22nd International Conference
ENGINEERING MECHANICS 2016
Svratka, Czech Republic, 9 – 12 May 2016

EXTREME VALUES CALCULATION OF MULTI-MODAL PEAK DISTRIBUTIONS

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Abstract: *To search for the extreme value of a measured quantity from the peak analysis of a time series it is common practice in the field of offshore engineering to perform the analysis by means of a Weibull distribution. Usually the peaks distribution is quite well described by a common two or at most three parameters Weibull. However, when severe sea states are investigated or the analyzed structure presents particular geometries, the representation on the Weibull plane can be nonlinear, even adopting a three parameters distribution. In such a case the standard Weibull distributions are no more suitable to describe the peaks population, which presents a multi-modal form. To overcome this problem, the Mixed Weibull distribution was here used to describe the populations and a procedure based on genetic algorithms has been established to fit the parameters of the multi-modal function. Thereafter the extreme value predictions by the developed procedure have been compared with the one coming from standard Weibull analysis.*

Keywords: Mixed Weibull distribution, extreme values, genetic algorithm.

1. Introduction

In ship design and offshore applications, the extreme values of motions or loads acting on the ship/structure are needed, hence it is usual to perform model tests or numerical simulations. Analyzing the time traces of the quantities under investigation and considering all the peaks obtained by a zero mean extraction process, it results that the extreme values can be evaluated by fitting a Weibull distribution on the obtained population. For standard quantities the use of a simple two parameters or at most a three parameters Weibull distribution is satisfactory to describe the peaks population and then evaluate the values of the extremes. However, for particular cases, considering severe sea states or very large and complex structures, the adoption of the standard Weibull distribution can be no more satisfactory, maybe leading to an excessive over/under estimation of the extreme values needed for the design application. This problem arises when the peaks are distributed with a multi-modal trend. For this purpose, the Mixed Weibull distribution has been used to fit the peaks population of the loads on a ship structure. A procedure based on genetic algorithm has been developed to find the regression coefficients of the multi-modal distribution to perform automatically the extreme values analysis. Through this paper an overview of the newly implemented procedure is given and the results are compared with the one obtained from the standard Weibull analysis for a sample time record. Proper regression analysis has avoided a highly over dimensioned structure and, therefore, a substantial cost reduction.

2. Two-parameters Weibull distribution

The simplest case of Weibull distribution is the two parameter distribution with a probability density function (PDF) defined as follows:

$$p(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-(x/\eta)^{\beta}} \qquad (1)$$

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The cumulative density function (CDF) results in:

$$F(x) = \int_0^{\infty} p(x)dx = 1 - e^{-(x/\eta)^{\beta}} \tag{2}$$

where $\beta \in (0,+\infty)$ is the *shape parameter* and $\eta \in (0,+\infty)$ is the *scale parameter*.

The two parameters Weibull distribution has another property. By adopting a representation of the distribution in a particular Q-Q plot (Weibull plane) where x axis is represented by $\ln x$ and y axis by $\ln(-\ln(1-F(x)))$, the function can be linearized in the following way:

$$\begin{aligned} &F(x) = 1 - e^{-(x/\eta)^{\beta}} \\ &\ln(-(\ln(1-F(x)))) = \beta \ln x - \beta \ln \eta \end{aligned} \tag{3}$$

At the end the obtained regression equation is of the type $y=mx+q$. That means, by using this particular plot (Weibull plane), it is possible to fit the peak distribution of a sample data with a straight line.

3. Three parameters Weibull distribution

In some particular cases, the Weibull plot representation of the analyzed population does not properly follow a straight line, but the data follow a curve with a single convexity or concavity. Thus it is possible to reconstruct the 2 parameters case by adopting a simple change of variable. To do this it is necessary to introduce a third parameter γ called location parameter, and the PDF of the distribution assumes the following form:

$$p(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \tag{4}$$

Consequently the CDF of the distribution is similar to equation (2) just adding the location parameter in the exponential. It must be noted that the three parameters Weibull distribution is defined for $x > \gamma$ with no limitation in sign for the location parameter.

