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THE USE OF SPECIAL ALGORITHM TO CONTROL THE FLIGHT OF ANTI-AIRCRAFT MISSILE

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Abstract: *The paper analyzes a possibility of using the special algorithm to control the flight of a homing air target missile. The method is based on the signum function and utilizes the phase trajectories of errors resulting from the adopted homing method (proportional navigation). The results obtained via Matlab package are presented in graphical form.*

Keywords: dynamics, phase trajectories, homing, missile, control.

1. Introduction

A constant rise in the number of combat operations involving air defence systems requires that the missile flight control methods be refined. Guidance to a moving target dictates that the missile should reach the target as fast as possible and meet the system performance requirements at the same time. The selection of the guidance method is an important factor as it has a profound effect on the likelihood of intercepting the target. Deviation of a missile motion from the ideal constraints is an error, which is used to formulate the guidance control rule. The special control algorithm proposed here is based on phase trajectories of control errors. The method utilizes the signum function for switching the control forces at appropriate points of the phase plane (Hsu & Meyer, 1968; Osiecki & Stefański, 2008). The flight of the anti-aircraft missile will be controlled with the use of aerodynamic forces. The homing of the flying object on the aerial target was executed with the proportional navigation method. The device providing the identification and tracing the target, and also precise and reliable lead the missile to it along the proper trajectory, could be the optical scanning-tracing head (Dziopa et al., 2015; Gapiński et al., 2014; Krzysztofik & Koruba, 2012), but in the paper this concept was not considered.

2. Missile flight dynamic and kinematic equations

Figure 1 summarizes the coordinate systems in which the missile flight equations were introduced. Figure 2 shows the forces that act on the moving missile. The following symbols are used (Koruba & Nocoń, 2015): α, β – attack angle and sideslip angle [rad]; ψ, ϑ, φ – pitch angle, yaw angle and roll angle of the missile [rad]; γ, χ – flight-path angle in vertical plane and horizontal plane – pitch angle and yaw angle of missile velocity vector [rad], $S\xi\eta\zeta$ – coordinate system for the missile; $Sxyz$ – velocity coordinate system; $Sx_g y_g z_g$ – coordinate system with the missile as an origin, parallel with the starting system $\vec{V}_m$ – missile velocity vector; $\vec{P}$ - thrust; $\vec{A}$ – resultant of aerodynamic forces; $\vec{G}$ – gravitational force; $\vec{Q}$ – control force; $\vec{M}$ – sum of moments of forces acting on the missile; ε, σ – pitch and yaw angles of the line-of-sight (LOS) [rad].

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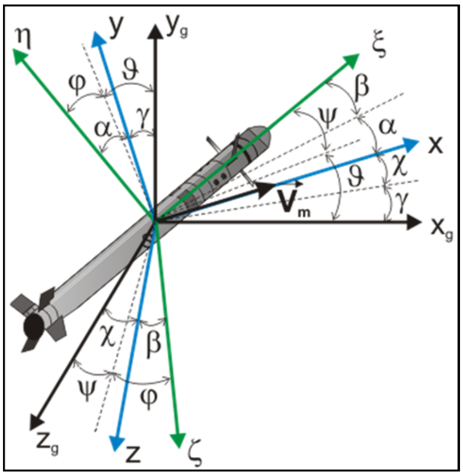

