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STABILITY ANALYSIS OF THE LATERAL OSCILLATIONS OF A FLEXIBLY SUPPORTED VISCOELASTIC JEFFCOTT ROTOR

P. Ferfecki*, J. Zapoměl, J. Kozánek***, V. Dekýš******

Abstract: *Material damping has a significant influence on vibrations of flexible rotors and can induce their self excited oscillations. The squeeze film dampers inserted in the rotor supports are frequently used to suppress occurrence of these undesirable operating conditions. Modelling the shaft by means of a Kelvin-Voigt material can arrive at the overestimation of the effect of internal damping on the rotor movement. This was a motivation to develop a procedure based on utilization of the Zener material (standard solid theoretical material) to represent the shaft of a Jeffcott rotor supported by squeeze film dampers. The development and testing of this procedure, the experimental determination of the appropriate material constants, and learning more on the influence of material damping on the stability and vibration attenuation of flexible rotors are the principal contributions of the presented article.*

Keywords: Jeffcott rotor, Zener material, squeeze film dampers, vibration amplitude and stability.

1. Introduction

Material damping has a significant effect on vibrations of flexible rotors. When the speed of their rotation exceeds a limit value, it becomes a source of self excited oscillations. The often used technological solution making it possible to suppress these undesirable operating conditions consists in adding the squeeze film dampers to the rotor supports.

A great attention must be paid to representing the shaft material in mathematical models of rotor systems. The experience shows that the most frequently used Kelvin-Voigt theoretical material characterized for a linear dependence of the specific damping capacity on the vibration frequency can overestimate the influence of internal damping on the rotor motion. The nonlinear dependence of the damping capacity is provided by the Zener material which is considered to be a standard solid material model.

In this paper there is presented a procedure developed for investigation of the stability and the vibration attenuation of a Jeffcott rotor excited by the disc unbalance. The shaft material is represented by a Zener one. The rotor is supported at both its ends by classical squeeze film dampers. The steady state response is determined by application of a trigonometric collocation method. The stability vibration is evaluated by means of the Floquet theorem. The pressure distribution in the gap of the squeeze film dampers is determined by solving the Reynolds equation. The development and testing of the new computational procedure, the experimental determination of the material constants, and learning more on the influence of internal damping on the vibration of flexible rotors are the principle contributions of this article.

* Ing. Petr Ferfecki, PhD.: IT4Innovations National Supercomputing Center, VŠB - Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba; CZ,
Department of Applied Mechanics, VSB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba; CZ
petr.ferfecki@vsb.cz

** prof. Ing. Jaroslav Zapoměl, DrSc.: Institute of Thermomechanics, Dolejškova 5; 182 00 Prague; CZ,
Department of Applied Mechanics, VSB-Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba; CZ
jaroslav.zapomel@vsb.cz

*** Ing. Jan Kozánek, CSc.: Institute of Thermomechanics, Dolejškova 5; 182 00, Prague; CZ
kozanek@it.cas.cz

**** doc. Ing. Vladimír Dekýš, CSc.: Department of Applied Mechanics, Zilina University, Univerzitná 8215/1, 010 26 Žilina; SK
vladimir.dekys@fstroj.uniza.sk

2. The investigated rotor system

The investigated rotor system consists of a shaft and of one disc (Fig 1). At both its ends it is supported by squeeze film dampers lubricated with classical oil. The rotor turns at constant angular speed, is loaded by its weight and excited by the disc unbalance. Both squirrel springs are prestressed to be eliminated their deflection caused by the rotor weight. The disc can be considered as thin and absolutely rigid and the whole system as symmetric relative to the disc middle plane.

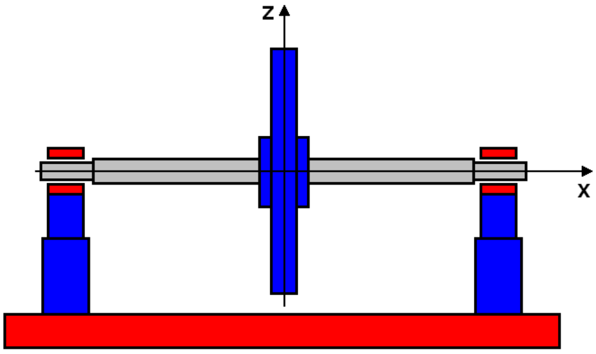


Fig. 1: The investigated rotor system

The shaft of the rotor is made of steel. The carried out measurements provided the value of the modulus of elasticity (isothermal) 200 GPa and dependence of the specific damping capacity on the vibration frequency. Based on analysis discussed in Zapoměl et al., 2015 behaviour of the investigated material is close to the viscoelastic Zener one (the standard solid material model). The corresponding specific damping capacity ψ is expressed by the relation (Zapoměl, 1998)

$$\psi = 2\pi\omega T_R \frac{\frac{E_S}{E_T} - 1}{1 + T_R^2 \omega^2 \frac{E_S}{E_T}}, \tag{1}$$

where T_R is the time relaxation constant, E_T, E_S are the isothermal and adiabatic Young's moduli, and ω is the angular frequency of the vibrations. The comparison of the measured data with those obtained by their approximation is depicted in Fig. 2. The values of the moduli ratio and the time relaxation constant were determined by application of the nonlinear least square method ($E_S/E_T = 1.008$, $T_R =$ 185 ms).

