mathrm{C}$. Locations of the measuring points $T_i(x,y,z)$ for $i=1,\ldots,5$ are presented in Fig. 1b with respect to the marked coordinate system.

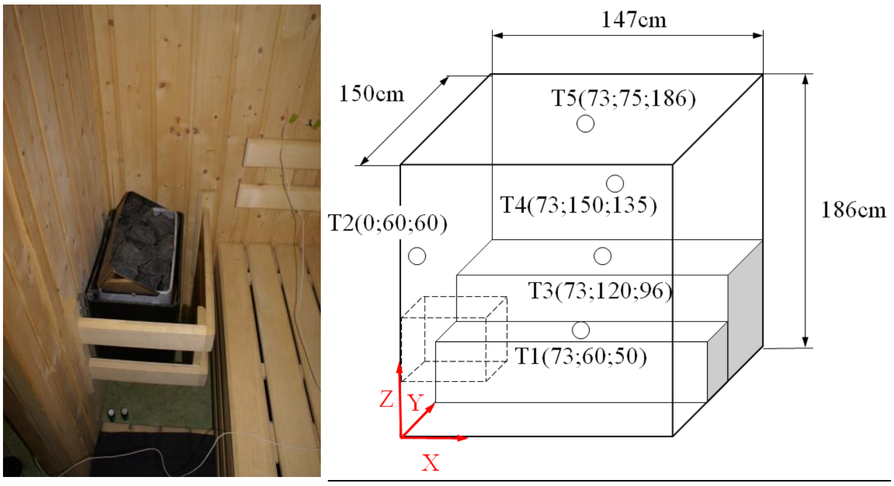

Fig. 1. Object of study: a) part of object with the heater, b) sensors placement

The research consisted of turning on the heating system of the sauna and recording the temperature of five selected measurement points. Time of the automatic sampling was 1 s. The heating system contains the control system, which provides automatic turning on and off of the heating process when the set temperature is reached, as well as its turning off after the set time.

3. Results

The results of experimental studies for selected measuring points are shown in Fig. 2. (Szmyt W. 2016). Each of presented curves consists of three intervals. The first interval (I.) consisting of heating the sauna to the operating temperature lasted 64 min. The second interval (II.), which lasted 196 min., occurred after reaching the operating temperature. Although, the temperature value was assumed to be constant during the control time, it was observed that the temperature was still increasing, however, much slower than during the heating time. The change of average temperature during the interval (II.) contradicts common on-off control system theory (Peszyński K., Siemieniako F. 2002), which does not consider transient states.

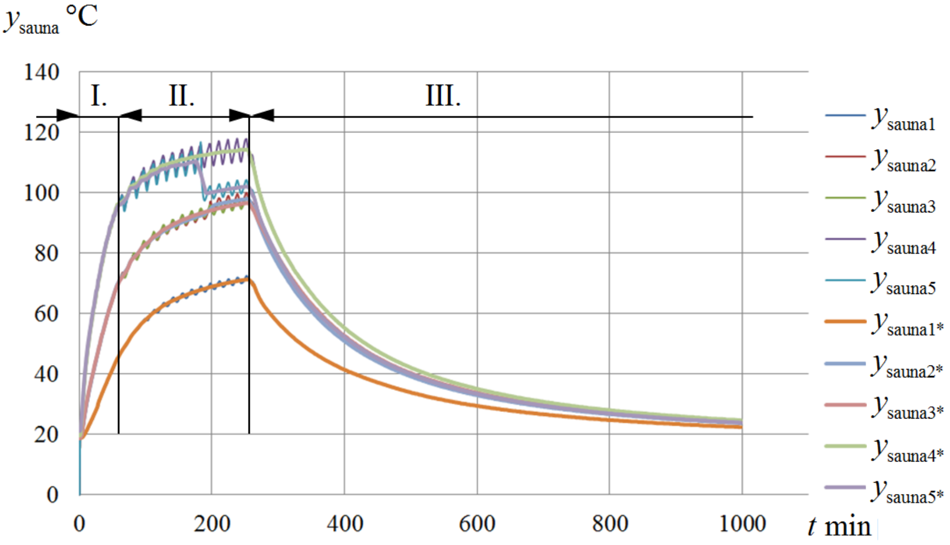

Fig. 2. Measurement results (measurement points as shown in Fig. 1b)

The phenomenon can be explained by the location of the temperature sensor, which controls on-off rely of the heater, near the heating element.