Fig. 1:Coordinate system with angles of rotation

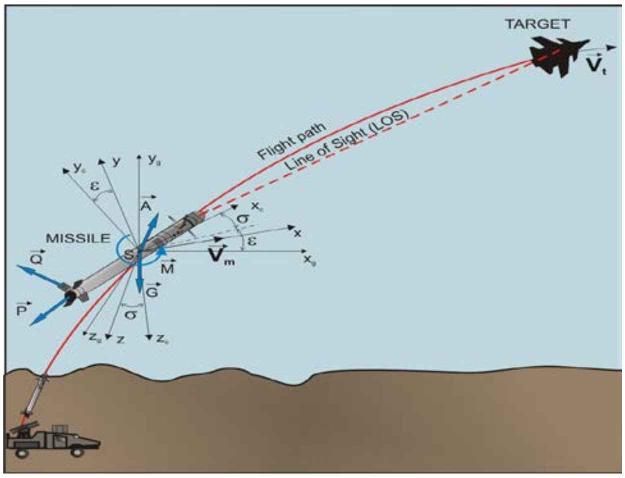

Fig. 2: Forces acting on a missile

For the purposes of these calculations, the rocket is assumed to be a rigid body which does not rotate around its longitudinal axis and the motion of the missile and the target is restricted to a common vertical plane. The flat motion (vertical plane) is considered in order to simplify the analysis of the process of homing the missile on the aerial target. In practice, if the target does not make rapid manoeuvres, the homing process departs only slightly from the vertical plane (also due to autopilot stabilization system), thereby the motion in the horizontal plane can be neglected. Thus, $\psi = 0$, $\varphi = 0$, $\beta = 0$; $\chi = 0$, $\sigma = 0$. With these assumptions applied, the missile dynamic equations are as follows (Koruba & Osiecki, 1999):

$$\dot{V}_m = \frac{P}{m}\cos\alpha - g\sin\gamma - \lambda_x V_m^2, \quad \dot{\gamma} = \frac{1}{V_m}\left(\frac{P\sin\alpha + Q_y}{m} - g\cos\gamma\right) + \lambda_y V_m \alpha \tag{1a}$$

$$\ddot{\vartheta} = -D_1 \frac{V_m^2}{l}\alpha - D_2 V_m \dot{\alpha} - D_3 V_m \dot{\vartheta} + \frac{Q_y e}{J_k}, \lambda_x = \frac{c_x S_x \rho}{2m}, \lambda_y = \frac{c_y S_y \rho}{2m}, D_{1,2,3} = \frac{C_i l}{J_k} \tag{1b}$$

where: l – length of the missile body [m]; ρ – air density [kg/m^3]; S_x – cross-sectional area of the missile; S_y – lifting area [m^2]; m – mass of the missile [kg]; J_k – moments of inertia of the missile in relation to its transverse axis [kgm^2]; γ – actual angle of missile flight-path [rad]; Q_y – missile flight control force [N]; e – distance between control force and aerodynamic pressure centre [m]; g – acceleration of gravity [m/s^2]; $\lambda_x, \lambda_y, D_{1,2,3}$ – relative aerodynamic coefficients of aerodynamic forces and moments [1/m] (Koruba & Osiecki, 1999); c_x, c_y – coefficients of aerodynamic forces; C_i – coefficients of moments of aerodynamic forces; V_m – flight velocity of the missile [m/s].

Kinematic relationships between the missile and target have the form (Koruba et al., 2010):

$$\dot{r} = V_t\cos(\varepsilon - \gamma_t) - V_m\cos(\varepsilon - \gamma_d), \qquad -r\dot{\varepsilon} = V_t\sin(\varepsilon - \gamma_t) - V_m\sin(\varepsilon - \gamma_d) \tag{2}$$

where: V_t – target velocity [m/s]; γ_t – angle of the target flight-path [rad]; γ_d – desired angle of the missile flight-path [rad]; r – distance between the missile and the target [m].

To satisfy the missile total kinematic overload requirement, the following equations are used

$$n_x = -\left(\frac{\dot{V}_m}{g} + \sin\gamma\right), \quad n_y = -\left(\dot{V}_m\dot{\gamma} + \cos\gamma\right), \quad n = \sqrt{n_x^2 + n_y^2} \tag{3}$$

3. The algorithm of homing and determination of the control force

To realize the homing guidance on the target, the following proportional navigation algorithm was used (Yanushevsky, 2011; Koruba & Osiecki, 1999):

$$\dot{\gamma}_d = a\dot{\varepsilon} \tag{4}$$

where: a – constant dimensionless coefficient of proportional navigation

To determine the control force for the rocket flight, the method based on phase trajectories of control errors was applied. This method consists in bringing the error to zero through switching the control force at appropriate points on the phase plane, as reported in detail by (Hsu & Meyer, 1968; Osiecki & Stefański, 2008). The control force was calculated from

$$Q_y = -u\left(p_1 \operatorname{sgn} e_1 + p_2 \operatorname{sgn} e_2 + p_3 \operatorname{sgn} e_3\right) \tag{5}$$

where: u, $p_{1,2,3}$ – control coefficients: $e_1 = \gamma - \gamma_d$, $e_2 = \dot{\gamma} - \dot{\gamma}_d$, $e_3 = \int_{t_0}^{t_k} (\gamma - \gamma_d) dt$

