## Void Fraction Based Two Phase Flow Model of Natural Draft Wet-Cooling Tower

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Abstract: The article deals with an improvement of recently developed model of natural draft wetcooling tower flow, heat and mass transfer [1]. The aim of present work is to make model more correct from the point of view of thermodynamics of two-phase flow. Void fraction of gas phase is included in governing equations. Homogeneous equilibrium model, where the two phases are well mixed and have the same velocity, is used. The main advantage of present approach is correct modelling of supersaturated moist air flow especially in the case where supersaturation occurs due to temperature decrease.

## **Mathematical Model**

Void fraction of gas phase is a fraction of gas volume in two phase mixture over the total volume of the mixture. Overall density of two phase mixture can be expressed by using void fraction of gas phase as [2]

$$\rho = \alpha \rho_g + (1 - \alpha) \rho_l,\tag{1}$$

where  $\rho_l$  is the density of water liquid and density of gas phase is a sum of the dry air density  $\rho_a$  and the density of water vapour  $\rho_v$ 

$$\rho_g = \rho_a + \rho_v. \tag{2}$$

If density is introduced in this way it is possible to calculate dry air mass fraction  $w_a$ , water vapour mass fraction  $w_v$  as and water liquid mass fraction  $w_l$  as

$$w_a = \frac{\alpha \rho_a}{\rho}, \ w_v = \frac{\alpha \rho_v}{\rho}, \ w_l = \frac{(1-\alpha)\rho_l}{\rho}.$$
(3)

It is possible to show that

$$w_a + w_v + w_l = 1. (4)$$

The ratio of gas mass fraction to liquid mass fraction is

$$\frac{\alpha\rho_g}{(1-\alpha)\rho_l} = \frac{\alpha(\rho_a + \rho_v)}{(1-\alpha)\rho_l} = \frac{w_a + w_v}{1 - w_a - w_v},\tag{5}$$

It is very interesting that it is possible to formulate the system of governing equations where there are only overall continuity equation, overall momentum balance, dry air continuity equation and overall equation of energy. The system of governing equations has to be closed by using the definition of internal energy and by using the equation of state. The system of governing equations can be written for the case of quasi one-dimensional flow as

$$\frac{\partial(\mathbf{W}A)}{\partial t} + \frac{\partial(\mathbf{F}A)}{\partial z} = \mathbf{Q},\tag{6}$$

where vector of conservative variables W, vector of fluxes F and vector of sources Q are

$$\mathbf{W} = \begin{bmatrix} \rho \\ \rho v \\ \rho w_a \\ \rho e \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} \rho v \\ \rho v^2 + p \\ \rho w_a v \\ (\rho e + p) v \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} A(z)\sigma_v(z) \\ p\frac{\mathrm{d}A}{\mathrm{d}z} - A(z)(\rho g + \zeta\rho\frac{v^2}{2}) \\ 0 \\ A(z)(\sigma_q(z) + \rho g v) \end{bmatrix}.$$
(7)

The density of overall energy e is

$$e = u + \frac{v^2}{2},\tag{8}$$

where internal energy u is defined as

$$u = c_{v_s}(p,T)t + w_v(p,T)\left(l_{TR} - \frac{p_{TR}}{\rho_{TR}}\right),\tag{9}$$

where  $l_{TR}$  is heat of vaporisation in the triple point,  $p_{TR}$  is the triple point pressure and  $\rho_{TR}$  is the density in triple point. Temperature in degrees of Celsius is t and temperature in Kelvins in T. Mixture specific heat capacity is

$$c_{v_s}(p,T) = w_a c_{v_a} + w_v(p,T) c_{v_v} t + [1 - w_a - w_v(p,T)] c_{v_w},$$
(10)

where  $c_{v_a}$ ,  $c_{v_v}$  and  $c_{v_w}$  are constant volume specific heat capacities of dry air, water vapour and water liquid respectively. Equation of state can be written as

$$p = r_s(p,T)\rho_g(p,T)T(p,T),$$
(11)

where  $r_s$  is gas constant of the mixture of dry air and water vapour and temperature in Kelvins can be expressed as

$$T(p,T) = \frac{u - w_v(p,T) \left( l_{TR} - \frac{p_{TR}}{\rho_{TR}} \right)}{c_{v_s}(p,T)} + 273.15.$$
(12)

The key part of this model of two phase flow is solving of temperature and pressure based on the Eq. 9 and Eq. 11. The two equations for pressure and temperature are non-linear mainly because water vapour mass fraction is the function of saturation pressure at given temperature

$$w_v(p,T) = 0.622w_a \frac{p_v(T)}{p - p_v(T)}.$$
(13)

The saturation pressure at given temperature can be calculated e.g. by using IAPWS formulation.

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## References

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