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MODELING A PNEUMATIC CYLINDER WITH FRICTION

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Abstract: *This article shows method of obtaining a mathematical model of a pneumatic actuator including changes in pressure, temperature, simplified flow of heat to the environment and friction. Position and velocity graph of real pneumatic drive and its mathematical model implemented in Matlab with LuGre and Stribeck friction was presented.*

Keywords: pneumatics, mathematical model, friction.

1. Introduction

One of the basic elements used in the industry is a pneumatic drive, in most cases is used for tasks that require dynamic pivot of action as well as a huge amount of work cycles. Pneumatic actuators are also used for positioning elements which do not require high precision or because of the operating environment is not possible to use a different drive. There are also application of pneumatic drives for manipulators with several degrees of freedom(Laski et al., 2014). In these cases, the pneumatic control requires the use of more stringent controls than other types of actuators.

2. Mathematical model of the pneumatic cylinder

The mathematical model of the pneumatic cylinder is based on the energy balance resulting from the assumptions: pneumatic actuator converts the energy of compressed air entering to the cylinder Q_i to work L, at the moment the compressed air in the chamber has a stored energy U and a stream of energy transmitted through the housing to the environment as a heat Q_T.

$$Q_i = Q_o + dL + dU + Q_T \tag{1}$$

Eq. 1: Energy equation; Q_i *– flowing energy,* Q_o *– outflow energy,* dL *– the work done by the actuator,* U *– changes in internal energy,* Q_T *– energy fed back into the environment.*

The energy entering and leaving in the form of compressed air can be written as the product of the air mass and its enthalpy: $Q = h \cdot dm$, assuming that the air supplied to the actuator has a constant pressure, enthalpy of the air express as a specific heat at a constant pressure with a known temperature $h = c_p \cdot T$. We assume that the air entering temperature is constant and equal to an ambient temperature and the outflow temperature is variable and corresponds to the actual temperature of air in the cylinder. The work done by the drive will be described as the product of pressure and volume change of the chamber: $L = p \cdot dV$ and internal energy as the product of specific energy and the current air mass: $dU = du \cdot m + dm \cdot u$, where $u = c_v \cdot T$ (c_v - specific heat at constant volume). The energy dissipated by the cylinder actuator housing will be described in a simplified form as a product of the coefficient of heat transmission α, heat casting surface $S = \pi D x$ (D – diameter of the piston, x – the current position of the piston) and the temperature difference between the actuator and the environment:

$$Q_T = \alpha\pi(Dx + 0,25D^2)\Delta \mathrm{T}\mathrm{dt}. \tag{2}$$

Using these dependencies we will write:

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$$c_pT_idm_i - c_pT_odm_o - pdV - c_vdTm - dmc_vT - \alpha\pi(Dx+0,25D^2)\Delta \mathrm{T}_o\mathrm{dt} = 0 \qquad (3)$$

For further considerations, we assume that the working medium is ideal gas which satisfies the Clapeyron equation in form: $mRT = pV$, heat capacity ratio $k = c_p / \mathrm{c}_v$ and the gas constant $R = c_p - c_v$.

Changes in internal energy, which depends on weight changes we describe in the form of: $dm = \frac{1}{RT}\left(dpV + dVp\right)$, and then, after simple transformations balance equation takes the form:

$$kRT_idm_i - kRTdm_o - kpdV - \frac{pVdT}{T} - dpV - (k-1)\alpha\pi(Dx+0,25D^2)\Delta \mathrm{Tdt} - = 0 \qquad (4)$$

And after pulling out pressure we obtain the differential equation describing the pressure changes:

$$\dot{p} = \frac{1}{V}\left(kRT_i\dot{m}_i - kRT\dot{m}_o - kp\dot{V} - \frac{pV}{T}\dot{T} - (k-1)\alpha\pi\left(Dx+0,25D^2\right)\Delta \mathrm{T}\right) \qquad (5)$$

The equation of the temperature in the chamber of the actuator is obtained from the differential equation of the ideal gas: $R \cdot dm \cdot dT = dV \cdot dp$, which after calculation and re-substitution Clapeyron equation instead of the air mass and $V = Ax$ where A is a surface of piston:

$$\frac{dT}{dt} = T\left(\frac{1}{p}\frac{dp}{dt} + \frac{1}{x}\frac{dx}{dt} - \frac{\dot{m}RT}{pV}\right) \qquad (6)$$

On the piston acts pressure difference between the chambers of the actuator, and the friction force F_T , from Newton's law we determine the equation:

$$\frac{d^2x}{dt^2} = \frac{1}{m}\left[A\left(p_1 - p_2\right) - F_T\right]. \qquad (7)$$

These equations describe the pressure and temperature in a single chamber into a full description of the operation of the actuator also requires a model of the second chamber. The realization of such a system in Matlab is shown in Figure 1. More complex model which takes into account the flow of heat between the chambers of the actuator, can be obtained from work relating to heat flow through the seal. (Blasiak et al., 2014)