4. Digital simulation results

Numerical simulations were conducted for a hypothetical missile attacking an aerial target from the front quarter. The following numerical values were used: starting missile position: x_{m0} = 0 [m], y_{m0} = 0 [m]; starting target position: x_{t0} = 5500 [m], y_{t0} = 3500 [m]; angle of a missile launch: γ_0 = 0.7667 [rad]; starting angle of pitch of a target velocity vector: γ_{t0} = 0.01 [rad]; starting missile velocity: V_{m0} = 20 [m/s]; target velocity: V_c = const = 300 [m/s]; l = 1.6 [m]; m = 10.8 [kg]; J_k = 2.304 [kgm^2]; λ_x = 0.000171 [1/m]; λ_y = 0.0051 [1/m]; D_1 = 0.081 [1/m], D_2 = 0.0821 [1/m], D_3 = 0.00041 [1/m]; a = 4; t – time. The flight path of the target was described as follows: $\gamma_t(t) = \pi - 0.003t$.

Graphical representation of the results is shown in Figs 3-8.

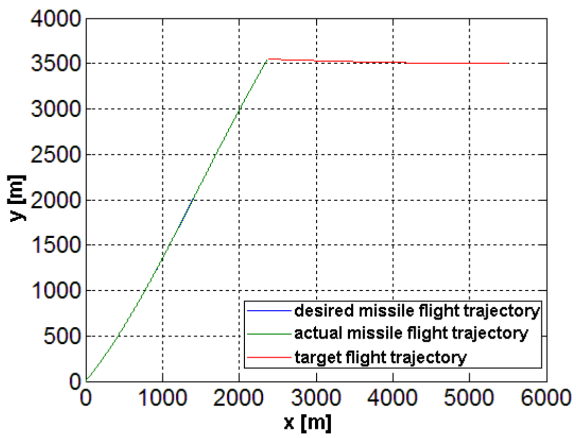

Fig. 3: The missile and target flight paths

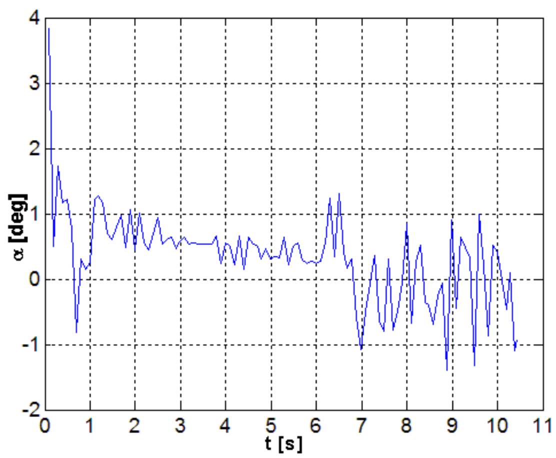

Fig. 4: The angle of attack realized during the missile flight

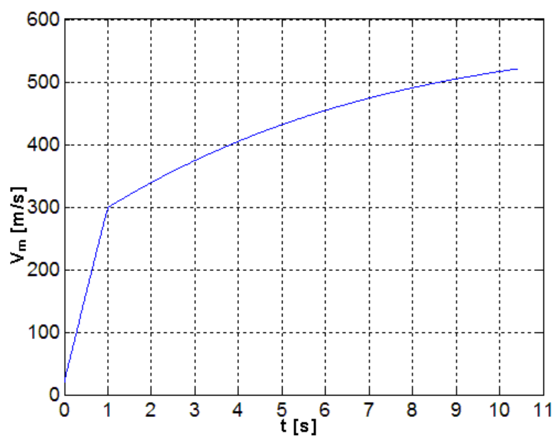

Fig. 5: Flight velocity of the missile

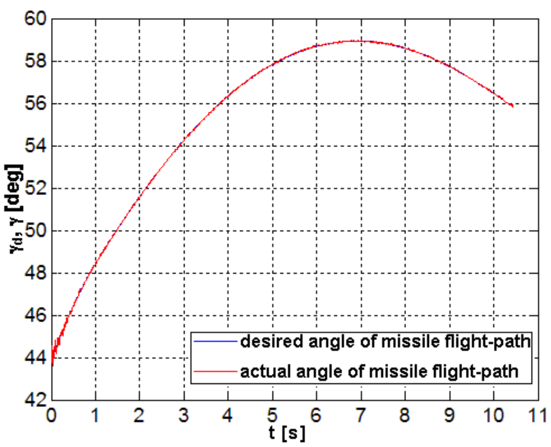

Fig. 6: Desired and actual angles of missile flight path

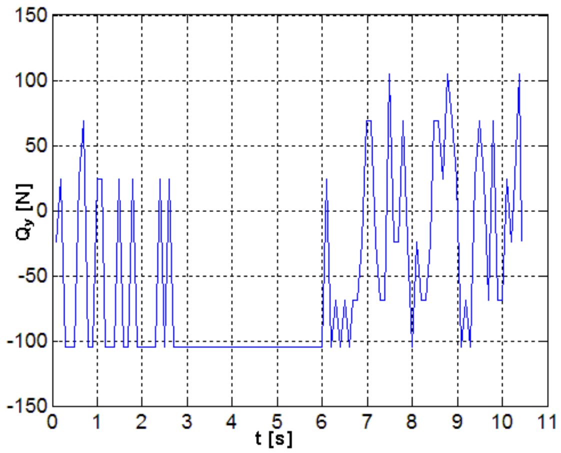

Fig. 7: Values of control forces required for homing the missile on the target

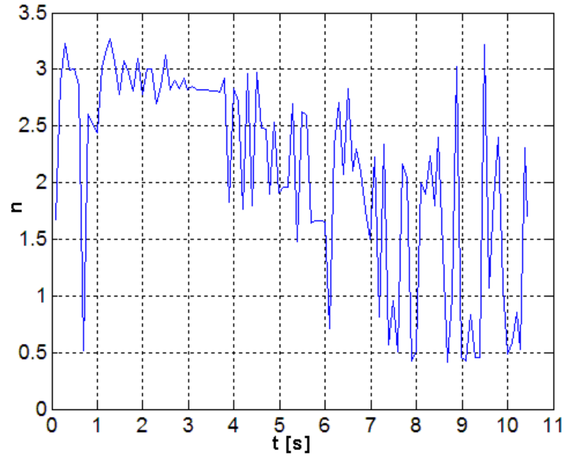

Fig. 8: Kinematic overloads acting on the missile during the flight

The results of the simulations are as follows: starting distance between the missile and the target: r_0 = 6519.2 [m]; starting values of pith angle of line-of-sight: $\varepsilon_0 = 0.5667$ [rad] (32.47 [deg]); final target position: x_{tk} = 2367 [m], y_{tk} = 3549 [m]; accuracy of missile hit: r_k = 5.04 [m]; time from a missile drop to hit a target: t_k = 10.44 [s].

5. Conclusions

The results of the simulations point to the following conclusions: the method proposed for the control of the air defence missile flight (with optimally selected controller gain rates (Gapiński et al., 2014)) allowed hitting the moving target; the hitting accuracy of 5.04 [m] is sufficient because most of the anti-aircraft missile are fitted with proximity fuses; minor oscillations of the rocket's actual flight path and angle result from the use of signum function - reduction of these oscillations should be considered; the control force reached low values, possible to attain in real conditions; overload acting on the flying rocket also reached acceptable values; the application of the proposed method, based on the phase trajectories of control errors, to the guidance of a missile on a target is possible and viable.

In the future, an attempt will have to be made to use the proposed method of missile control in a spatial approach.

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