4. Analysis of measurement results

Seeking the mathematical model which describes the thermal properties it is assumed that the sauna is an object with one heat capacity (interior) determined by the transfer function

$$G_{\mathrm{sauna}}(s)=\frac{k}{Ts+1} \tag{1}$$

where k is the gain of the sauna and T is time constant.

Designation the parameters of k and T is difficult because of the measurement characteristic. The curves presented in Fig. 2 only during the initial phase (interval I.) and cooling time (interval III.) qualitatively correspond to the curves calculated from the formula (1). During the interval II., the on-off control is carried out and involves the periodic switching on and off the heating element. This type of control is difficult for mathematical description, especially during the transitional period when the average temperature changes. The simplification consisting of determining the average time for each switching on and off periods enables to determine the course of temperature presented in Fig. 2 through the curves designated as $y_{\mathrm{sauna}i*}$, where $i=1,2,\ldots,5$ is a number of the measuring point. Power of the heating element, as a result of part time heater operation, is assumed to be an input value. The above analysis enables the assumption of the following mathematical model of the sauna

$$\Delta y_{\mathrm{sauna}}=\begin{cases} y=0 & t<0 \\ y_{\mathrm{I}}=\Delta y_{\mathrm{sauna}}^{\infty}\left(1-e^{-\frac{t}{T}}\right) & 0\le t<t_{c\mathrm{P}} \\ y_{\mathrm{II}}=y_{\mathrm{I}}-c\cdot\Delta y_{\mathrm{sauna}}^{\infty}\left(1-e^{-\frac{t-t_{c\mathrm{P}}}{T}}\right) & t_{c\mathrm{P}}\le t<t_{0\mathrm{P}} \\ y_{\mathrm{III}}=y_{\mathrm{II}}-(1-c)\Delta y_{\mathrm{sauna}}^{\infty}\left(1-e^{-\frac{t-t_{0\mathrm{P}}}{T}}\right) & t\ge t_{0\mathrm{P}} \end{cases} \tag{2}$$

The period fulfilment coefficient c, shown in the formula (2), is calculated from the dependence

$$c=\frac{t_{\mathrm{on}}}{t_{\mathrm{cycle}}} \tag{3}$$

where the period of oscillation equals $t_{\mathrm{cycle}}=t_{\mathrm{on}}+t_{\mathrm{off}}$, time of heater operation t_{on}.

Because of enforcing periods and the responses of the controlled object are always the same, the output raising time y_{sauna} is assumed as the time of heater operation t_{on}, while the dropping time of the value (temperature) as the off time t_{off}.

Another problem was to determine the gain of the sauna $\Delta y_{\mathrm{sauna}}^{\infty}$, included in the formula (2), which corresponds to the coefficient k of the model written as the formula (1). $\Delta y_{\mathrm{sauna}}^{\infty}$ cannot be found in the diagrams because the heater was switched off before reaching the set state. The measurement without switching off the heater was impossible, because the sauna would have been destroyed by the temperature above 200°C. A solution to the problem has been found by using ANSYS-FLUENT software and numerical modelling. Only the steady state resulting from switching on the heater to the max has been modelled. The sauna is enclosed by four walls with different thicknesses, which are made of different materials, ceiling and floor, which are also made of different materials, the outside temperatures are also different, therefore it was assumed that heat flows through an averaged wall. The thermal parameters of the wall were chosen iteratively (Szmyt W. 2016), in the way to ensure the outflow of thermal energy complies with the average outflow during interval I. of heating the sauna. The temperature distribution in the sauna in steady state is presented in Fig. 3. It should be noted that the temperature in Fig. 3 is shown on Kelvin scale.