4. Mixed Weibull distribution

For particularly complicated systems, the sample data on the Weibull plane could be not only far away of a straight line fit, but could also present more than one convexity. In such a case the linear regression is no more sufficient to correctly describe the population even with the three parameters extension. This is the typical case of multi-modal responses, i.e. sample data could contain more than one population. To give a reliable description of the population, use can be made of the so-called Mixed Weibull distribution. This distribution is a combination of two or more standard Weibull distributions (with 2 or 3 parameters). Then the PDF and CDF assume the following forms:

$$p(x) = \sum_{i=1}^{N} p_i \frac{\beta_i}{\eta_i}\left(\frac{x-\gamma_i}{\eta_i}\right)^{\beta_i-1} e^{-\left(\frac{x-\gamma_i}{\eta_i}\right)^{\beta_i}} \tag{5}$$

$$F(x) = 1 - \sum_{i=1}^{N} p_i e^{-\left(\frac{x-\gamma_i}{\eta_i}\right)^{\beta_i}} \tag{6}$$

where N is the number of subpopulations and p_i is the percentile of the subpopulation in the total population ($\sum p_i = 1$). The other parameters are the same of the previously mentioned Weibull distributions. There are no limitations on the number of subpopulations, just the number of parameters increases. To give a rough example, to fit a 2 subpopulations Mixed Weibull using 3 parameters Weibull distributions, 7 parameters should be estimated, in case of 3 subpopulations the parameters became 12 and so on.

