# Frequency Analysis of Torsion Vibration of Hard Rubbers under Finite Deformations

Petr Šulc<sup>a\*</sup>, Luděk Pešek<sup>b</sup>, Vítězslav Bula<sup>c</sup>, Jan Cibulka<sup>d</sup>, Jan Košina<sup>e</sup> Institute of Thermomechanics AS CR, v. v. i., Dolejškova 1402/5, 182 00 Praha 8, Czech Republic

<sup>a</sup>sulc@it.cas.cz, <sup>b</sup>pesek@it.cas.cz, <sup>c</sup>bula@it.cas.cz, <sup>d</sup>cibulka@it.cas.cz, <sup>e</sup>kosina@it.cas.cz

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**Abstract:** The paper deals with experimental analysis of hard rubber dynamic tests under finite torsion deformations. The experimental test rig based on controlled servo-motor was proposed and built for torsion straining of the rubbers under defined amplitudes and frequencies and temperature of angle vibration. The paper brings results of the tests for different excitation frequencies under different levels of torsion deformations. The mathematical rheological model of rubber is also presented for phenomenological description and parametric identification of hard rubbers.

#### Introduction

Contrary to the most standard construction materials, the time variable material behavior of hard rubbers occurs during their dynamic loading due to deformation level, creep, temperature and aging [1]. Therefore, we have been dealing with the thermo-mechanical behavior of hard rubbers in our

dynamic laboratory. Recently we investigated behavior of rubbers under small deformations and latest we start to deal with their behavior under larger finite deformation (up to 10%) [2].

## Experimental set-up for torsional tests

For finite deformations, an experimental set-up for torsional dynamic tests of rubber samples of circular cross-section has been designed and assembled. Furthermore methodology and programs for evaluation rubber material constants from the measured force and response signals of the test rig depending on frequency, amplitude and temperature were developed, too. Excitation is directly controlled by the control unit of the engine. The control unit provides one analogue output signal, either position or speed. Analogue signals were digitized A/D converter NI PCI-6035 and processed by DAQ toolbox and numerical programs in the environment Matlab 2007 b.



Fig. 1: Test rig for torsional tests of hard rubbers

#### The rheological model of hard rubber

Mathematical rheological model was proposed for phenomenological description of rubber behaviour under torsional straining. Parameters of the model are to be identified from the comparison of the analytical and experimental hysteresis curves. The type of rheological model was designed to fit the experimental results of hysteresis of the rubber at finite (up to 10%) straining.

Total inner force of the rheological model as a sum of forces acting in its parallel branches is expressed

$$F_{cel} = k_0 \cdot \varphi + F_{mt\alpha} + F_{t0} \cdot \frac{2}{\pi} \cdot \operatorname{arctg}(\alpha \cdot \dot{\varphi})$$
(1)

where the first member equation (1) represents the stiffness force,  $F_{mt\alpha}$  is force of differential equations obtained from force equilibrium of the Maxwell member

$$\dot{F}_{mt\alpha}(t) \quad \frac{F_{mt\alpha}(t)}{\tau_{\alpha}} = k_{\alpha} \cdot \dot{\phi}(t) \tag{2}$$

and the third member of equation (1) is a friction force of the rheological model. After substituting the inner force in the equation of torsion motion we get

$$I_0\ddot{\varphi} + k_0 \cdot \varphi + F_{mt\alpha} + F_{t0} \cdot \frac{2}{\pi} \cdot arctg(\alpha \cdot \dot{\varphi}) = M \cdot \sin(\omega \cdot t) \qquad , \qquad (3)$$

where an external excitation is realized by harmonic torque moment M with excitation frequency  $\omega$ .

The equation of motion (3) and equation (2) were transferred into the system of differential equations of the first order for numerical solution by Runge-Kutta method.

## Sumary

This paper brings first results of evaluation of hard rubber material constants (Fig.2) from the torsion The material characteristics tests. show higher dependence on the level of straining than on the frequency. Since the loss factor is ratio of the dissipation and deformation energy at a given strain it leads to the conclusion that the dissipation energy of hard rubbers under larger strains is related to the deformation energy by multiplication constant similarly as at the dissipation by small strains.



Fig. 2: Characteristics of the shear modulus (dashed) and loss factor (solid lines) on strains at different excitation frequencies (5, 10, 15, 20)Hz

# References

- [1] David 1. G. Jones, Viscoelastic Vibration Damping. John Wiley and Sons Ltd, Chichester, 1988.
- [2] P. Šulc, L. Pešek, V. Bula, J. Cibulka, J. Košina, Experimentální analýza torzních kmitů tvrdých pryží při velkých deformacích, Colloquim Dymamesi (2014) 73-80.