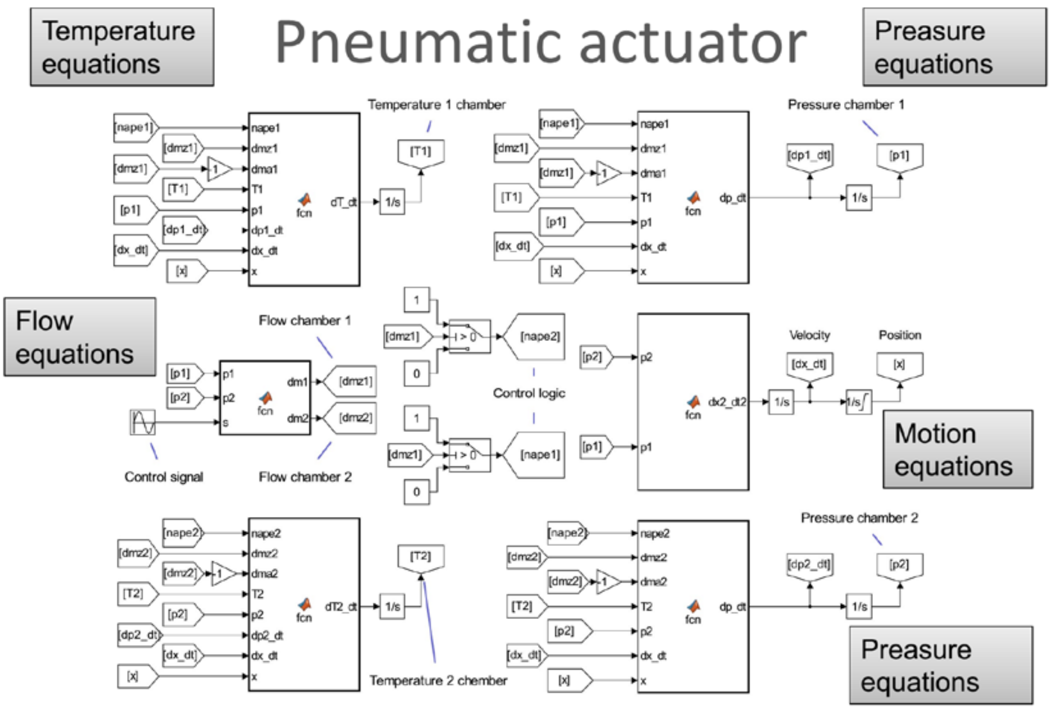

Fig. 1: Pneumatic actuator model implemented in Matlab - Simulink.

3. Model of friction in pneumatic drives

The forces of friction in pneumatic drives are especially important at low speeds as well as during starting when the transition from static friction to kinetic occurs. Mathematical models describing the physical phenomena occurring in the sealing elements have been described, for example. in the works: (Blasiak, 2015),(Blasiak & Zahorulko, 2016). There are several basic models of friction, which allows the actuator to create a model suitable for simulation, selection of controls and control systems. One such model is the model of friction Lund – Grenoble (Olsson, 1996). This model besides typical representation of drive operation with friction, also allows simulations of well-known and very negative phenomenon of stick-slip, which makes precise positioning of pneumatic drives for low speeds practically impossible. Mathematical notation of friction equations is as follows*:*

$$\frac{dz}{dt} = v - \sigma_0 \cdot \frac{|v|}{g(v)} \cdot z \qquad g(v) = F_c + (F_s - F) \cdot e^{-(v/v_s)^2} \qquad F_f = \sigma_0 z + \sigma_1 \dot{z} + F_v v \qquad (8)$$

where z – average deviation bristles, v – velocity of the piston, σ_0 – stiffness of bristles, σ_1 – damping,

F_v – viscous friction, F_c – Coulomb force, F_s – stiction force.

The realization of friction LuGre is shown in Fig. 2.

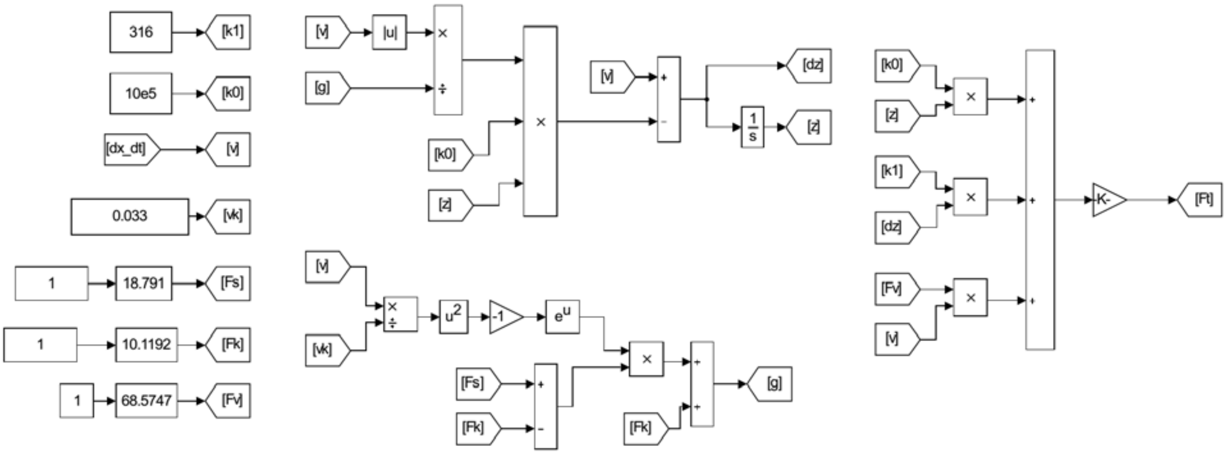

Fig. 2: Implementation of LuGre friction model in Matlab – Simulink .

