# Determining the Distribution of Forces in Reinforcing Bars in Slab-Column Connections of Reinforced Concrete Structures

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**Abstract:** The behaviour of reinforced concrete slab-column structures under the impact of accidental loading is very significant due to safety reasons. The failure of the support zone by punching and lack of proper structural integrity reinforcement can lead to a progressive collapse. However, the instructions on how to prevent such situations are not very detailed. According to the guidelines of standard EC2, the structural integrity reinforcement should be continuous throughout the length and consist of at least two bars above the column in each perpendicular direction. The article presents a theoretical model of calculation that permits a more detailed analysis of internal forces in reinforcing bars located directly above the column. Adopting a solution in the form of exact equations makes it possible to take into account the influence of a non-linear change of the bar rigidity and considerable deflections. The calculation model is based on the results of the experimental investigations.

### Introduction

The designing of slab-column connections in reinforced concrete structures requires the application of proper reinforcement in the support zone. In case of the failure of this zone by punching the upper reinforcement is detached off the slab, causing it to fall down. One of the options to counteract this kind of situation is introducing an additional bottom reinforcement in the form of bars passing directly above the column. The primary aim is to counteract the possibility of a progressive collapse in case of a local damage of the structure. Current standards do not directly state any guidelines in this issue, pointing only at the necessity of using this type of reinforcement.

In order to investigate the problem in more detail, a research was conducted including a detailed analysis of phenomena occurring in connections of this type. The research included full-scale models constituting a part of the reinforced concrete structure [1]. The numerical analysis of this range was presented in [2].

## **Description of the problem**

During the research [1] an actuator was used to impose a load of the force P in the middle of the span of the bar L. After that the induced vertical deflections f of the bar were measured. The deflections depend not only on the dimensions of the bar, the material properties, the way of mounting the ends of the bar and the value of the load. As a result of deflection, the centre line of the bar becomes elongated. For this reason the value of deflection f depends also on the value of force N, causing the tensioning of the bar. In case of considerable deflections the value of force N is significant and cannot be neglected in the calculations.

The conducted experimental investigations primarily revealed the necessity of modifying axial forces and bending stiffness of the bars. This is a problem of considerable importance. Such detailed analysis of the changes in internal forces in the bar in the course of its deformation is possible only in theoretical methods and numerical calculations.

#### **Description of the method**

The observation of the behaviour of the reinforcing bar during the experimental investigations was the basis adopting a substitute static scheme (Fig.1). The differential equation of the deformed axis of the bar subjected to tension with force N and the transverse load P has the form Eq. 1. The solution of Eq. 1 taking into account the boundary conditions is presented in Eq. 2. Assuming the load of value P corresponds to a deflection of value f, the dependence Eq. 2 is the basis for obtaining Eq. 3. The value a obtained from this Eq. 3 allows to determine the axial force  $N=EI a^2$  in a deflected bar.

$$P \qquad EI \frac{d^4 w}{dx^4} = N \frac{d^{42} w}{dx^2}$$
(1)

$$w(x) = \frac{P}{2EI a^2} \left( x - \frac{1}{a} \frac{\sinh(ax)}{\cosh(aL/2)} \right), \quad a = \sqrt{\frac{P}{EI}} \quad (2)$$

$$\frac{4EIf}{PL}a^{3} - a + \frac{2}{L}tanh(aL/2) = 0$$
(3)

Fig. 1: Static scheme

#### Calculation

♥ <sub>w(x)</sub>

The calculations were conducted on the basis of the introduced dependences (2) and (3) for a bar with the diameter 16 mm and length of L=2.4 m. The dependence between the applied load P and the deflection f corresponding to it (Fig.2) was adopted in accordance with the results of the experimental investigations [1]. Additionally, each stage of the calculations incorporates a non-linear change in stiffness *EI* in accordance with the material tests [2] (Fig.3). The graph illustrating the changes in the value of the force N at each level of the load is presented in Fig.4.



#### Summary

The developed method of theoretical determination of distribution of internal forces in a reinforcing bar in the course of its loading leads to the results presented in the form of equations. In the case of considerable deflections an additional tensile force N associated with the elongation of the bar is present. This force should be taken into account in the calculations. The presented solution permits to directly calculate the sought force N. Moreover, the solution takes into account a non-linear change in the bending stiffness of the bar which has a significant impact on the value of the force N, especially in the case of minor spans of the bar. The conducted calculations suggest a correlation between the imposed load P and the tensile force N.

#### References

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