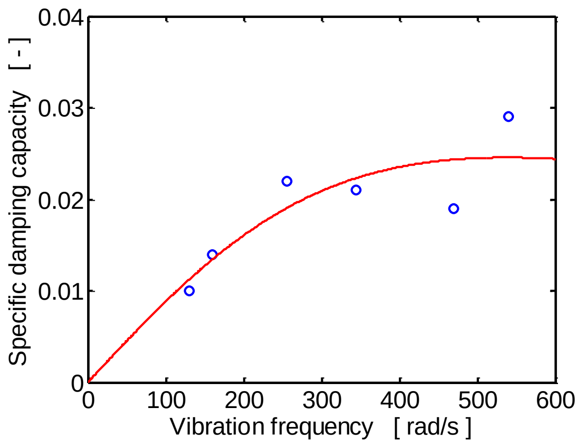


Fig. 2: The dependence of the specific damping capacity on the vibration frequency

3. The motion equations

In the computational model the rotor is considered to be a Jeffcott one. The shaft is massless represented by a viscoelastic Zener material and the squeeze film dampers are implemented by springs with no itertia effects and by force couplings (Zapoměl, 2007, Szeri, 1980).

Because of the system symmetry, the rotor vibration is governed by four nonlinear motion equations that have the form with respect to the fixed (inertial) frame of reference

$$m\ddot{y}_R + b_P \dot{y}_R = -F_{my} + m e_T \omega^2 \cos \omega t \,, \tag{2}$$

$$m\ddot{z}_R + b_P \dot{z}_R = -F_{mz} + m e_T \omega^2 \sin \omega t - mg \,, \tag{3}$$

$$0 = F_{my} - 2k_D y_D + 2F_{hy} + 2F_{psy} \,, \tag{4}$$

$$0 = F_{mz} - 2k_D z_D + 2F_{hz} + 2F_{psz} \,. \tag{5}$$

m is the disc mass, b_P is the coefficient of external disc damping (damping caused by the environment), F_{my}, F_{mz} are the y and z components of the viscoelastic force by which the flexibly deformed shaft acts on the disc, e_T is the eccentricity of the disc centre of mass, k_D is the stiffness of one squirrel spring, F_{hy}, F_{hz} are the y and z components of the hydraulic force acting on the rotor journal (Zapoměl, 2007, Szeri, 1980), F_{psy}, F_{psz} are the y and z components of the prestress force, g is the gravity acceleration, y_R, z_R are the y and z displacements of the disc centre, y_D, z_D are the y and z displacements of the journal centre, t is the time, and $(\dot{\ }),(\ddot{\ })$ denote the first and second derivative with respect to time.

The components $F_{m\eta}$, $F_{m\zeta}$ of the viscoelastic force by which the shaft acts on the disc can be expressed in the coordinate system rotating together with the rotor

$$F_{m\eta} = k_T(\eta_R - \eta_D) + \int_0^t b_M \mu (\dot{\eta}_R - \dot{\eta}_D) e^{-\mu(t-\vartheta)} d\vartheta \,, \tag{6}$$

$$F_{m\zeta} = k_T(\zeta_R - \zeta_D) + \int_0^t b_M \mu (\dot{\zeta}_R - \dot{\zeta}_D) e^{-\mu(t-\vartheta)} d\vartheta \,. \tag{7}$$

k_T, k_S are the shaft isothermal and adiabatic stiffnesses, η_R, ζ_R, η_D, ζ_D are the η and ζ displacements of the disc and of the journal centre, respectively, μ is the inverse value of the time relaxation constant, ϑ is the time parameter, and b_M is the coefficient of the shaft material damping (Zapoměl, 1998)

$$b_M = T_R(k_S - k_T) \,. \tag{8}$$

After performing a series of manipulations one obtains a matrix equation referred to the fixed frame of reference that governs the Jeffcott rotor vibrations

$$\begin{bmatrix} \mathbf{M} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{g}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_\mathrm{P} + \mathbf{B}_\mathrm{M} & -\frac{1}{\mu}\mathbf{B}_\mathrm{M} \\ -\mu\mathbf{I} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{g}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} - \omega\mathbf{K}_\mathrm{C} & \frac{\omega}{\mu}\mathbf{K}_\mathrm{C} \\ \mu\mathbf{\Omega} & \mu\mathbf{I} - \mathbf{\Omega} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{o} \end{bmatrix} . \tag{9}$$

$\mathbf{M}$, $\mathbf{K}$, $\mathbf{B}_\mathrm{P}$, $\mathbf{B}_\mathrm{M}$ are the mass, stiffness, external and material damping matrices, $\mathbf{K}_\mathrm{C}$ is the circulation matrix, $\mathbf{I}$ is the unity matrix, $\mathbf{\Omega}$ is the matrix of the rotor angular velocity, $\mathbf{O}$ is the zero matrix, $\mathbf{x}$ is the vector of physical displacements, $\mathbf{f}$ is the vector of applied forces, $\mathbf{g}$ is the vector of auxiliary (internal) coordinates, and $\mathbf{o}$ is the zero vector.

