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INFLUENCE OF WALL FLEXIBILITY OF LIQUID STORAGE TANKS ON HYDRODYNAMIC PRESSURES INDUCED DURING SEISMIC ACTIVITY

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Abstract: *The paper deals with the seismic analysis of the circular vertical ground-supported tanks containing liquid with the aim to determine the distribution of hydrodynamic pressures along the tank height during seismic excitation. Circular tanks are used for storage of liquids in various industrial sectors. Hence, it is a request for satisfactory performance during dynamic loadings. In general, these tanks are usually made of reinforced concrete or steel. For determination of distribution of hydrodynamic pressures there must be distinguished between rigid and flexible tanks due to different responses of tank-liquid systems. In cases of flexible tanks, the wall flexibility is taken into account. The paper is also dedicated to compare results of distribution of hydrodynamic pressure for both cases assuming models with identical geometric proportions. For computation and subsequent comparison Eurocode 8 standard and other procedures are used.*

Keywords: circular liquid storage tank, hydrodynamic pressures, flexibility, seismic excitation, Eurocode 8

1. Introduction

Liquid storage tanks are important components in various industrial sectors and are used as part of lifeline transmission and distribution systems. The main purpose of these systems is to store a variety of liquids (e.g. with toxic explosive nature) before subsequent treatment or utilization. Liquid storage tanks can be subjected to loadings of various nature which may threaten ordinary operation. In addition to static loading, which is usually represented by hydrostatic pressure, tanks may be subjected to the dynamic effects (e.g. seismic excitation) as well. There are many negative consequences which may be caused by dynamically loaded systems and can take one of the following ways: buckling of shells due to excessive axial compression taking the form of a bulge called elephant's foot; roof damage caused by sloshing of upper zone of contained liquid with inadequately designed freeboard between liquid surface and roof etc. There were proposed procedures for seismic resistance of tank-liquid systems. One of the basic seismic characteristics is the distribution of hydrodynamic pressures along the tank height. Time-dependent hydrodynamic pressures induce time-dependent stresses in tanks which can significantly affect their performance. Hence, estimate of distribution and magnitude of hydrodynamic pressures during loading are crucial in tank design in order to preserve their functionality before, during and after seismic event. These pressures and relative stresses depend on type of motion, liquid properties, proportions and material of tank.

2. Basic concepts and theory

Following the past earthquakes like El Centro, San Fernando and others, there was a huge effort to describe tank behavior in a more common way. The most widely used procedure for evaluating seismic effects in cylindrical liquid storage tanks is the one based on a spring-mass equivalent model (Housner, 1954). The proposed method shows that hydrodynamic pressures can be expressed as the sum of two contributions called impulsive and convective, separately. The impulsive pressures are those associated with inertial forces produced by accelerations of the walls of the container and are directly proportional to

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these accelerations. The convective pressure is produced by oscillation of liquid and is a consequence of the impulsive pressure. However, Housner's method is limited to the rigid tanks. It is usually unconservative to consider all tanks to be rigid due to the wall deflection. Whereas walls of rigid tanks move in unison as the ground, motion of walls of flexible tanks is different. The wall deflection configuration of these tanks do not remain the same but vary from the base to the top of the tank (Veletsos, 1974). From procedures it follows, maximum acceleration in a given direction of a point on a rigid tank is the same as maximum acceleration of the ground, whereas in cases of flexible tanks it can be smaller than, equal to or greater than the maximum ground acceleration depending on tank flexibility. All these knowledge are reflected in seismic characteristics (base shear, overturning moment, stresses etc.). Tank flexibility affects the hydrodynamics effects and these may be increased significantly. Following investigation in the past, there were proposed procedures which underlie international standards in relation to design of structure for seismic resistance (e.g. Eurocode 8).

Following subchapters of paper are focused on the comparison of procedures for estimate of distribution of hydrodynamic pressures in rigid and flexible tanks in accordance with Eurocode 8 and other approaches. An investigated liquid storage tank is illustrated in Fig. 1. It is the cylindrical tank of radius R (8 m), height H (16 m) and thickness t (0,004 m). Tank is filled to the height H_L (16 m) with liquid (water). It is assumed that the base of the tank is continuously fixed to gravel-sand foundation, which is excited by a horizontal component of ground acceleration of 1 ms^{-2} with component importance factor III in accordance with Eurocode 8.