Figure 3 shows the speed and movements of a real cylinder, on this graph is possible to observe the phenomenon of stick-slip. The figure also shows simulation results of pneumatic piston model with LuGree friction.

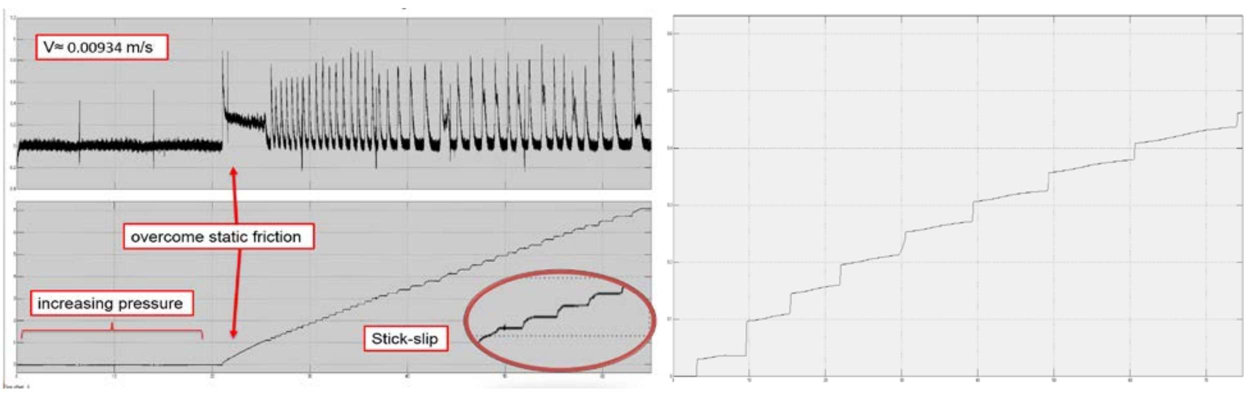

Fig. 3: Graph of speed and the actual position of the pneumatic cylinder and the simulation model of the actuator with LuGre friction (right side).

Another much simpler approach to modeling the pneumatic actuator with friction is the combination of Coulombs friction with Stribeck curve. Studies have shown that this model pretty well simulates the behavior of a pneumatic drive and with comparison to the model of LuGre is much easier to implement in control systems.

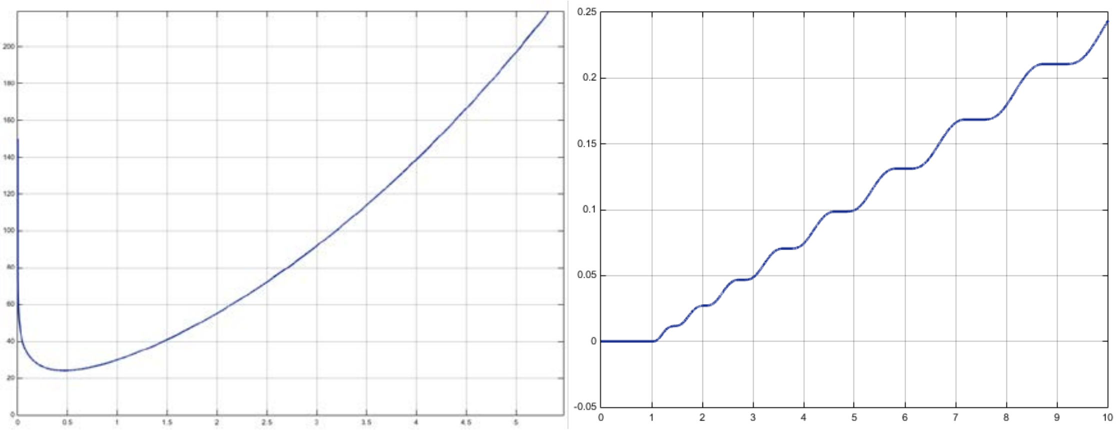

Fig. 4: The dependence of friction force on the relative speed for the combined Stribeck and Coulomb friction. On the right side of the graph movement for a very small constant flow.

4. Conclusions

The selection of controllers and control systems for compressed air devices requiring precision work often requires the development of accurate mathematical model and carry out large amounts of simulations. An important element in creating a model is to include relevant operating parameters, for fast and less precise movements can be taken a simplified model without the influence of temperature or friction, while for relatively slow processes essential element may be predictive controller for which is required knowledge of the friction influence to control the pneumatic drive (Schindele & Aschemann, 2009). Another approach to control pneumatic actuators is the use of special fuzzy logic controllers that allow you to skip the creation of a mathematical model but require more expertise knowledge (Takosoglu et al., 2012).

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