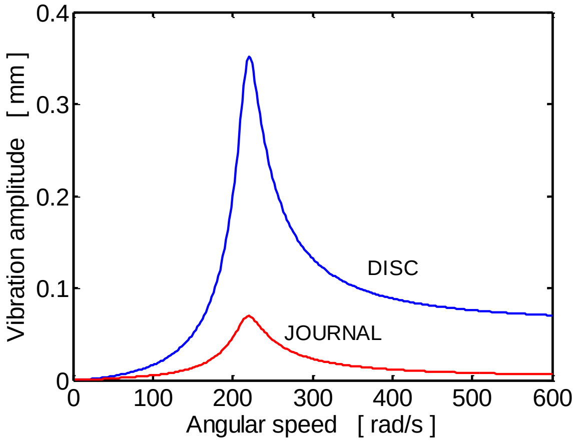
As the forces induced by the disc unbalance are of a harmonic time history, the trigonometric collocation method was used to determine the steady state component of the rotor vibration. Its stability was evaluated by application of the Floquet theorem.

4. Results of the simulations

The technological and operating parameters of the investigated rotor system are: the disc mass 130 kg, the shaft stiffness (isothermal) 7 MN/m, the stiffness of one squirrel spring 3 MN/m, the coefficient of external damping of the disc 4 kg/s, the disc unbalance 7.8 kg.mm, the mean diameter and the length of the damper 120 mm, 40 mm, the width of the damper clearance 200 μm, the dynamic viscosity of the lubricant 0.08 Pa.s, the oil input pressure 300 kPa, the specified angular speed range 0 ÷ 600 rad/s.

The (isothermal) stiffness of the shaft 7 MN/m, the measured values of the time relaxation constant 185 ms and the adiabatic/isothermal stiffness ratio 1.008 give the magnitude of the material damping coefficient of 101.8 kg/s.

The frequency responses of the disc and of the rotor journal referred to the vibration in the horizontal direction are drawn in Fig. 3. The maximum displacements correspond to the rotor critical speed the value of which is approximately 220 rad/s. Amplitude of the disc vibrations approaches with rising angular speed of the rotor rotation to the disc centre of mass eccentricity.



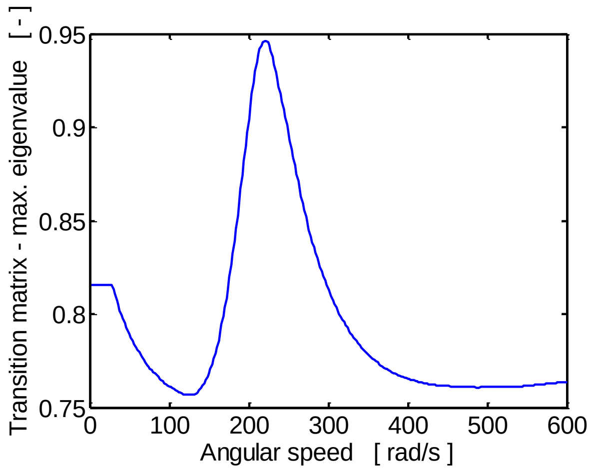


Fig. 3: The rotor frequency response

Fig. 4: The vibration stability evaluation

The dependence of the maximum eigenvalue of the transition matrix set up over the span of time of one period on the rotor angular speed is depicted in Fig. 4. The results show that all magnitudes of the maximum eigenvalues are less than 1 which implies the rotor vibration is stable in the specified range of the angular speed.

5. Conclusions

The lateral vibration of rotors is significantly influenced by the shaft elastic properties and material damping. It implies a great attention must be paid to the choice of the theoretical material to implement the shaft in the computational model. The approach for investigation of the oscillation amplitude and evaluation of the vibration stability of a Jeffcott rotor supported by squeeze film dampers is presented in this paper. The rotor shaft is made of a Zener material which is characterized for a nonlinear dependence of the specific damping capacity on the vibration frequency. A trigonometric collocation method was applied to determine the steady state response of the rotor. The stability of the forced vibration was evaluated by means of the Floquet theorem. The development of a new computational procedure, the experimental determination of the appropriate material constants, and learning more on the effect of the internal damping and the squeeze film dampers on the vibration attenuation of flexible rotors are the principle contributions of this article.

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