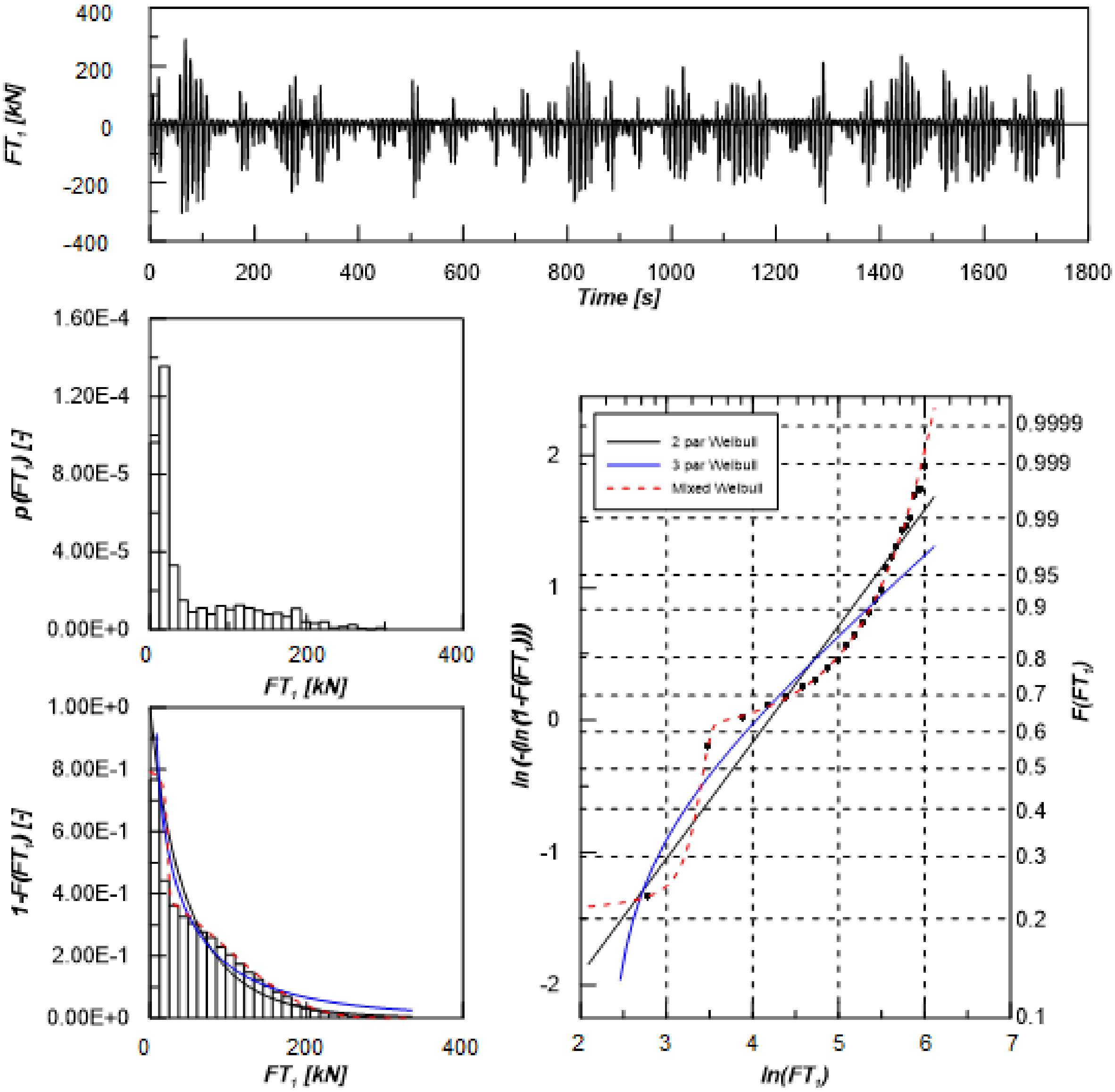

Fig. 1 Weibull data analysis

5. Parameters determination

To estimate the parameters of a Weibull distribution different methods can be adopted like least square fitting, method of moments, maximum likelihood and so on. All these methods have been implemented for the 2 parameters and thereafter extended for the 3 parameters distribution, but the scope of this work is not centered on the differences between the estimation methods. Due to the high number of unknowns, once the Mixed Weibull distribution should be analyzed, the above mentioned methods cannot be directly used, and it is a common practice to perform a 'manual fitting' of data. To build up an automatic procedure for the parameter estimation a general evolutionary algorithm has been here used, performing an extended version of the last square fitting method. The same estimation method has been used in this work also for the standard 2 and 3 parameters Weibull distribution as well.

6. Extreme values calculation

After the parameters determination process, the extreme values of the population can be predicted from the fitted distribution. Usually the values of interest are the events with the following probability p, i.e., 3%, 1% and 0.1%. To know these values, one should use the quantiles (inverse cumulative distribution) of the selected Weibull or Mixed Weibull distribution. For the different distributions the quantiles have the following forms, respectively:

$$Q(p,\eta,\beta)=\eta\left(-\ln(1-p)\right)^{\frac{1}{\beta}} \quad (7)$$

$$Q(p,\gamma,\eta,\beta)=\gamma+\eta\left(-\ln(1-p)\right)^{\frac{1}{\beta}} \quad (8)$$

$$Q(p,p_i,\gamma_i,\eta_i,\beta_i)=\sum_{i=1}^{N}p_i\left(\gamma_i+\eta_i\left(-\ln(1-p)\right)^{\frac{1}{\beta_i}}\right) \quad (9)$$

which are representative of the 2, 3 parameters and Mixed Weibull distribution respectively.

7. Numerical example

To highlight the differences between the predictions of extreme values, different distributions have been used to analyze the time series of a generic structural load which was recorded during a seakeeping test of an offshore vessel in severe sea state conditions. Data shown in Figure 1 present clearly a bi-modal shape, and for this reason a 2 population Mixed Weibull distribution has been selected for fitting. The results of the time series analysis are presented in Table 1.

Tab. 1 Weibull analysis results

Regression Type	Force T1 [kN]		
	p 3%	p 1%	p 0.1%
2 parameters	281	376	580
3 parameters	296	465	921
Mixed	264	294	342

The standard 2 and 3 parameters regressions are estimating much higher extreme values than the Mixed Weibull distribution. Analyzing the Weibull plot in Figure 1 it can be noted that the standard regressions (2 and 3 parameters Weibull) are not fitting well the data points for large force values. This fact is generating a huge overestimate of the extremes.

8. Conclusions

The Mixed Weibull distribution is reproducing well the behavior of a multi modal population and therefore is suitable to perform an accurate prediction of the extreme values of load responses. The parameters determination procedure based on a genetic algorithm allows to perform quickly and automatically the parameters computation to be used for fitting the population distribution. Further study must be carried out to determine the regression technique sensitivity with the sample interval.

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