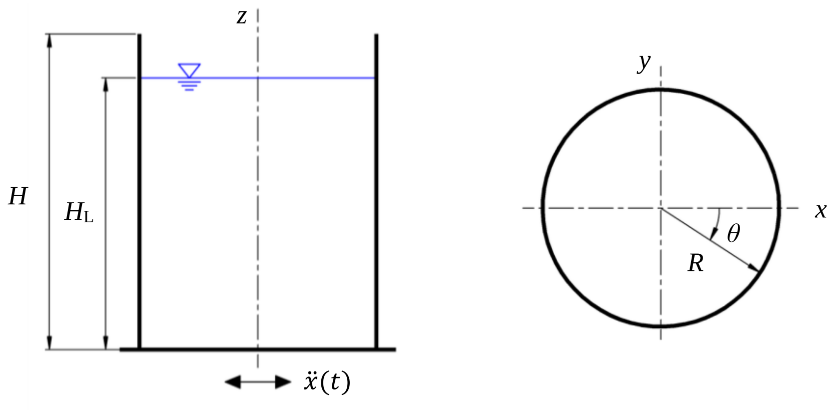

Fig. 1: Liquid storage tanks under horizontal seismic excitation

2.1. Hydrodynamic pressures in rigid tanks

During lateral base excitation, tank wall is subjected to the lateral hydrodynamic pressure. For rigid tanks in accordance with Eurocode 8, hydrodynamic pressures consists of two components analogous to Housner's theory. The spatial-temporal distribution of the rigid impulsive pressure is given by expression

$$p_{\mathrm{i}}(\xi,\varsigma,\theta,t) = C_{\mathrm{i}}(\xi,\varsigma)\rho_{\mathrm{L}}H_{\mathrm{L}}A_{\mathrm{g}}(t)\cos\theta \tag{1}$$

where C_i gives the distribution along the height of p_i

$$C_{\mathrm{i}}(\xi,\varsigma) = 2\sum_{n=0}^{\infty}\frac{(-1)^n}{I_1'(v_n/\gamma)v_n^2}\cos(v_n\varsigma)I_1\left(\frac{v_n}{\gamma}\xi\right)$$

in which

$$v_n = \frac{2n+1}{2}\pi;\ \gamma = \frac{H_{\mathrm{L}}}{R};\ \ \xi = \frac{r}{R};\ \varsigma = \frac{z}{H_{\mathrm{L}}}$$

I_1 and I_1' represent a modified Bessel function of first kind and its derivative, $A_{\mathrm{g}}(t)$ denotes the ground acceleration time-history, ξ,ς are non-dimensional geometric parameters and γ denotes slenderness parameter (liquid height H_{L} to tank radius R). The part of liquid that does not move as a rigid body with the tank experiences sloshing effect. This motion can be expressed as a linear combination of the corresponding natural modes of liquid vibration. The spatial-temporal distribution of the convective pressure component is given by expression

$$p_{\mathrm{c}}(\xi,\varsigma,\theta,t) = \rho_{\mathrm{L}}\sum_{n=1}^{\infty}\psi_n\cosh(\lambda_n\gamma\varsigma)J_1(\lambda_n\xi)\cos\theta A_{cn}(t) \tag{2}$$

where

$$\psi_n = \frac{2R}{(\lambda_n^2-1)J_1(\lambda_n)\cosh(\lambda_n\gamma)}$$

J_1 is a Bessel function of the first order and λ_n are values for which the first derivative of J_1 is zero, $A_{cn}(t)$ is the acceleration time-history of SDOF oscillator with natural frequency ω_{cn}. To withstand roof damage caused by sloshing in the tank, a sufficient freeboard is required. The sloshing wave height may be given from the following expression

$$d(r,\theta,t) = R\sum_{n=1}^{\infty}\frac{2}{\lambda_n^2-1}\frac{J_1\left(\lambda_n\frac{r}{R}\right)}{J_1(\lambda_n)}\frac{S_e(T_{cn})}{g}\cos\theta \tag{3}$$

where $S_e(T_{cn})$ is the elastic spectral acceleration at first convective frequency. In Fig. 2, distributions of maximum impulsive and convective pressure along the tank height of a described model are shown.