Time constant T of the model, the formulas (1) and (2), has been estimated from the courses of measured values (Fig. 2).

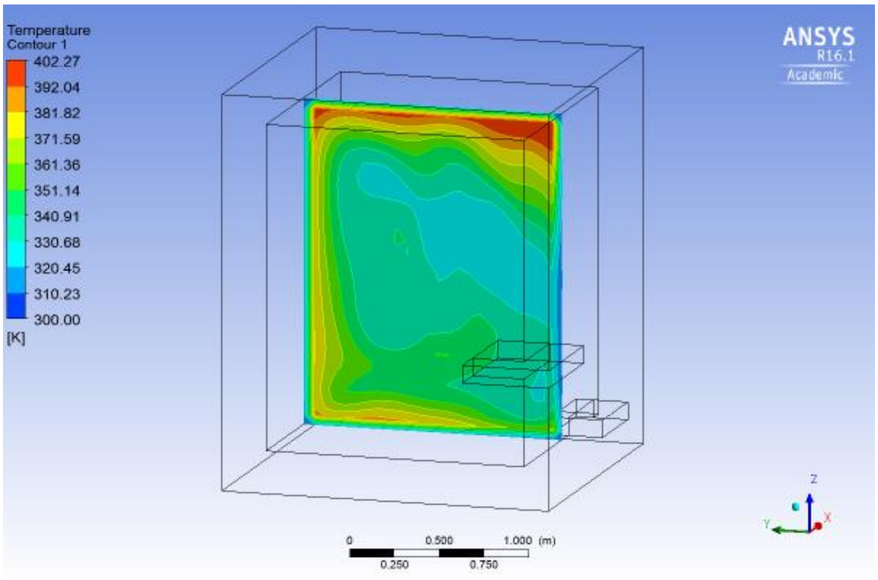

Fig. 3: Results of temperature simulation.

5. Conclusions

The mathematical model determined by the formula (2) enabled finding the courses of temperature changes in the sauna. SCILAB–Xcos computing environment has been used for determining the courses. The following parameters have been adopted the simulation: $\Delta y_{\text{sauna}}^{\infty} = 220°\text{C}$, $t_{\text{cP}} = 64\,\text{min}$, $t_{0\text{P}} = 260\,\text{min}$, $T = 160\,\text{min}$, $c = 0.476$.

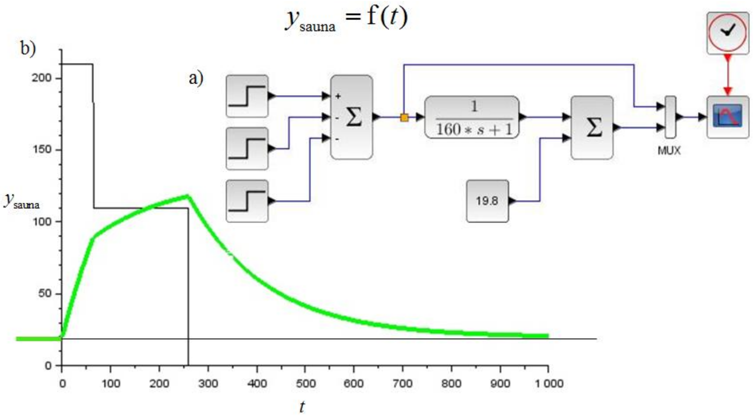

Fig. 4: Sauna temperature simulation: a) block diagram b), course of input power and output temperature

The courses presented in Fig. 2 and Fig. 4 are qualitatively compatible. Precise quantitative matching will be investigated in the next step of modelling. The modelling techniques (measurements, numerical simulations in ANSYS-FLUENT and SCILAB-Xcos computing environment) have been very useful for identification of thermal objects.

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