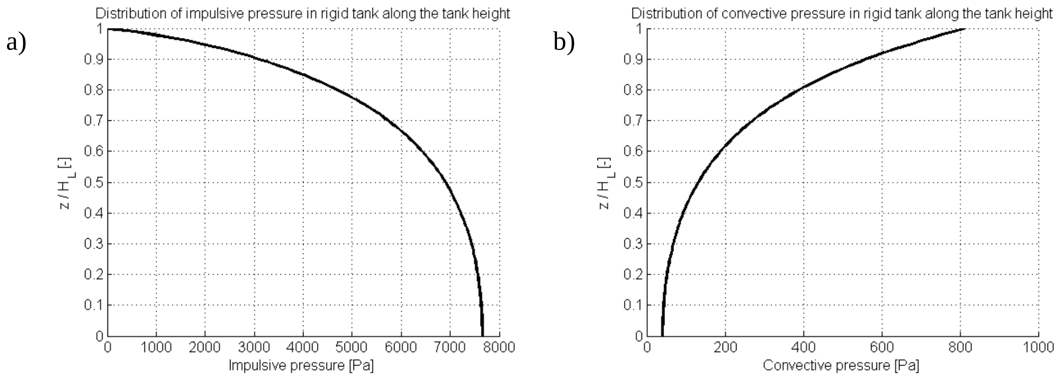

Fig. 2: Distribution of impulsive and convective pressures in the model of rigid tank

2.2. Hydrodynamic pressures in flexible tanks

According to the standard Eurocode 8, hydrodynamic pressure of flexible tanks is usually expressed as the sum of three contributions namely as rigid impulsive, flexible and convective. Flexible contribution satisfies the conditions of radial velocity of liquid along the height to be equal to the deformation velocity of the tank wall, zero velocity at the tank bottom and zero pressure at the free surface of liquid. Due to weak coupling between the second and the third component of hydrodynamic pressure, the flexible component may be determined independently. Weak coupling is a consequence of large differences between natural frequencies of sloshing liquid and tank-liquid system. The rigid impulsive and sloshing component in (1) and (2) remain unaffected. Assuming the modes of vibration $f(\varsigma)$, the flexible pressure distribution on the walls takes form

$$p_f(\varsigma,\theta,t) = \rho_L H_L \psi\cos\theta\sum_{n=0}^{\infty} d_n\cos(\nu_n\varsigma)\,A_{fn}(t) \tag{4}$$

where

$$d_n = 2\frac{\int_0^1 f(\varsigma)\cos(\nu_n\varsigma)d\varsigma}{\nu_n}\frac{I_1(\nu_n/\gamma)}{I_1'(\nu_n/\gamma)}$$

$$\psi = \frac{\int_0^1 f(\varsigma)\left[\frac{\rho_S t(\varsigma)}{\rho_L H_L}+\sum_{n=0}^{\infty} b_n'\cos(\nu_n\varsigma)\right]d\varsigma}{\int_0^1 f(\varsigma)\left[\frac{\rho_S t(\varsigma)}{\rho_L H_L}f(\varsigma)+\sum_{n=0}^{\infty} d_n\cos(\nu_n\varsigma)\right]d\varsigma}$$

$$b_n' = 2\frac{(-1)^n}{\nu_n^2}\frac{I_1(\nu_n/\gamma)}{I_1'(\nu_n/\gamma)}$$

ρ_S is the mass density of the tank, $t(\varsigma)$ is the tank thickness and $A_{fn}(t)$ is the pseudo-acceleration function corresponding to the natural frequency of the tank-liquid system. In most cases of flexible tanks, the pressure p_f provides predominant contribution to the total pressure, due to the fact, that while the rigid impulsive pressure (1) varies with ground acceleration, the flexible component varies with response acceleration $A_{fn}(t)$, which may be significantly greater than maximum ground acceleration. Procedure for flexible tanks proposed by A. S. Veletsos (Veletsos, 1974) assumes the tank-liquid system vibrates in a fixed configuration along its height with no distortion of its cross-section. For deflection, it takes three different configurations between base and free surface liquid, like half-sine wave function $\psi_A = \sin\left(\frac{\pi}{2}\frac{z}{H_L}\right)$, linear function $\psi_B = \frac{z}{H_L}$ and reversed half-sine function $\psi_C = 1\text{-}\cos\left(\frac{\pi}{2}\frac{z}{H_L}\right)$, where z represents coordinate along the tank height. Fig. 3 a) represents mode of vibration of the tank-liquid system when liquid oscillates in unison with the tank and represents unfavorable response to the system which behaves like uniform cantilever flexural beam. Pressure distribution of flexible contribution of impulsive pressure along the tank height is shown in Fig. 3 b). For estimate of pressure distribution, maximum response acceleration of the tank-liquid system having the period of impulsive mode is assumed. It compares pressure distributions according to the Eurocode 8 and Veletsos's approach. Good conformity can be seen when comparing Eurocode 8 and deflection configuration ψ_A. Using another mentioned deflection configuration in Eurocode 8 computations, similar results as those shown in Fig. 3 b) will be attained.

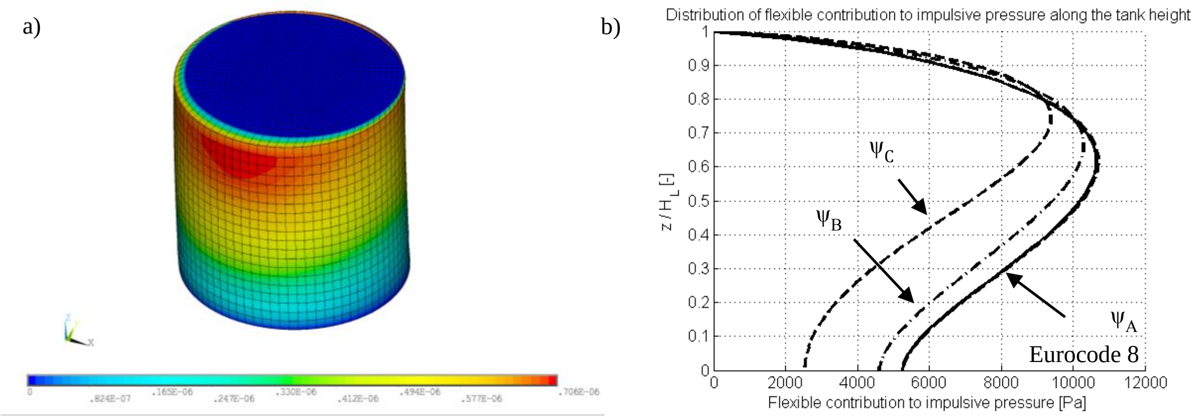

Fig. 3: Impulsive mode and distribution of flexible pressure contribution in the model of flexible tank

3. Conclusions

The aim of this paper was to estimate hydrodynamic pressures in the model of liquid storage tank and to compare acquired results when assuming rigid and flexible tank walls, respectively. It can be concluded that impulsive pressure in rigid tank increases from zero at liquid surface to a maximum at the base, whereas convective pressure reaches maximum at the liquid surface and decreases with depth. It was shown that flexibility of tank wall has impact on the impulsive liquid that experiences accelerations that are greater than peak ground acceleration. Therefore there is a different impulsive pressure distribution assuming additional flexible component which has tendency to shift a peak ordinate from the base to the top as γ increases. Convective pressure due to a weak coupling is not affected by wall flexibility. For design purposes, these effects can be evaluated by considering the tank to be rigid. To sum up, the base shear and overturning moment of flexible tanks calculated by assuming the tank as rigid can be non-conservative.

Acknowledgement

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